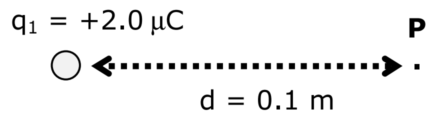
Electric Fields:

1. Consider a fixed point charge of +2.0 μC (q_1).



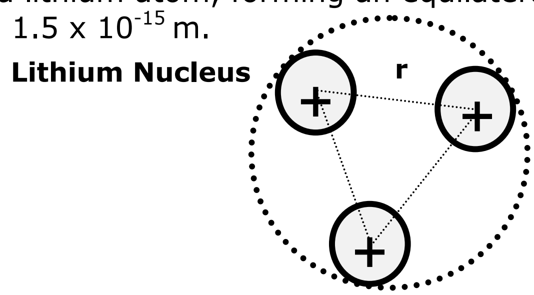
a. What is the magnitude and direction of the electric field at a point P, a distance of 0.1 m?

Ans. $\vec{E} = \left(\frac{1}{4\pi\varepsilon_o}\right)\frac{q_1}{d^2}\hat{i} = \left(1.8\text{x}10^6 \frac{N}{C}\right)\hat{i}$

b. A 2nd charge (q_2 = -2.0 μC) is placed at point P. What is the magnitude and direction of the electric force exerted on q_3?

Ans. $\vec{F}_E = \left(\frac{1}{4\pi\varepsilon_o}\right)\frac{q_1 q_2}{d^2}\hat{i} = -q_2\vec{E} = -(3.6\text{ N})\hat{i}$ {an attractive force}

2. Three protons are present in the nucleus of a lithium atom, forming an equilateral triangle. The distance, r, between each pair of protons is 1.5 x 10^{-15} m.



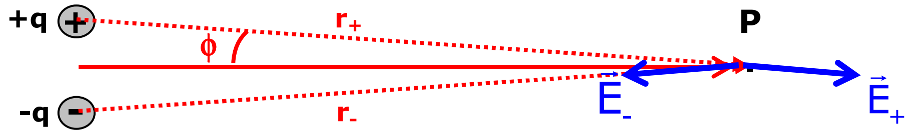
a. What is the magnitude of the electric field vector due to the lithium nucleus at point P, a distance of $2.5\text{x}10^{-10}$m from the center of the nucleus?

Ans. Since d>>r: $\left|\vec{E}\right| \simeq \left(\frac{1}{4\pi\varepsilon_o}\right)\frac{3q_p}{d^2} = 6.9\text{x}10^{10}\frac{N}{C}$ {radial outward from nucleus}

b. What is the magnitude and direction of the total electric force exerted on an electron positioned at point P.

Ans. $\left|\vec{F}_E\right| = q_e\left|\vec{E}\right| = e\left|\vec{E}\right| = 1.1\text{x}10^{-8}$ N {inward towatd nucleus}

3. Consider an electric dipole, q = 0.1 μC, separated by a distance (d) of $5\text{x}10^{-6}$ m.



a. Sketch the electric field lines for the electric field vector due to the dipole.

b What is the magnitude and direction of the electric field vector due to the dipole at point P, a distance (r) of 0.5 m to the <u>right</u> of the dipole?

$$\vec{E} = \vec{E}_+ + \vec{E}_- = \left(\frac{q}{4\pi\varepsilon_o r_+^2}\right)\left(\cos\phi\ \hat{i} - \sin\phi\hat{j}\right) + \left(\frac{q}{4\pi\varepsilon_o r_-^2}\right)\left(-\cos\phi\ \hat{i} - \sin\phi\hat{j}\right)$$

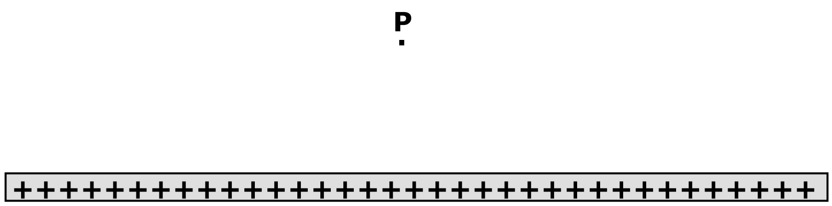
Ans.

$$\vec{E} \cong \left(\frac{-2q}{4\pi\varepsilon_o r^2}\right)\sin\phi\ \hat{j} = \left(\frac{-q}{2\pi\varepsilon_o r^2}\right)\left(\frac{\left(\frac{d}{2}\right)}{\sqrt{r^2+\left(\frac{d}{2}\right)^2}}\right)\hat{j} = \left(\frac{-p}{4\pi\varepsilon_o r^3}\right)\hat{j} = -3.6\times10^{-2}\ \tfrac{N}{C}\ \hat{j}$$

c. What is the magnitude and direction of the electric field due to the dipole at a distance of 0.5 m directly above the dipole?

