Phy 213: General Physics III

Chapter 22: Electric Fields
Lecture Notes

The Electric Field

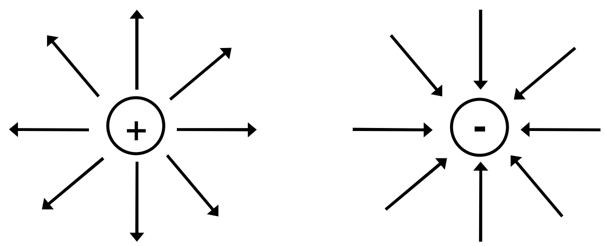
1. The vector field associated with the presence of electric charge is called electric field
2. The electric field is the influence of electric charge on space (or the medium) itself
3. The direction of the electric field is determined by the direction of electric force on a positive "test" charge (q_o):

$$\vec{E} = \frac{\vec{F}_E}{q_o}$$

4. The units of electric field are N/C
5. The electric field for a point charge: $\vec{E} = \left(\frac{1}{4\pi\varepsilon_o} \cdot \frac{q}{r^2}\right)\hat{r}$

Electric Field Lines

1. The ability of a charge to influence other charges in its vicinity its electric field
2. The electric field is a vector property
   - **a.** **E** fields due to multiple charges add as vectors
   - **b.** **E** field lines originate at + charges & terminate at - charges
3. The direction of an electric field vector (at a point in space) is the direction of electric force that would be exerted by on a positive charge at that location



A Point Charge in an Electric Field

1. The electric field (magnitude) for a point charge: $\left|\vec{E}\right| = \frac{1}{4\pi\varepsilon_o} \cdot \frac{|q|}{r^2}$
2. Expressed as a vector: $\vec{E} = \left(\frac{1}{4\pi\varepsilon_o} \cdot \frac{q}{r^2}\right)\hat{r}$
3. A non-point charge can be treated as a point charge as well, when the separation distance is much greater than the radius of the charged object, i.e. $r_{object} << r_{separation}$
4. Strategy for finding **E** using the point charge definition:
   – Since the electric field vectors for individual charges are additive, treat small charge elements within a continuous charge distribution as individual point charges (dq):

$$dE = \left(\frac{1}{4\pi\varepsilon_o r^2}\right)dq \Rightarrow E = \int dE = \left(\frac{1}{4\pi\varepsilon_o r^2}\right)\int dq$$

A Point Charge in an Electric Field

1. The electric force (F_E) acting on a point charge (q_o) in an electric field is given by: $\vec{F}_E = q_o\vec{E}$

2. The direction of F_E is in the direction of E for positive q_o & opposite E for negative q_o

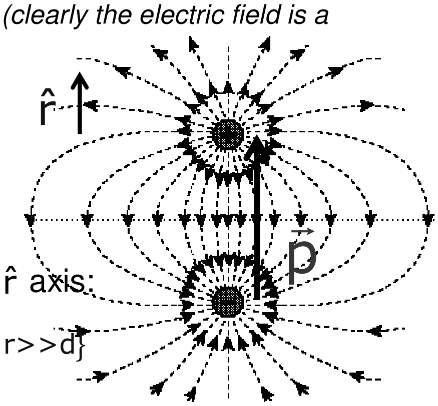
Electric Field (due to an electric dipole)

1. An electric dipole consists of 2 identical but opposite sign electric charges
2. Electric field lines emanate from the positive pole and terminate at the negative pole *(clearly the electric field is a complicated function). In general:*

$$\vec{E}_{dipole} = \vec{E}_+ + \vec{E}_-$$

$$\vec{E}_{dipole} = \left(\frac{1}{4\pi\varepsilon_o}\cdot\frac{q}{r_-^2}\right)\hat{r}_- - \left(\frac{1}{4\pi\varepsilon_o}\cdot\frac{q}{r_+^2}\right)\hat{r}_+$$

$$\vec{E}_{dipole} = \frac{q}{4\pi\varepsilon_o}\left[\left(\frac{1}{r_-^2}\right)\hat{r}_- - \left(\frac{1}{r_+^2}\right)\hat{r}_+\right]$$



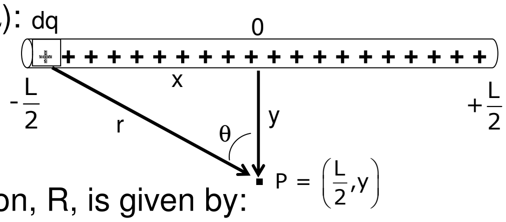
3. The magnitude of E_{dipole} along $\hat{r}$ axis:

$$\left|\vec{E}_{dipole}\right| \cong \left(\frac{qd}{2\pi\varepsilon_o r^3}\right) = \left(\frac{p}{2\pi\varepsilon_o r^3}\right) \quad \{\text{for } r>>d\}$$

or $\vec{E}_{dipole} \cong \left(\frac{\vec{p}}{2\pi\varepsilon_o r^3}\right)$ where $\vec{p} = q\vec{d}$ (directed from - to +)

Electric Field (due to a line of charge)

1. Consider a line of charge (length, L) with a continuous, uniform charge density (λ):



2. The electric field at position, R, is given by:

$$d\vec{E} = d\vec{E}_x + d\vec{E}_y = \left(\frac{dq}{4\pi\varepsilon_o r^2}\right)\sin\theta\,\hat{i} + \left(\frac{dq}{4\pi\varepsilon_o r^2}\right)\cos\theta\,\hat{j}$$

$$\Rightarrow\ \vec{E} = \int d\vec{E} = \vec{E}_x + \vec{E}_y = \left(\frac{\lambda}{4\pi\varepsilon_o}\right)\left(\frac{L}{y\sqrt{\frac{L^2}{4} + y^2}}\right)\hat{j}$$

