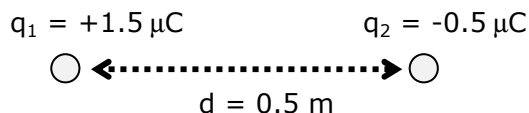


Coulomb's Law & Electric Fields:

1. Consider a fixed point charge of $+1.5 \mu\text{C}$ (q_1). A 2nd charge of $-0.5 \mu\text{C}$ (q_2) is placed at a distance of 0.5 m from q_1 .



a. What is the magnitude and direction of the electric force exerted on q_2 due to q_1 ?

Ans.

$$\vec{F}_E = -k \frac{q_1 q_2}{d^2} \hat{i} = -\left(8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right) \frac{(1.5 \times 10^{-6} \text{C})(0.5 \times 10^{-6} \text{C})}{(0.5 \text{m})^2} \hat{i} = -0.0270 \text{N} \hat{i}$$

b. A 3rd charge of $-1.0 \mu\text{C}$ (q_3) is placed 0.25 m directly to the right of q_2 . What is the magnitude and direction of the electric force exerted on q_3 ?

Ans.

$$\vec{F}_E = \left(-k \frac{q_1 q_3}{d_{1 \rightarrow 3}^2} + k \frac{q_2 q_3}{d_{2 \rightarrow 3}^2} \right) \hat{i} =$$

$$\vec{F}_E = \left(8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \right) (1.0 \times 10^{-6} \text{C}) \left(-\frac{(1.5 \times 10^{-6} \text{C})}{(0.75 \text{m})^2} + \frac{(0.5 \times 10^{-6} \text{C})}{(0.25 \text{m})^2} \right) \hat{i} = 0.0479 \text{N} \hat{i}$$

c. If q_3 is exactly halfway between q_1 and q_2 , what is the magnitude and direction of the electric force exerted on q_3 ?

Ans.

$$\vec{F}_E = \left(-k \frac{q_1 q_3}{d_{1 \rightarrow 3}^2} - k \frac{q_2 q_3}{d_{2 \rightarrow 3}^2} \right) \hat{i} =$$

$$\vec{F}_E = \left(8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \right) (1.0 \times 10^{-6} \text{C}) \left(-\frac{(1.5 \times 10^{-6} \text{C})}{(0.25 \text{m})^2} - \frac{(0.5 \times 10^{-6} \text{C})}{(0.25 \text{m})^2} \right) \hat{i} = -0.288 \text{N} \hat{i}$$

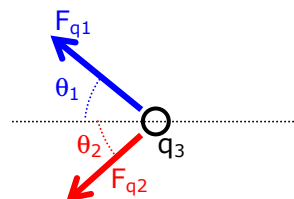
d. If q_3 is moved 0.3 m downward but still equidistant to q_1 and q_2 , what is the magnitude and direction of the electric force exerted on q_3 ?

Ans.

$$\vec{F}_E = k \frac{q_3}{d^2} \left[(q_1 \cos \theta_1 + q_2 \cos \theta_2) \hat{i} + (q_1 \sin \theta_1 - q_2 \sin \theta_2) \hat{j} \right]$$

$$\vec{F}_E = \left(8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \right) \frac{(-1.0 \times 10^{-6} \text{C})}{(0.391 \text{m})^2} \left[(1.5 \times 10^{-6} \text{C} + 0.5 \times 10^{-6} \text{C}) \cos(50.2^\circ) \hat{i} + (1.5 \times 10^{-6} \text{C} - 0.5 \times 10^{-6} \text{C}) \sin(50.2^\circ) \hat{j} \right]$$

$$\vec{F}_E = -0.075 \text{N} \hat{i} + 0.045 \text{N} \hat{j}$$



e. Where would q_3 need to be placed such that there is no electrical force acting on the charge?

Ans. {Assume all charge positions are referenced to the origin at q_3 }

$$\vec{F}_E = \left(-k \frac{q_1 q_3}{d_{x1 \rightarrow 3}^2} - k \frac{q_2 q_3}{d_{x2 \rightarrow 3}^2} \right) \hat{i} + \left(-k \frac{q_1 q_3}{d_{y1 \rightarrow 3}^2} - k \frac{q_2 q_3}{d_{y2 \rightarrow 3}^2} \right) \hat{j} = 0 \text{ N } \hat{i} + 0.0 \text{ N } \hat{j}$$

$$\text{in x: } \left| k \frac{q_1 q_3}{d_{x1 \rightarrow 3}^2} \right| = \left| k \frac{q_2 q_3}{d_{x2 \rightarrow 3}^2} \right| \Rightarrow \left| \frac{d_{x2 \rightarrow 3}}{d_{x1 \rightarrow 3}} \right| = \sqrt{\frac{d_{x2 \rightarrow 3}^2}{d_{x1 \rightarrow 3}^2}} = \sqrt{\frac{q_2}{q_1}} = 0.58$$

