Phy 213: General Physics III

Chapter 24: Electric Potential
Lecture Notes

Electric Potential Energy

1. When charge is in an electric field, the electric force exerted upon it, as it moves from one point (A) to another point (B) can do work:

$$dW = \vec{F}_E \cdot d\vec{s} = q\vec{E} \cdot d\vec{s}$$

$$W_{AB} = \int dW = \int_A^B \vec{F}_E \cdot d\vec{s} = q\int_A^B \vec{E} \cdot d\vec{s}$$

2. The total work performed to move a charge from point A to B in an electric field is then:

$$W_{AB} = U_A - U_B$$

3. Since the electric force is a conservative force, the work performed is equal to the difference in electric potential energy between points A & B:

Electric Potential

1. The electric potential energy for a charge at a point in space can be expressed in per unit charge, this is the electric potential at this location:

$$V = \frac{U_E}{q}$$

   a. In this manner, V associated with a position can be expressed without concern for the specific charge present
   b. The SI units are joules/coulomb (J/C), called the volt (V)

2. The work performed on an electric charge is related to the change in electric potential (ΔV) and its charge (q):

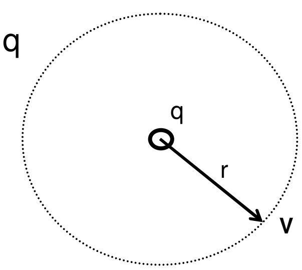
$$-W_{AB} = q\Delta V \quad \underline{\text{or}} \quad \Delta V = -\frac{W_{AB}}{q}$$

3. Expressing W in terms of V we can combine them to obtain a relation between E and V:

$$-\frac{W_{AB}}{q} = \Delta V = -\int_A^B \vec{E}\cdot d\vec{s} \quad \underline{\text{or}} \quad \vec{E} = -\frac{\partial V}{\partial s}$$

Potential due to a Point Charge

Consider a point charge, q



1. The electric field is radial from q: $\vec{E} = \dfrac{q}{4\pi\varepsilon_o r^2}\hat{i}_r$

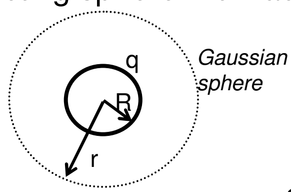
2. The electric potential a distance r from q is:

$$V = -\int_\infty^r \vec{E}\cdot d\vec{s} = -\frac{q}{4\pi\varepsilon_o}\int_\infty^r \frac{dr}{r^2} = \frac{q}{4\pi\varepsilon_o r}$$

This relationship is extremely useful for evaluating more complex charge geometries…

Potential due to a Charged Sphere

For a hollow conducting sphere with radius R & charge, q:



1. The electric field outside R is : $\vec{E} = \dfrac{q}{4\pi\varepsilon_o r^2}$
2. The electric potential for points outside R is :

$$V = -\int_\infty^r \vec{E}\cdot d\vec{s} = -\frac{q}{4\pi\varepsilon_o}\int_\infty^r \frac{dr}{r^2} = \frac{q}{4\pi\varepsilon_o r}$$

3. The electric field inside the sphere is:

$$\vec{E} = 0$$

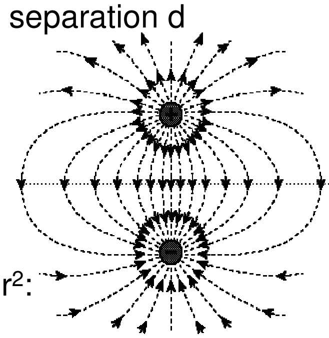
4. The electric potential inside the sphere is constant:

$$\left|\vec{E}\right| = -\frac{\partial V}{\partial s} = 0 \Rightarrow V_f - V_i = \int E\cdot ds = 0 \Rightarrow V_f = V_i = \text{constant}$$

