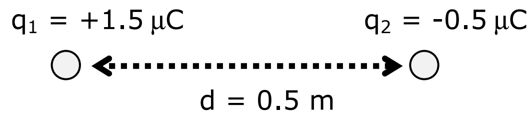
Coulomb's Law & Electric Fields:

1. Consider a fixed point charge of +1.5 μC (q_1). A 2nd charge of -0.5 μC (q_2) is placed at a distance of 0.5 m from q_1.



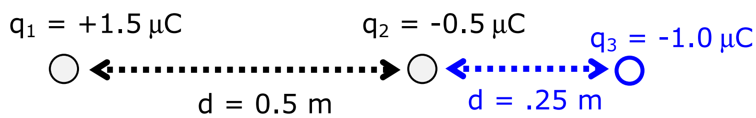
a. What is the magnitude and direction of the electric force exerted on q_2 due to q_1?

Ans.

$$\vec{F}_E = -k\frac{q_1q_2}{d^2}\hat{i} = -\left(8.99\times10^9\,\tfrac{N\times m^2}{C^2}\right)\frac{\left(1.5\times10^{-6}C\right)\left(0.5\times10^{-6}C\right)}{(0.5m)^2}\hat{i} = -0.0270N\ \hat{i}$$

b. A 3rd charge of -1.0 μC (q_3) is placed 0.25 m directly to the right of q_2. What is the magnitude and direction of the electric force exerted on q_3?

Ans.

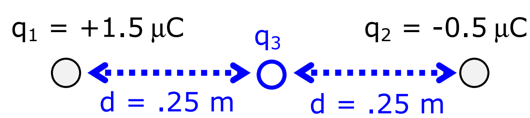


$$\vec{F}_E = \left(-k\frac{q_1q_3}{d^2_{1\to3}} + k\frac{q_2q_3}{d^2_{2\to3}}\right)\hat{i} =$$

$$\vec{F}_E = \left(8.99\times10^9\,\tfrac{N\times m^2}{C^2}\right)\left(1.0\times10^{-6}C\right)\left(-\frac{\left(1.5\times10^{-6}C\right)}{(0.75m)^2} + \frac{\left(0.5\times10^{-6}C\right)}{(0.25m)^2}\right)\hat{i} = 0.0479N\ \hat{i}$$

c. If q_3 is exactly halfway between q_1 and q_2, what is the magnitude and direction of the electric force exerted on q_3?

Ans.

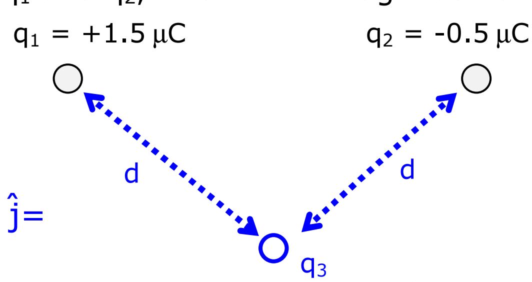


$$\vec{F}_E = \left(-k\frac{q_1q_3}{d^2_{1\to3}} - k\frac{q_2q_3}{d^2_{2\to3}}\right)\hat{i} =$$

$$\vec{F}_E = \left(8.99\times10^9\,\tfrac{N\times m^2}{C^2}\right)\left(1.0\times10^{-6}C\right)\left(-\frac{\left(1.5\times10^{-6}C\right)}{(0.25m)^2} - \frac{\left(0.5\times10^{-6}C\right)}{(0.25m)^2}\right)\hat{i} = -0.288N\ \hat{i}$$

d. If q_3 is moved 0.3 m downward but still equidistant to q_1 and q_2, what is the magnitude and direction of the electric force exerted on q_3?

Ans.



$$\vec{F}_E = \left(-k\frac{q_1q_3}{d^2_{x\,1\to3}} - k\frac{q_2q_3}{d^2_{x\,2\to3}}\right)\hat{i} + \left(-k\frac{q_1q_3}{d^2_{y\,1\to3}} - k\frac{q_2q_3}{d^2_{y\,2\to3}}\right)\hat{j} =$$

$$\vec{F}_E = -\,0.288N\ \hat{i} + \left(8.99\times10^9\,\tfrac{N\times m^2}{C^2}\right)\left(1.0\times10^{-6}C\right)\left(\frac{\left(1.5\times10^{-6}C\right)}{(0.30m)^2} - \frac{\left(0.5\times10^{-6}C\right)}{(0.30m)^2}\right)\hat{j}$$

$$\vec{F}_E = -\,0.288N\ \hat{i} - 0.010N\ \hat{j}$$

e. Where would q_3 need to be placed such that there is no electrical force acting on the charge?