$$\text{when } \vec{x}_{q_1} = -0.75 \text{ m } \hat{i} \text{ then } \vec{x}_{q_2} = -0.43 \text{ m } \hat{i}$$

$$\text{in y: } \left| k \frac{q_1 q_3}{d_{y1 \rightarrow 3}^2} \right| = \left| k \frac{q_2 q_3}{d_{y2 \rightarrow 3}^2} \right| \Rightarrow \left| \frac{d_{y2 \rightarrow 3}}{d_{y1 \rightarrow 3}} \right| = \sqrt{\frac{d_{y2 \rightarrow 3}^2}{d_{y1 \rightarrow 3}^2}} = \sqrt{\frac{q_2}{q_1}} = 0.58$$

$$\text{when } \vec{y}_{q_1} = +0.75 \text{ m } \hat{j} \text{ then } \vec{y}_{q_2} = +0.43 \text{ m } \hat{j}$$

2. Consider a simple model of an atomic nucleus. In this case, three protons are present in the nucleus of a lithium atom, forming an equilateral triangle. The distance, r , between each pair of protons is $1.5 \times 10^{-15} \text{ m}$.

a. What is the magnitude of the electric force exerted between any two of the protons?

$$\text{Ans. } |\vec{F}_E| = k \frac{e^2}{d^2} = (8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}) \frac{(1.602 \times 10^{-19} \text{ C})^2}{(1.5 \times 10^{-15} \text{ m})^2} = 103 \text{ N}$$

b. What is the magnitude and direction of the total electric force exerted on a proton (due to its neighboring protons). You will need to assign an appropriate coordinate system.

Ans. Using the bottom charge as a reference and applying the Law of Cosines to the forces:

$$\vec{F}_E = \sqrt{\left(k \frac{e^2}{d^2} \right)^2 + \left(k \frac{e^2}{d^2} \right)^2 - 2 \left(k \frac{e^2}{d^2} \right)^2 \cos 150^\circ} = 178 \text{ N}$$

Applying the Law of Sines to obtain the direction of the resultant force:

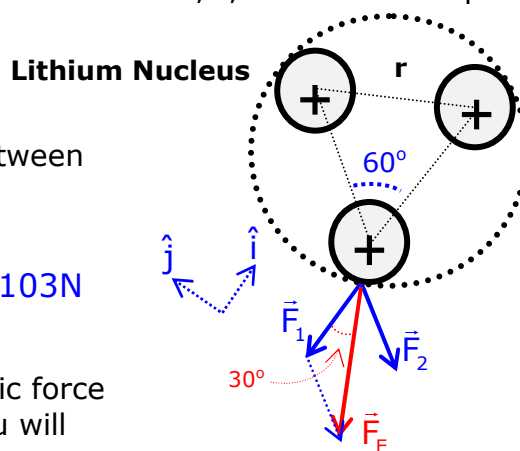
$$\frac{\sin \phi_{\vec{F}_E}}{103 \text{ N}} = \frac{\sin 120^\circ}{178 \text{ N}} \Rightarrow \sin \phi_{\vec{F}_E} = 103 \text{ N} \left(\frac{\sin 120^\circ}{178 \text{ N}} \right) = 0.501$$

$$\phi_{\vec{F}_E} = \sin^{-1}(0.501) = 30^\circ \text{ \{south of } -\hat{i}\} \text{ or } 210^\circ \text{ w/r to } \hat{i}$$

c. Express the total electric force vector, in component form.

Ans. Using the bottom charge as a reference:

$$\vec{F}_E = -k \frac{e^2}{d^2} \left((1 + \cos 60^\circ) \hat{i} + \sin 60^\circ \hat{j} \right) = -155 \text{ N } \hat{i} - 89.2 \text{ N } \hat{j}$$



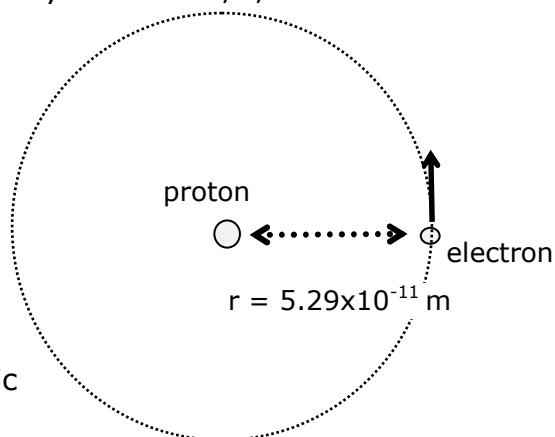
d. What is the net force acting on the subject proton in (b) & (c)? Explain.

