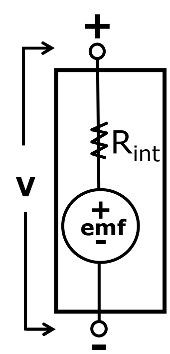
Phy 213: General Physics III

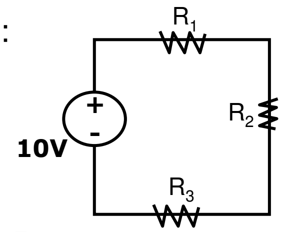
Chapter 27: Electric Circuits
Lecture Notes

Electromotive Force & Internal Resistance

1. The intrinsic potential difference associated with a power source is referred to as the "electromotive force" or emf
   a. In an ***ideal*** power source, the voltage across its terminals is its emf
   b. For a ***real*** power source, such as a battery, the emf is determined by the net electrochemical potential due to its internal redox reaction **BUT** the actual voltage across its terminals is slightly lower due to its internal resistance (R_{int}).
2. Real power sources are limited in their ability to deliver power output, due to factors such as the maximal rate of the internal chemical reaction, the input power (in an AC plug-in DC power source), etc…
   a. For a battery, the rate of reaction is dependent on the conditioning and corrosion of electrodes and depletion of internal reactants. This results establishes an effective internal resistance, within the voltage source.
3. As an electrochemical power source is utilized and is run down, the decline in performance output is reflected in the increased in internal resistance
   a. The output voltage will wane as more of the potential drops across R_{int} even though the emf remains constant



Kirchoff's Voltage & Current Laws



Kirchoff's Voltage Law (aka the Loop Law):

- For any closed loop in a circuit, the total voltage around the loop is equal to zero:

$$\sum_{i=1}^{N} V_i = 0$$

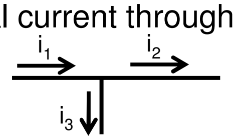
Example: A single loop circuit where $R_1 = R_2 = R_3 = 10\Omega$

$$\sum_{i=1}^{N} V_i = V - V_{R_1} - V_{R_2} - V_{R_3} = 0 \Rightarrow V_{R_1} + V_{R_2} + V_{R_3} = 3V_R = 10V$$

$$V_R = 3.33V \quad \text{and} \quad i_{loop} = \frac{V_R}{R} = 0.333A$$

Kirchoff's Current Law (aka the Node Law):

- The total current through any node is equal to zero:



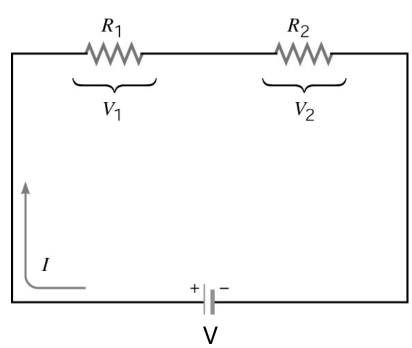
$$\sum_{i=1}^{N} i_i = 0$$

Series Circuits

1. In series wiring, circuit elements (loads) are connected end to end
2. The combined load or resistance (R_{eq}) in the series is

$$R_{eq} = R_1 + R_2 + \ldots + R_N = \sum_{i=1}^{N} R_i$$

3. Across each resistance, the potential difference (V) drops



4. The current i that flows through R_1 also flows through R_2

$$V = V_1 + V_2 = iR_1 + iR_2$$

or

$$V = i(R_1 + R_2) = iR_{eq}$$

Parallel Circuits

1. Circuit elements (loads) are connected with ends attached
2. The combined load or resistance (R_p) in the parallel is

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_N} = \sum_{i=1}^{N} \frac{1}{R_i}$$

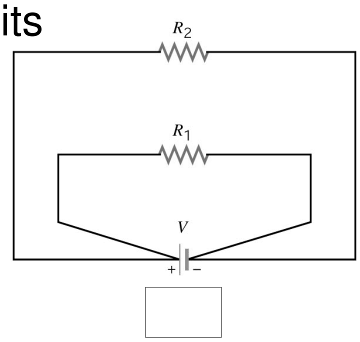
3. Across each resistance, the potential difference (V) is the same

