Wavelength & Index of Refraction:

1. The wave speed for 590 nm light is $2.25\text{x}10^8$ m/s in water.

a) What is the index of refraction for water at this wavelength?

Ans. $n = \frac{c}{v} = \frac{3.0\text{x}10^8 \frac{m}{s}}{2.25\text{x}10^8 \frac{m}{s}} = 1.33$

b) If 590 nm light has an incident angle of 30° onto a settled surface of water, what is the angle of the refracted light?

Ans. Re-arranging Snell's Law: $\theta = \sin^{-1}\left(\frac{(1.0)\sin 30^\circ}{1.33}\right) = 22.1^\circ$

c) If 590 nm light were shined up through water toward the surface, at what angle would all of the light be reflected and not transmitted into the air? *This is referred to as the critical angle for total internal reflection.*

Ans. The condition for total internal reflection is $\theta_C = \sin^{-1}\left(\frac{(1.0)\sin 90^\circ}{1.33}\right) = 48.8^\circ$

2. A monochromatic light source (λ=450 nm) is shined onto a transparent material (n = 1.5).

a) How fast does the light travel through this material?

Ans. $v = \frac{c}{n} = \frac{3.0\text{x}10^8 \frac{m}{s}}{1.5} = 2.0\text{x}10^8 \frac{m}{s}$

b) What is the frequency of the light in this material?

Ans. $f = \frac{c}{\lambda_{air}} = \frac{3.0\text{x}10^8 \frac{m}{s}}{4.50\text{x}10^{-7}m} = 6.7\text{x}10^{14}\text{Hz}$

c) What is the wavelength of light in this material?

Ans. $\lambda_{material} = \frac{\lambda_{air}}{n} = \frac{4.50\text{x}10^{-7}m}{1.5} = 3.0\text{x}10^{-7}m$

Young's Experiment (Two Slit Interference):

3. Two parallel slits (width = $1.0\text{x}10^{-5}$ m) are side-by-side and separated by a distance of 1.5 x 10^{-4} m. A monochromatic light source with a wavelength of 600 nm is transmitted through the slits.

a) Determine the angular location of the 1st interference maxima from the center.

Ans. $\theta = \sin^{-1}\left(\frac{6.0\text{x}10^{-7} \frac{m}{s}}{1.6\text{x}10^{-4}m}\right) = 0.21^\circ$ Note: the center to center distance is $1.6\text{x}10^{-4}$ m.

b) A screen is placed 2.0 m directly in front of the slits. What is the distance from the center of the central maxima to the 1st interference maxima?

Ans. $y = (2m)\cdot\tan(0.21^\circ) = 0.0073m$

c) What is the distance between the 1st and 2nd maxima?

Ans. $\Delta y = (2m)\cdot\left[\tan(0.43^\circ) - \tan(0.21^\circ)\right] = 0.0077m$

Intensity for Two Slit Interference:

4. In a double slit experiment, a viewing screen is located at L=4.00 m from the slits. The wavelength for the light source is 500 nm and the separation distance between the slits is $1.5\text{x}10^{-4}$ m.

a) How far away (in m) from the center of the central intensity maxima is the m=2 maxima?

Ans. $$y = (4.00\text{m})\tan\left[\sin^{-1}\left(\frac{2(5.00\text{x}10^{-7}\text{ m})}{1.5\text{x}10^{-4}\text{m}}\right)\right] = 0.027\text{m}$$

b) Calculate the ratio of the intensities, $I_{m=2}$ to $I_{m=0}$.

Ans.

$$\frac{I_{m=2}}{I_{m=0}} = \frac{4I_o\cos^2\left(\frac{\pi d}{\lambda}\sin(\theta_{m=2})\right)}{4I_o\cos^2\left(\frac{\pi d}{\lambda}\sin(\theta_{m=0})\right)} = \frac{\cos^2\left(\frac{\pi d}{\lambda}\cdot\frac{2\lambda}{d}\right)}{\cos^2\left(\frac{\pi d}{\lambda}\sin(0^\circ)\right)}$$

$$\frac{I_{m=2}}{I_{m=0}} = \frac{\cos^2(2\pi)}{\cos^2(0)} = 1$$

Diffraction:

5. A diffraction grating has a spacing of $1.5 \text{ x } 10^{-6}$ m. A monochromatic light source with a wavelength of 500 nm is transmitted through the grating.

a) Determine the angular location of the 1st interference maxima.

Ans. $$\theta = \sin^{-1}\left(\frac{5.0\text{x}10^{-7}\ \frac{m}{s}}{1.5\text{x}10^{-6}\text{m}}\right) = 19.5^\circ$$

b) If the grating is located 2.5 m from the screen, what is the distance from the center of the central maximum to the 1st interference maximum?

Ans. $$y = (2.5\text{m})\tan(19.5^\circ) = 0.89\text{m}$$

6. Consider 2 diffraction gratings (spacing = $1.0 \text{ x } 10^{-6}$m and $2.0 \text{ x } 10^{-6}$m).

a) Which grating will produce the greatest angular separation between its maxima?

Ans. The smaller grating spacing will produce the larger the angular separation.

b) What is the angular separation between the 1st maxima produced by the two gratings? The light source has a wavelength of 500 nm.