Ans. {Assume all charge positions are referenced to the origin at q_3}

$$\vec{F}_E = \left(-k\frac{q_1q_3}{d^2_{x\,1\to3}} - k\frac{q_2q_3}{d^2_{x\,2\to3}}\right)\hat{i} + \left(-k\frac{q_1q_3}{d^2_{y\,1\to3}} - k\frac{q_2q_3}{d^2_{y\,2\to3}}\right)\hat{j} = 0\text{ N }\hat{i} + 0.0\text{ N }\hat{j}$$

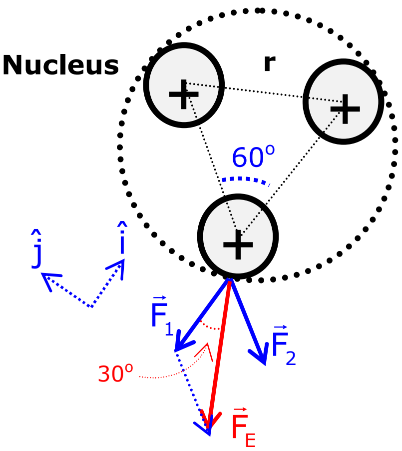
$$\text{in x: } \left|k\frac{q_1q_3}{d^2_{x\,1\to3}}\right| = \left|k\frac{q_2q_3}{d^2_{x\,2\to3}}\right| \Rightarrow \left|\frac{d_{x\,2\to3}}{d_{x\,1\to3}}\right| = \sqrt{\frac{d^2_{x\,2\to3}}{d^2_{x\,1\to3}}} = \sqrt{\frac{q_2q_3}{q_1q_3}} = 0.58$$

when $\vec{x}_{q_1} = -0.75\text{ m}\hat{i}$ then $\vec{x}_{q_2} = -0.43\text{ m}\hat{i}$

$$\text{in y: } \left|k\frac{q_1q_3}{d^2_{y\,1\to3}}\right| = \left|k\frac{q_2q_3}{d^2_{y\,2\to3}}\right| \Rightarrow \left|\frac{d_{y\,2\to3}}{d_{y\,1\to3}}\right| = \sqrt{\frac{d^2_{y\,2\to3}}{d^2_{y\,1\to3}}} = \sqrt{\frac{q_2q_3}{q_1q_3}} = 0.58$$

when $\vec{y}_{q_1} = +0.75\text{ m }\hat{j}$ then $\vec{y}_{q_2} = +0.43\text{ m }\hat{j}$

2. Consider a simple model of an atomic nucleus. In this case, three protons are present in the nucleus of a lithium atom, forming an equilateral triangle. The distance, r, between each pair of protons is 1.5×10^{-15} m.



a. What is the magnitude of the electric force exerted between any two of the protons?

Ans. $$\left|\vec{F}_E\right| = k\frac{e^2}{d^2} = \left(8.99\times10^9\,\tfrac{N\times m^2}{C^2}\right)\frac{\left(1.602\times10^{-19}C\right)^2}{\left(1.5\times10^{-15}m\right)^2} = 103N$$

b. What is the magnitude and direction of the total electric force exerted on a proton (due to its neighboring protons). You will need to assign an appropriate coordinate system.

Ans. Using the bottom charge as a reference and applying the Law of Cosines to the forces:

$$\vec{F}_E = \sqrt{\left(k\frac{e^2}{d^2}\right)^2 + \left(k\frac{e^2}{d^2}\right)^2 - 2\left(k\frac{e^2}{d^2}\right)^2\cos150^\circ} = 178\text{ N}$$

Applying the Law of Sines to obtain the direction of the resultant force:

$$\frac{\sin\phi_{\vec{F}_E}}{103N} = \frac{\sin 120^\circ}{178\text{ N}} \Rightarrow \sin\phi_{\vec{F}_E} = 103N\left(\frac{\sin 120^\circ}{178\text{ N}}\right) = 0.501$$

$$\phi_{\vec{F}_E} = \sin^{-1}(0.501) = 30^\circ\ \{\text{south of } -\hat{i}\}\text{ or } 210^\circ\text{ w/r to }\hat{i}$$

c. Express the total electric force vector, in component form.

Ans. Using the bottom charge as a reference:

$$\vec{F}_E = -k\frac{e^2}{d^2}\left(\left(1 + \cos60^\circ\right)\hat{i} + \sin60^\circ\right)\hat{j} = -155\text{ N }\hat{i} - 89.2\text{ N }\hat{j}$$

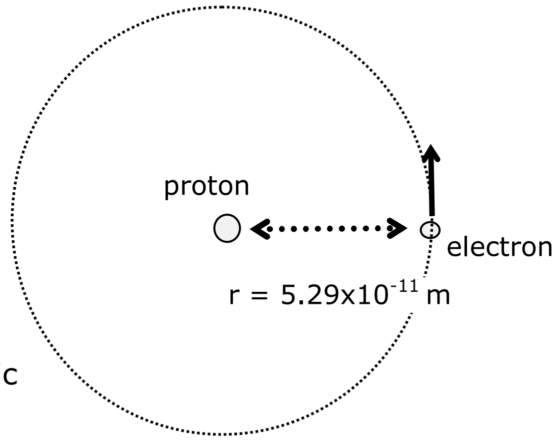
d. What is the net force acting on the subject proton in (b) & (c)? Explain.