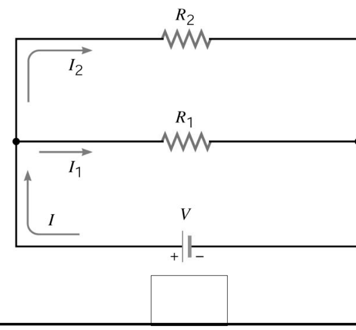
$$V = \frac{i_1}{R_1} = \frac{i_2}{R_2}$$

4. The total current drawn through the circuit is: $i = i_1 + i_2$

or

$$i = \frac{V}{R_1} + \frac{V}{R_2} = V\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{V}{R_{eq}}$$





Analyzing Circuits 1 (using Kirchoff's Laws)

Consider the following 2 loop circuit, with 6 equal value resistors ($R=R_1=R_2=R_3=R_4=R_5=R_6=10\Omega$):

1. What is the current & voltage for each R?
   a. Kirchoff's Laws:

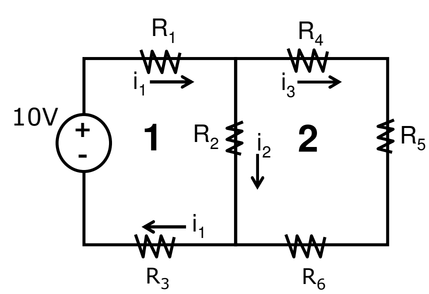
Loop 1: $V_{R_1} + V_{R_2} + V_{R_3} = 10V$

$i_1R_1 + i_2R_2 + i_1R_3 = 10V$

Loop 2: $V_{R_4} + V_{R_5} + V_{R_6} = V_{R_3}$

$i_3(R_4 + R_5 + R_6) = i_2R_2$

Node: $i_1 = i_2 + i_3$



solving for i's:

$$i_3 = \tfrac{1}{11}A = 0.091\ A \Rightarrow V_{R_4} = V_{R_5} = V_{R_6} = (10\Omega)(0.091A) = 0.91V$$

$$i_2 = 3i_3 = 0.273\ A \Rightarrow V_{R_2} = (10\Omega)(0.273A) = 2.73V$$

$$i_1 = i_2 + i_3 = 0.364\ A \Rightarrow V_{R_1} = V_{R_3} = (10\Omega)(0.364A) = 3.64V$$

Analyzing Circuits 2 (using equivalent resistances)

Consider the same circuit:

2. What is the current & voltage for each R?

   a. Solve for R_{eq1}:

   $R_{eq1} = R_4 + R_5 + R_6 = 30\Omega$

   b. Solve for R_{eq2}:

   $$R_{eq2} = \frac{R_2 R_{eq1}}{R_2 + R_{eq1}} = 7.5\Omega$$

   c. Solve for R_{eq}: $R_{eq} = R_1 + R_{eq2} + R_3 = 27.5\Omega$

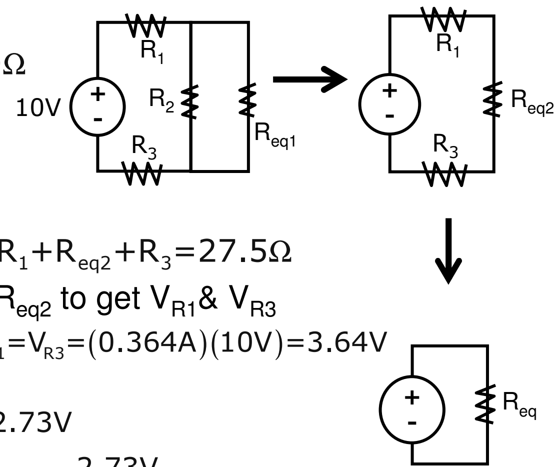
   d. Use R_{eq} to get i_1 & R_{eq2} to get V_{R1}& V_{R3}

   $$i_1 = \frac{10V}{27.5\Omega} = 0.364A \Rightarrow V_{R1} = V_{R3} = (0.364A)(10V) = 3.64V$$

   e. Solve for the rest:

   $V_{R2} = V_{eq2} = (10-7.27)V = 2.73V$

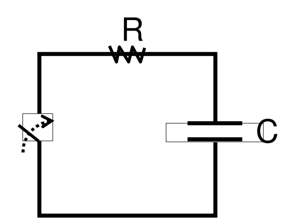
   $$\Rightarrow i_2 = \frac{2.73V}{10\Omega} = 0.273A \text{ and } i_3 = \frac{2.73V}{30\Omega} = 0.091A$$



RC Circuits

1. A circuit containing a capacitor and resistor(s) is called an RC circuit
2. A resistor in series with a capacitor will limit the rate (not quantity) at which charge accumulates in the capacitor
3. When V is constant across a capacitor ($\frac{dV}{dt}$ = 0) no current will flow through this branch of the circuit since:

   $$i = \frac{dq}{dt} = C\frac{dV}{dt} = 0$$



4. When a fully charged capacitor is discharged, the rate of charge loss is limited by the voltage across it and is limited by on the resistance:

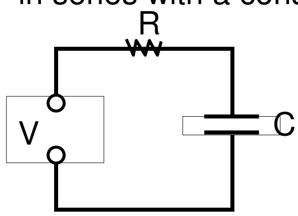
   $$q = CV = CiR = RC\frac{dq}{dt} \Rightarrow \frac{dq}{q} = RCdt$$

   integrating both sides $\Rightarrow \int_{q_{max}}^{q} \frac{dq}{q} = \frac{1}{RC}\int_0^t dt \Rightarrow \ln\left(\frac{q}{q_{max}}\right) = \frac{1}{RC}t$

   solving for q $\Rightarrow q(t) = q_{max}e^{-\frac{t}{RC}}$

Charging a Capacitor

The voltage equation around a loop with resistor and capacitor in series with a constant voltage source is given by:



$$V - V_R - V_{cap} = 0$$

$$V - iR - \frac{q}{C} = 0 \Rightarrow \frac{dq}{dt} + \frac{q}{RC} = \frac{V}{R}$$

$$\left\{ \text{where } V_{cap} = \frac{q}{C} \text{ and } i = \frac{dq}{dt} \right\}$$

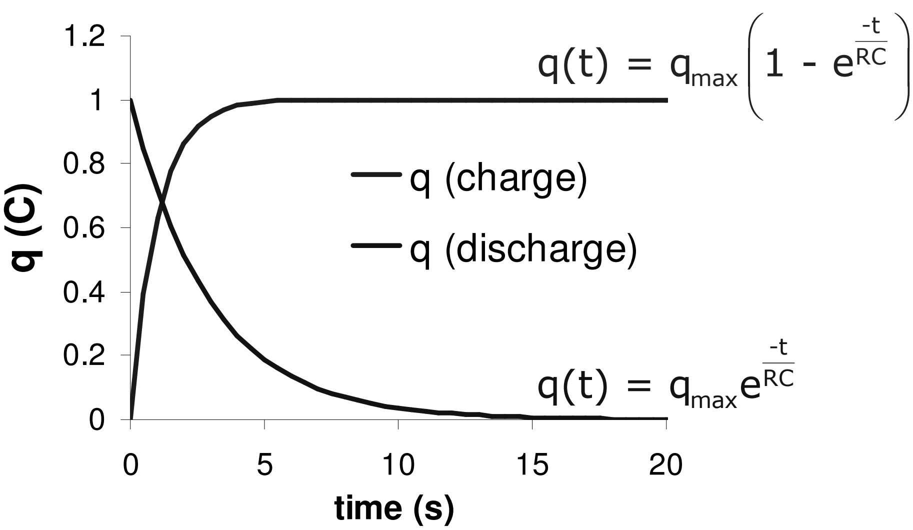
Re-arranging the equation leads to a 1st order linear non-homogeneous differential equation. This can be solved by applying the separation of variables technique:

$$\frac{dq}{dt} = \frac{V}{R} - \frac{q}{RC} = \frac{q_{max}}{RC} - \frac{q}{RC} \Rightarrow \frac{dq}{q_{max} - q} = \frac{1}{RC} dt$$

$$\int_0^q \frac{dq}{q_{max} - q} = \frac{1}{RC} \int_0^t dt \Rightarrow \ln\left(\frac{q_{max} - q}{q_{max}}\right) = \frac{t}{RC} \Rightarrow \quad q(t) = q_{max}\left(1 - e^{-\frac{t}{RC}}\right)$$

Capacitive Charging & Discharging

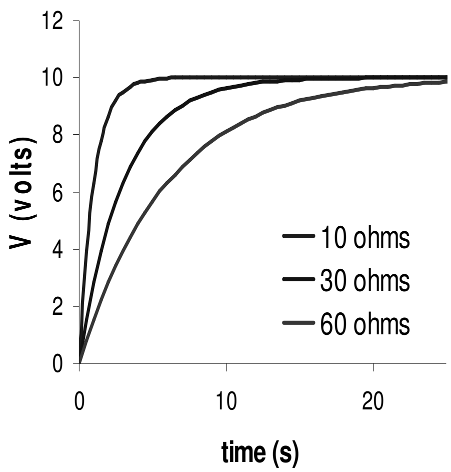
($C=0.1F$, $V_{max}=10V$ & $R=10\Omega$)



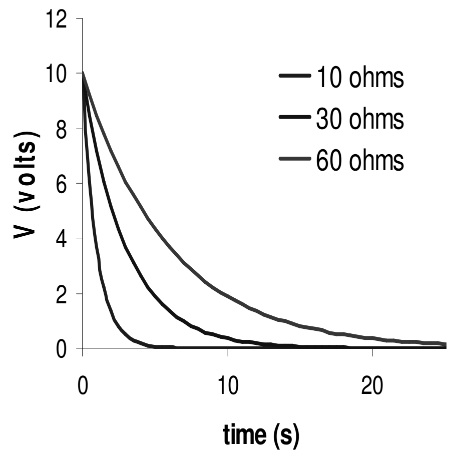
Capacitive Charging in RC circuit

(Effects of increasing R on V_{cap})

Charging



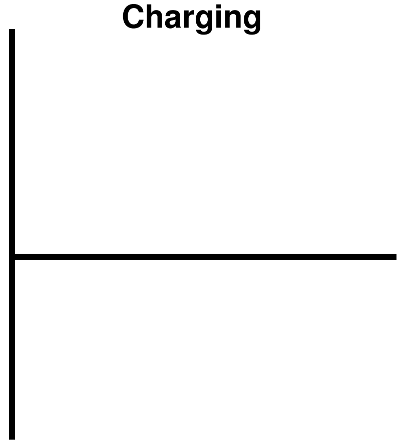
Discharging



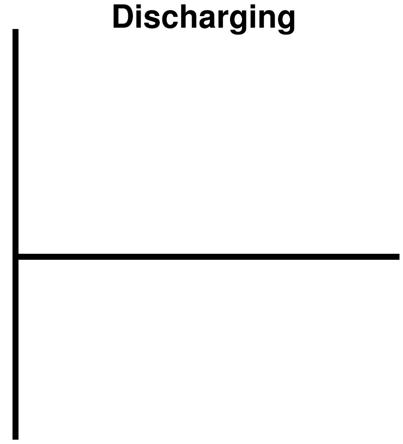
Capacitive Charging

(i vs t)

Charging



Discharging



What should these graphs should look like?