Ans. $\vec{E} \simeq \left(\frac{1}{2\pi\varepsilon_o}\right)\frac{p}{r^3}\ \hat{j} = 7.2\text{x}10^{-2}\ \tfrac{N}{C}\ \hat{j}$

4. Consider a uniform line of charge with a total charge of 5 pC and a total length of 0.5 m.



a. What is the magnitude of the linear charge density?

Ans. $\lambda = \frac{5\ \text{pC}}{0.5\ \text{m}} = 1\text{x}10^{-11}\ \tfrac{C}{m}$

b. Sketch the field lines for the line of charge in the diagram above.

c. Derive an equation for the electric field vector at point P, located 0.3 m above the line and centered between the ends.

Ans. Since all but the vertical component of E cancels out:

$$d\vec{E}_y = d\left(\vec{E}\cos\phi\right) = \left(\frac{1}{4\pi\varepsilon_o}\right)\frac{dq}{r^2}\cos\phi\ \hat{j} = \left(\frac{1}{4\pi\varepsilon_o}\right)\left(\frac{\lambda dx}{x^2+y^2}\right)\left(\frac{y}{\left(x^2+y^2\right)^{\frac{1}{2}}}\right)\hat{j}$$

Solving for E:

$$\vec{E} = \int d\vec{E} = \left(\frac{y\lambda}{4\pi\varepsilon_o}\right)\int_0^L \left(\frac{dx}{\left(x^2 + y^2\right)^{\frac{3}{2}}}\right)\hat{j}$$

$$\vec{E} = \frac{\lambda L}{4\pi\varepsilon_o y\left(L^2 + y^2\right)^{\frac{1}{2}}}\hat{j}$$ {in general, radially outward from line}

d. What is the magnitude of the electric field vector at point P?

Ans. $\left|\vec{E}\right| = 0.44\frac{N}{C}$ {directed away from the line of charge}

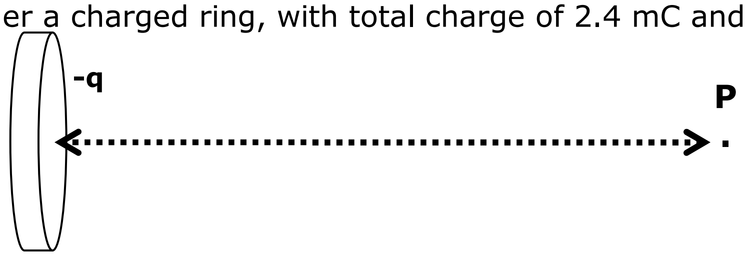
e. Determine the electric force vector exerted on an electron placed at point P.

Ans. $\vec{F}_E = q_e\vec{E} = e\vec{E} = -7.0\text{x}10^{-20}\ N\ \hat{j}$ {inward toward the line}

f. What is the acceleration of the electron located at point P?

Ans. $\vec{a} = \frac{\vec{F}_E}{m_e} = -7.7\text{x}10^{10}\,\frac{m}{s^2}\ \hat{j}$ {inward toward the line}

5. Consider a charged ring, with total charge of 2.4 mC and radius of 0.3 m.



a. What is the magnitude of the surface charge density?

Ans. $\sigma = \frac{2.4\text{x}10^{-3}C}{\pi(0.3\ m)^2} = 8.5\text{x}10^{-3}\,\frac{C}{m^2}$

b. Derive an equation for the electric field vector at point P, located 1.5 m to the right of the center of the disc.

Ans. Since all but the radial component of E cancels out, establish dEcosϕ in terms of dq:

$$d\left(\vec{E}\cos\phi\right)=\left(\frac{1}{4\pi\varepsilon_o}\right)\frac{dq}{d^2}\cos\phi\hat{i}=\left(\frac{1}{4\pi\varepsilon_o}\right)\left(\frac{\sigma rdr}{x^2+r^2}\right)\left(\frac{x}{\left(x^2+r^2\right)^{\frac{1}{2}}}\right)\hat{i}=\left(\frac{1}{4\pi\varepsilon_o}\right)\left(\frac{\sigma xrdr}{\left(x^2+r^2\right)^{\frac{3}{2}}}\right)\hat{i}$$

Solving for E:

$$\vec{E} = \int d\vec{E} = \left(\frac{\sigma x}{4\pi\varepsilon_o}\right)\int_0^R\left(\frac{rdr}{\left(x^2+r^2\right)^{\frac{3}{2}}}\right)\hat{i}$$

$$\vec{E} = \left(\frac{\sigma x}{4\pi\varepsilon_o}\right)\frac{-1}{\left(x^2+R^2\right)^{\frac{1}{2}}}\Bigg|_0^R\hat{i} = \left(\frac{\sigma x}{4\pi\varepsilon_o}\right)\left(\frac{1}{x}-\frac{1}{\left(x^2+R^2\right)^{\frac{1}{2}}}\right)\hat{i}$$ {away from ring}

d. What is the magnitude and direction of the electric field vector at point P?