3. For L=∞: $\vec{E} = \left(\frac{\lambda}{2\pi\varepsilon_o y}\right)\hat{j}$

(The Derivation: Line of Charge)

Note the following identities:

$$r^2 = x^2 + y^2,\ \cos\theta = \frac{y}{x^2 + y^2} \text{ and } \sin\theta = \frac{x}{x^2 + y^2}$$

Solving for the electric field:

$$d\vec{E} = d\vec{E}_x + d\vec{E}_y = \left(\frac{dq}{4\pi\varepsilon_o r^2}\right)\left(\sin\theta\,\hat{i} + \cos\theta\,\hat{j}\right) = \left(\frac{\lambda}{4\pi\varepsilon_o}\right)\left(\frac{xdx}{\left(x^2+y^2\right)^{\frac{3}{2}}}\hat{i} + \frac{ydx}{\left(x^2+y^2\right)^{\frac{3}{2}}}\hat{j}\right)$$

$$\vec{E} = \int d\vec{E} = \left(\frac{\lambda}{4\pi\varepsilon_o}\right)\int_{-\frac{L}{2}}^{\frac{L}{2}}\left(\frac{xdx}{\left(x^2+y^2\right)^{\frac{3}{2}}}\hat{i} + \frac{ydx}{\left(x^2+y^2\right)^{\frac{3}{2}}}\hat{j}\right) = \left(\frac{\lambda}{4\pi\varepsilon_o}\right)\left(\frac{-1}{\left(x^2+y^2\right)^{\frac{1}{2}}}\hat{i} + \frac{yx}{y^2\left(x^2+y^2\right)^{\frac{1}{2}}}\hat{j}\right)\Bigg|_{-\frac{L}{2}}^{\frac{L}{2}}$$

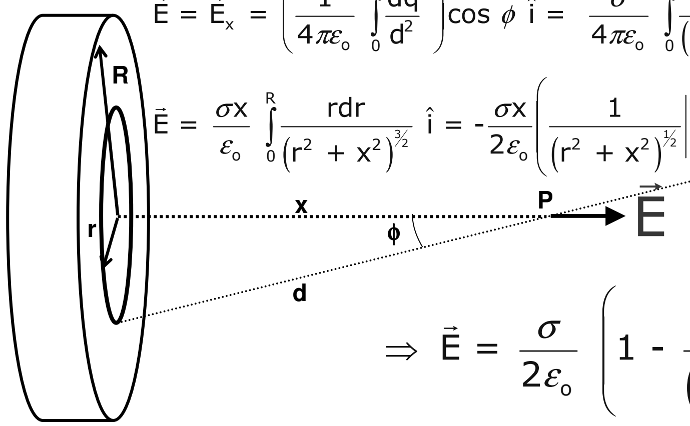
$$\vec{E} = \int d\vec{E} = \left(\frac{\lambda}{4\pi\varepsilon_o}\right)\left(0\,\hat{i} - \frac{L}{y\left(\frac{L^2}{4} + y^2\right)^{\frac{1}{2}}}\hat{j}\right) \Rightarrow \mathbf{\vec{E}_x = \left(0\tfrac{N}{C}\right)\hat{i}}\ \&\ \mathbf{\vec{E}_y = \left(\frac{\lambda}{4\pi\varepsilon_o}\right)\frac{L}{y\sqrt{\frac{L^2}{4} + y^2}}\hat{j}}$$

Electric Field (due to a flat disc)

1. For a charged disc of uniform charge, dq can be defined by: $dq = \sigma(2\pi r dr)$
2. The electric field at P is:

$$\vec{E} = \vec{E}_x = \left(\frac{1}{4\pi\varepsilon_o}\int_0^R \frac{dq}{d^2}\right)\cos\phi\ \hat{i} = \frac{\sigma}{4\pi\varepsilon_o}\int_0^R \frac{2\pi r dr}{\left(r^2 + x^2\right)}\frac{x}{\left(r^2 + x^2\right)^{1/2}}\hat{i}$$

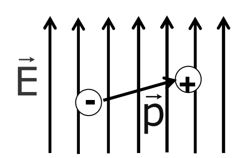
$$\vec{E} = \frac{\sigma x}{\varepsilon_o}\int_0^R \frac{r dr}{\left(r^2 + x^2\right)^{3/2}}\ \hat{i} = -\frac{\sigma x}{2\varepsilon_o}\left(\left.\frac{1}{\left(r^2 + x^2\right)^{1/2}}\right|_0^R\right)\hat{i}$$



$$\Rightarrow \vec{E} = \frac{\sigma}{2\varepsilon_o}\left(1 - \frac{x}{\left(R^2 + x^2\right)^{1/2}}\right)\hat{i}$$

A Dipole in an Electric Field

1. A dipole located within an $\vec{E}$ field will tend to align itself in the direction of the field vector, this rotation is a result of the torque exerted on the dipole by the field
2. The torques exerted on the dipole is:



$$\vec{\tau}_{dipole} = 2\left(\vec{\ell}\times\vec{F}_E\right)$$

$$\vec{\tau}_{dipole} = 2q_{dipole}\left(\vec{\ell}\times\vec{E}\right) = 2q_{dipole}\left(\frac{\vec{d}}{2}\times\vec{E}\right) \text{ or } \vec{\tau}_{dipole} = \vec{p}\times\vec{E}$$

3. The magnitude of the torque on the dipole is:

$$\left|\vec{\tau}_{dipole}\right| = p\cdot E\cdot\cos\phi$$

4. The potential energy, U, for the dipole is:

$$U_{dipole} = -\vec{p}\cdot\vec{E} = -p\cdot E\cdot\cos\phi$$