Potential due to an Electric Dipole

1. Consider an electric dipole with charge separation d and dipole moment p
2. The electric field for a dipole is:

$$\vec{E}_{dipole} = \left(\frac{\vec{p}}{2\pi\varepsilon_o r^3}\right)$$



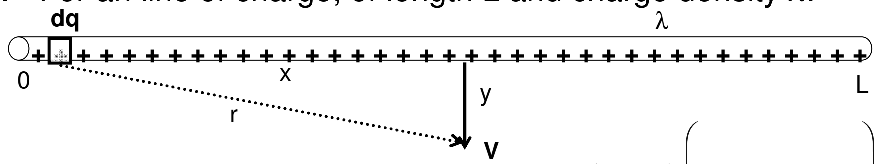
3. The potential decreases inversely with r^2:

$$V = -\int \vec{E}\cdot d\vec{s} = \int_\infty^r \left(\frac{\vec{p}}{2\pi\varepsilon_o r^3}\right)\cdot d\vec{r}$$

$$V = -\left(\frac{p\cdot\cos\theta}{2\pi\varepsilon_o}\right)\int_\infty^r \frac{dr}{r^3} = \frac{p\cdot\cos\theta}{4\pi\varepsilon_o r^2}$$

Potential due to a Line of Charge

1. For an line of charge, of length L and charge density λ:



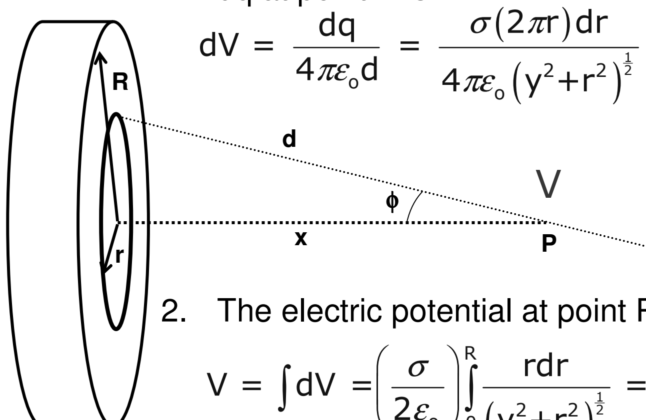
2. The electric field is described by: $\vec{E} = \left(\dfrac{\lambda}{4\pi\varepsilon_o}\right)\left(\dfrac{L}{y\sqrt{\dfrac{L^2}{4} + y^2}}\right)\hat{i}_r$

3. The electric potential is:

$$V = \int dV = \frac{1}{4\pi\varepsilon_o}\int\frac{dq}{r} = \frac{\lambda}{4\pi\varepsilon_o}\int_0^L\frac{dx}{\left(x^2 + y^2\right)^{1/2}}$$

$$V = \left(\frac{\lambda}{4\pi\varepsilon_o}\right)\ln\left(\frac{L + \sqrt{L^2 + y^2}}{y}\right)$$

Potential due to a Charged Disc



1. The electric potential due to a thin ring of charge, dq at point P is:

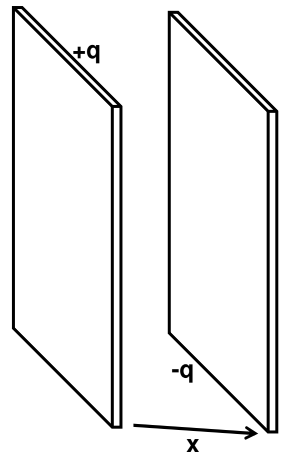
$$dV = \frac{dq}{4\pi\varepsilon_o d} = \frac{\sigma(2\pi r)dr}{4\pi\varepsilon_o\left(y^2+r^2\right)^{\frac{1}{2}}} = \left(\frac{\sigma}{2\varepsilon_o}\right)\frac{rdr}{\left(y^2+r^2\right)^{\frac{1}{2}}}$$

2. The electric potential at point P is:

$$V = \int dV = \left(\frac{\sigma}{2\varepsilon_o}\right)\int_0^R\frac{rdr}{\left(y^2+r^2\right)^{\frac{1}{2}}} = \left(\frac{\sigma}{2\varepsilon_o}\right)\left[\left(y^2+R^2\right)^{\frac{1}{2}} - y\right]$$

Potential between 2 Parallel Charged Plates

Two oppositely charged conducting plates, with charge density σ:



1. The electric field between the plates is constant:

$$\left|\vec{E}\right| = \frac{\sigma}{\varepsilon_o} = -\frac{\partial V}{\partial x}$$

2. The corresponding electric potential is linear between the plates:

$$V = -\int \vec{E} \cdot d\vec{x} = \frac{\sigma}{\varepsilon_o}\int_0^x dx = \frac{\sigma}{\varepsilon_o} x$$