Ans.

$$\vec{E} = \frac{(8.5\text{x}10^{-3}\,\tfrac{C}{m^2})(1.5m)}{4\pi\varepsilon_o}\left(\frac{1}{(1.5m)} - \frac{1}{\left((1.5m)^2 + (0.3m)^2\right)^{\frac{1}{2}}}\right)\hat{i}$$

$$\vec{E} = 1.5\text{x}10^{6}\,\tfrac{N}{C}\ \hat{i}\ \{\text{away from ring}\}$$

5. The plates of a parallel plate capacitor have a uniform charge of +1.0 C and -1.0 C, respectively. Each plate is a square with sides of length of 0.5 m.

a. What is the magnitude of the surface charge density on each plate?

Ans. $\sigma = \dfrac{1.0C}{(0.5\ m)^2} = 4.0\ \tfrac{C}{m^2}$

b. What is the magnitude of the electric field vector inside the plates of the capacitor?

Ans. Treat each plate as a charged flat disc of infinite R, i.e. x << R, then the E field due to each plate respectively is then:

$$\vec{E}_{plate} = \left(\frac{\sigma}{2\varepsilon_o}\right)\hat{i}\ \{\text{both fields will point toward the right}\}$$

The total electric field between the plates is therefore:

$$\vec{E}_{between\ plates} = \vec{E}_{plate\ 1} + \vec{E}_{plate\ 2} = \left(\frac{\sigma}{\varepsilon_o}\right)\hat{i} = 4.5\text{x}10^{11}\,\tfrac{N}{C}\,\hat{i}$$

c. What is the electric force exerted on an electron placed inside the plates of the capacitor?

Ans. $\vec{F}_E = q_e\vec{E} = e\vec{E} = -7.2\text{x}10^{-8}\ N\ \hat{j}$ {to the left}

d. What is the acceleration of the electron while it is in between the capacitor plates?

Ans. $\vec{a} = \dfrac{\vec{F}_E}{m_e} = -7.9\text{x}10^{22}\,\tfrac{m}{s^2}\ \hat{j}$ {to the left}

e. For a 0.2 m plate separation, calculate the final speed of an electron (initially at rest), that is placed at the negative charged plate and just reaches the positive plate.

Ans.

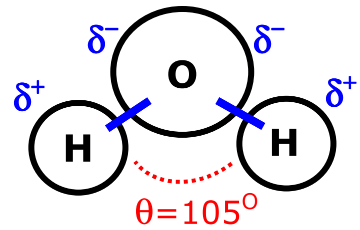
$$\vec{v} = \sqrt{v^2} = -\sqrt{2ad}\ \hat{i} = -\sqrt{2\left(-7.9\text{x}10^{22}\,\tfrac{m}{s^2}\right)(-0.2m)}\ \hat{i}$$

$$\vec{v} = -1.8\text{x}10^{11}\ \hat{i}\ \{\text{a bit unrealistic eh??}\}$$

Dipole Moment of Water:

6. Water is a polar molecule that consists of 2 H atoms attached to a central oxygen atom. The H atoms are oriented at an angle of approximately 105°. Each of the H-O bonds has a dipole moment (p_{OH}) associated with it and together the 2 dipole moments have a resulting net dipole moment (p_{H_2O}) of $6.2\text{x}10^{-30}$ C·m. The effective separation distance between the respective positive (δ^+) and negative (δ^-) charges in each O-H bond is $3.9\text{x}10^{-12}$ m.

   a. What is the magnitude of the effective dipole moment of the individual O-H bond for water?



Ans.

$$\vec{p}_{H_2O} = \vec{p}_{OH_1} + \vec{p}_{OH_2} = 2qd\cos(52.5^o)\hat{j} =$$

$$\left|\vec{p}_{OH}\right| = \left|\vec{p}_{OH_1}\right| = \left|\vec{p}_{OH_2}\right| = \left|\frac{\vec{p}_{H_2O}}{2\cos(52.5^o)}\right| = 5.1\text{x}10^{-10}\text{C}\cdot\text{m}$$

   b. Determine the magnitude of the charge (δ) for the individual O-H dipole.

Ans. $\left|\vec{p}_{OH}\right| = qd = \delta d = 5.1\text{x}10^{-30}\text{C}\cdot\text{m} \Rightarrow \delta = 1.3\text{x}10^{-18}\text{C}$

   c. Calculate the torque on a water molecule due to the electric field of a charged sphere ($q_{sphere} = -1.5\text{x}10^{-6}$ C) at a separation distance of 0.05 m, where the direction of the electric field is oriented at an angle of 30° to p_{H_2O}.

Ans.

$$\left|\vec{E}\right| = \left(\frac{1}{4\pi\varepsilon_o}\right)\frac{q_{sphere}}{r^2} = \left(5.4\text{x}10^6\ \tfrac{\text{N}}{\text{C}}\right)$$

$$\left|\tau_E\right| = \left|\vec{p}\times\vec{E}\right| = pE\sin30^o = 1.7\text{x}10^{-23}\ \text{N}\cdot\text{m}$$

   d. What is the potential energy of the water molecule in this electric field?

Ans. $U = \vec{p}\cdot\vec{E} = pE\cos30^o = 2.9\text{x}10^{-23}\ \text{J}$