Ans. Since the protons are fixed (static equilibrium), there must be at least 1 additional force acting on each nucleon to oppose the net electric (net) force:

$$\vec{F}_{\text{Net}} = \vec{F}_E + \vec{F}_{??} = 0 \text{ N } \hat{i} + 0 \text{ N } \hat{j}$$

This is was the justification for proposing a "strong nuclear force" within the nucleus!

3. Consider the "Bohr Model" of the atom, where an electron moves around a proton in a circular orbit. Assume that the proton and electron are separated by a distance, r , of $5.29 \times 10^{-11} \text{ m}$.



a. What is the magnitude and direction of the electric force exerted on electron?

$$\text{Ans. } |\vec{F}_E| = k \frac{e^2}{d^2} = (8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}) \frac{(1.602 \times 10^{-19} \text{C})^2}{(5.29 \times 10^{-11} \text{m})^2} = 8.24 \times 10^{-8} \text{N}$$

b. What is the centripetal force exerted on the electron such that it maintains this orbit?

$$\text{Ans. } |\vec{F}_{\text{net}}| = |\vec{F}_c| = |\vec{F}_E| = 8.24 \times 10^{-8} \text{N}$$

c. What is the speed of the electron?

Ans.

$$|\vec{F}_c| = \frac{mv^2}{r} = 8.24 \times 10^{-8} \text{N} \Rightarrow v = \sqrt{\frac{|\vec{F}_c| r}{m}} = \sqrt{\frac{(8.24 \times 10^{-8} \text{N})(5.29 \times 10^{-11} \text{m})}{9.11 \times 10^{-31} \text{kg}}} = 2.19 \times 10^6 \frac{\text{m}}{\text{s}}$$

d. What is the kinetic and potential energy, respectively, for the electron in J and eV?

Ans.

$$K = \frac{mv^2}{2} = 2.18 \times 10^{-18} \text{J} = 13.6 \text{ eV}$$

$$U = |\vec{F}_E| \cdot r \cdot \cos 180^\circ = -(8.24 \times 10^{-8} \text{N})(5.29 \times 10^{-11} \text{m}) = -4.36 \times 10^{-18} \text{J} = -27.2 \text{ eV}$$

e. What is the total energy of the electron in J and eV?

Ans.

$$E_{\text{tot}} = K + U = 2.18 \times 10^{-18} \text{J} + (-4.36 \times 10^{-18} \text{J}) = -2.18 \times 10^{-18} \text{J}$$

or

$$E_{\text{tot}} = K + U = 13.6 \text{ eV} - 27.2 \text{ eV} = -13.6 \text{ eV}$$

f. Calculate the orbital radius for an electron with a total energy (K+U) of -3.40 eV.

Ans.

$$E_{\text{tot}} = K + U = -3.40 \text{ eV} = -5.45 \times 10^{-19} \text{ J}$$

$$E_{\text{tot}} = \frac{mv^2}{2} + |\vec{F}_E| \cdot r \cdot \cos 180^\circ = \frac{mv^2}{2} + \left(\frac{mv^2}{r} \right) \cdot r \cdot \cos 180^\circ =$$

$$E_{\text{tot}} = \frac{mv^2}{2} - mv^2 = -\frac{mv^2}{2} = -5.45 \times 10^{-19} \text{ J}$$

$$\text{or } mv^2 = 1.09 \times 10^{-18} \text{ J}$$

since $|\vec{F}_c| = |\vec{F}_E|$ you can solve for r:

$$\frac{mv^2}{r} = k \frac{e^2}{r^2} \Rightarrow r = k \frac{e^2}{mv^2} = \frac{(8.99 \times 10^9 \frac{\text{J} \cdot \text{m}^2}{\text{C}^2})(1.602 \times 10^{-19} \text{ C})^2}{(1.09 \times 10^{-18} \text{ J})} = 2.12 \times 10^{-10} \text{ m}$$

4. What is the total electric charge of 1.0 moles of electrons?

$$\text{Ans. } Q = Ne = (6.022 \times 10^{23} \frac{\text{electrons}}{\text{mol}})(1.602 \times 10^{-19} \frac{\text{C}}{\text{electron}}) = 96400 \frac{\text{C}}{\text{mol}}$$

5. How many electrons are present in $-1.5 \mu\text{C}$?

Ans.

$$N = \frac{Q}{e} = \frac{(1.5 \times 10^{-6} \text{ C})}{(1.602 \times 10^{-19} \frac{\text{C}}{\text{electron}})} = 9.36 \times 10^{12} \text{ electrons}$$

or

$$n = \frac{(1.5 \times 10^{-6} \text{ C})}{96400 \frac{\text{C}}{\text{mol}}} = 1.56 \times 10^{-11} \text{ mol}$$