Ans. Since the protons are fixed (static equilibrium), there must be at least 1 additional force acting on each nucleon to oppose the net electric (net) force:

$$\vec{F}_{Net} = \vec{F}_E + \vec{F}_{??} = 0\text{ N }\hat{i} + 0\text{ N }\hat{j}$$

This is was the justification for proposing a "strong nuclear force" within the nucleus!

3. Consider the "Bohr Model" of the atom, where an electron moves around a proton in a circular orbit. Assume that the proton and electron are separated by a distance, r, of 5.29x10^{-11} m.



a. What is the magnitude and direction of the electric force exerted on electron?

Ans. $$\left|\vec{F}_E\right| = k\frac{e^2}{d^2} = \left(8.99\times10^{9}\ \tfrac{N\times m^2}{C^2}\right)\frac{\left(1.602\times10^{-19}C\right)^2}{\left(5.29\times10^{-11}m\right)^2} = 8.24\times10^{-8}N$$

b. What is the centripetal force exerted on the electron such that it maintains this orbit?

Ans. $$\left|\vec{F}_{net}\right| = \left|\vec{F}_c\right| = \left|\vec{F}_E\right| = 8.24\times10^{-8}N$$

c. What is the speed of the electron?

Ans.

$$\left|\vec{F}_c\right| = \frac{mv^2}{r} = 8.24\times10^{-8}N \Rightarrow v = \sqrt{\frac{\left|\vec{F}_c\right|r}{m}} = \sqrt{\frac{\left(8.24\times10^{-8}N\right)\left(5.29\times10^{-11}m\right)^2}{9.11\times10^{-31}kg}} = 2.19\times10^{6}\ \tfrac{m}{s}$$

d. What is the kinetic and potential energy, respectively, for the electron in J and eV?

Ans.

$$K = \frac{mv^2}{2} = 2.18\times10^{-18}J = 13.6\text{ eV}$$

$$U = \left|\vec{F}_E\right|\cdot r\cdot\cos 180^{o} = -\left(8.24\times10^{-8}N\right)\left(5.29\times10^{-11}m\right) = -4.36\times10^{-18}J = -27.2\text{ eV}$$

e. What is the total energy of the electron in J and eV?

Ans.

$$E_{tot} = K + U = 2.18\times10^{-18}J + \left(-4.36\times10^{-18}J\right) = -2.18\times10^{-18}J$$

or

$$E_{tot} = K + U = 13.6\text{ eV} - 27.2\text{ eV} = -13.6\text{ eV}$$

f. Calculate the orbital radius for an electron with a total energy (K+U) of -3.40 eV.

Ans.

$$E_{tot} = K + U = -3.40 \text{ eV} = -5.45\times10^{-19}\text{J}$$

$$E_{tot} = \frac{mv^2}{2} + \left|\vec{F}_E\right|\cdot r\cdot\cos 180^\circ = \frac{mv^2}{2} + \left(\frac{mv^2}{r}\right)\cdot r\cdot\cos 180^\circ =$$

$$E_{tot} = \frac{mv^2}{2} - mv^2 = -\frac{mv^2}{2} = -5.45\times10^{-19}\text{J}$$

or $mv^2 = 1.09\times10^{-18}\text{J}$

since $\left|\vec{F}_c\right| = \left|\vec{F}_E\right|$ you can solve for r:

$$\frac{mv^2}{r} = k\frac{e^2}{r^2} \Rightarrow r = k\frac{e^2}{mv^2} = \frac{\left(8.99\times10^9\ \frac{\text{J}\cdot\text{m}^2}{\text{C}^2}\right)\left(1.602\times10^{-19}\text{C}\right)^2}{\left(1.09\times10^{-18}\text{J}\right)} = 2.12\times10^{-10}\text{m}$$

4. What is the total electric charge of 1.0 moles of electrons?

Ans. $Q = Ne = \left(6.022\times10^{23}\ \frac{\text{electrons}}{\text{mol}}\right)\left(1.602\times10^{-19}\ \frac{\text{C}}{\text{electron}}\right) = 96400\ \frac{\text{C}}{\text{mol}}$

5. How many electrons are present in -1.5 μC?

Ans.

$$N = \frac{Q}{e} = \frac{\left(1.5\times10^{-6}\text{C}\right)}{\left(1.602\times10^{-19}\ \frac{\text{C}}{\text{electron}}\right)} = 9.36\times10^{12} \text{ electrons}$$

or

$$n = \frac{\left(1.5\times10^{-6}\text{C}\right)}{96400\ \frac{\text{C}}{\text{mol}}} = 1.56\times10^{-11} \text{ mol}$$