Ans. $$\Delta\theta = \sin^{-1}\left(\frac{\lambda}{d_1}\right) - \sin^{-1}\left(\frac{\lambda}{d_2}\right) = \sin^{-1}\left(\frac{5.00\text{x}10^{-7}\text{m}}{1.0\text{x}10^{-6}\text{m}}\right) - \sin^{-1}\left(\frac{5.00\text{x}10^{-7}\text{m}}{2.0\text{x}10^{-6}\text{m}}\right) = 15.5^\circ$$

c) If both gratings are located 2.0 m from the screen, what is the distance between the 2 maxima (on the same side of the central maxima?

Ans. $$\Delta d = L\cdot\tan(\Delta\theta) = (2\text{m})\cdot\tan(15.5^\circ) = 0.55\text{ m}$$

Diffraction and Compact Discs:

7. The surface of a CD contains closely spaced grooves (similar to an old style phonograph). When incident light is reflected off these grooves, the reflected light undergoes diffraction. The relationship between angle (θ) and groove spacing (d) agrees with that of the equation for double slit diffraction and the diffraction grating, $\mathbf{d \cdot \sin\theta = m\lambda}$. Consider a 700 nm light source ($\theta_{incidence}=0^o$) that is reflected off a CD surface, at a region where the grooves are approximately parallel.

   a) The angular location of the 1st interference maxima is 23.5°. Determine the groove spacing for this CD.

   Ans. $d = \dfrac{m\lambda}{\sin(23.5^o)} = \dfrac{7.00\text{x}10^{-7}\text{m}}{\sin(23.5^o)} = 1.76\text{x}10^{-6}\text{m}$

   b) The data storage capacity for a CD is given by: Data Storage (in bytes) $\propto \dfrac{A_{CD}}{d_{CD}^2}$, where A_{CD} is the recordable region of the CD. If the outer and inner diameters of a CD are 11.4 cm and 4.3 cm, respectively, estimate the data storage capacity for a CD?

   Ans.

$$\text{Storage Capacity (in bits)} \sim \frac{\pi\left(\left(\frac{0.114}{2}\right)^2 - \left(\frac{0.043}{2}\right)^2\right)}{\left(1.76\text{x}10^{-6}\text{m}\right)} \sim 2.83\text{x}10^9 \text{ bits } (353\text{x}10^6 \text{ bytes})$$

Thin Film Interference:

8. What happens to light (traveling in air) that reflects off the surface of a material where
   a. $n_{material} > n_{air}$? The reflected light wave undergoes a $\lambda/2$ phase shift.
   b. $n_{material} < n_{air}$? The reflected light wave does not undergo a phase shift.

9. Consider a light wave (traveling in air, λ_{air} = 550 nm) incident on a thin layer of soap (n_{soap}=1.33, both sides of the soap film are in contact with air).

   a. What is the minimum thickness of the film that will result in destructive interference?

Ans. $d_{min} = \dfrac{m\lambda}{2n_{soap}} = \dfrac{5.50\text{x}10^{-7}\text{m}}{2\cdot 1.33} = 2.07\text{x}10^{-7}\text{m}$

   b. What is the minimum thickness of the film that will result in constructive interference?

Ans. $d_{min} = \dfrac{(m+\frac{1}{2})\lambda}{2n_{soap}} = \dfrac{(\frac{1}{2})5.50\text{x}10^{-7}\text{m}}{2\cdot 1.33} = 1.03\text{x}10^{-7}\text{m}$

10. Consider a light wave traveling through water (n_{water} = 1.33 and λ_{air} = 600 nm) incident on a thin layer of glass (n_{glass} = 1.5). The glass layer is completely submerged in water.

   a. What is the minimum thickness of the glass that will result in destructive interference?

Ans. This problem is treated the same as the above problem, except that the incident wave travels in water not air. The wavelength of the wave in water is: $\lambda_{water} = \lambda_{air}/n_{water}$

$$d_{min} = \frac{\lambda_{water}}{2n_{glass}} = \frac{\lambda_{air}}{2n_{water}n_{glass}} = \frac{6.00\text{x}10^{-7}\text{m}}{2\cdot(1.33)(1.5)} = 1.50\text{x}10^{-7}\text{m}$$

   b. What is the minimum thickness of the glass that will result in constructive interference?

Ans. $$d_{min} = \frac{\lambda_{water}}{4n_{glass}} = \frac{\lambda_{air}}{4n_{water}n_{glass}} = \frac{6.00\text{x}10^{-7}\text{m}}{4\cdot(1.33)(1.5)} = 7.50\text{x}10^{-8}\text{m}$$

   c. If the glass is 300 nm in thickness, which wavelength(s) of visible light, and corresponding m value(s), will result in destructive interference?

Ans. $$\lambda_{air} = \frac{2d_{min}n_{water}n_{glass}}{m} = \frac{\left(2(3.00\text{x}10^{-7}\text{m})(1.33)(1.5)\right)}{m} = \frac{1.20\text{x}10^{-6}\text{m}}{m}$$

The wavelength range for visible light is roughly 400 nm < λ_{air} < 700 nm,

m = 2: λ_{air} = 599 nm

m = 3: λ_{air} = 400 nm

   d. If the glass is 300 nm in thickness, which wavelength(s) of visible light, and corresponding m value(s), will result in constructive interference?

Ans. $$\lambda_{air} = \frac{2d_{min}n_{water}n_{glass}}{m} = \frac{\left(2(3.00\text{x}10^{-7}\text{m})(1.33)(1.5)\right)}{(m + \frac{1}{2})} = \frac{1.20\text{x}10^{-6}\text{m}}{(m + \frac{1}{2})}$$

The wavelength range for visible light is roughly 400 nm < λ_{air} < 700 nm,

m = 2: λ_{air} = 480 nm