Thermal Expansion:

1. A wedding ring composed of pure gold (inner diameter = 1.5 x 10^{-2} m) is placed on a person's finger (diameter = 1.5 x 10^{-2} m). Both the ring and the finger are at 27.0°C.

a. The finger and ring are placed in cold running water (5.0°C). Determine the contraction of the ring and the finger, respectively. Determine the ratio of $\Delta L_{finger}/\Delta L_{ring}$. Based on your calculation, is the ring looser or tighter?

b. The finger and ring are placed in warm running water (40.0°C). Determine the ratio of $\Delta L_{finger}/\Delta L_{ring}$. Based on your calculation, is the ring looser or tighter?

c. Based on (a) and (b), which approach, warm vs. cold water, would be the most effective for removing a tight ring from a finger? Why?

Heat Capacity & Heat Transformation:

2. A lead bullet (m=0.050 kg & V= 5.00×10^{-6} m^3) at 20.0°C impacts a block, made of an ideal thermal insulator, and comes to rest at its center. After impact, the temperature of the bullet is 327 °C.

a. How much heat was needed to raise the bullet to its final temperature?

Ans. $Q = cm\Delta T = \left(128 \frac{J}{kg°C}\right)(0.050kg)(307°C) = 1965\ J$

b. What is the coefficient of volume expansion (β) for the bullet?

Ans. $\beta = 3\alpha = 3\left(29\times10^{-6}\,°C^{-1}\right) = 87\times10^{-6}\,°C^{-1}$

c. What is the volume of the bullet after it comes to rest?

Ans. $\Delta V = V_o\beta\Delta T = \left(5.00\times10^{-6}m^3\right)\left(87\times10^{-6}\,°C^{-1}\right)(307°C) = 1.33\times10^{-7}m^3$

$V_o = V_o + \Delta V = 5.00\times10^{-6}m^3 + 1.3\times10^{-7}m^3 = 5.13\times10^{-6}m^3$

d. How much additional heat would be needed to melt the bullet?

Ans. $Q = mL_f = (0.050kg)\left(23.2\times10^3 \frac{J}{kg°C}\right) = 11.6\times10^3\ J$

e. How fast was the bullet traveling prior to hitting the block? Assume that all mechanical energy in the bullet is translational kinetic energy, prior to contact with the block and no energy is lost to the block.

Ans. Assuming that all heat gain by bullet is due to loss of kinetic energy, ΔK=1965J, and the final kinetic energy for the bullet is K_f=0J,

then $K_i = \Delta K - K_f$ =1965J-0J=1965J

3. A 0.500 kg piece of copper at an initial temperature of 20.0°C is placed in a water bath and the temperature of the metal is raised to 100.0°C.

a. How much heat was required to raise the temperature of the copper?

Ans. $Q = cm\Delta T = \left(386\,\frac{J}{kg^{\circ}C}\right)(0.500kg)(80^{\circ}C) = 1.54\times10^{4}\ J$

b. How much more heat would be required to raise the copper to its melting point?

Ans. The melting point for copper is $T_{melt}=1083^{o}C$:

$$Q = cm\Delta T = \left(386\,\frac{J}{kg^{\circ}C}\right)(0.500kg)(983^{\circ}C) = 1.90\times10^{5}\ J$$

c. How much heat would be required to completely melt the piece of copper, from an initial temperature of 100.0°C?

Ans.
$$Q_{tot} = Q_{\Delta T}+Q_{melt} = 1.90\times10^{5}\ J + (0.500kg)\left(2.07\times10^{5}\,\frac{J}{kg}\right)$$
$$Q_{tot} = 1.90\times10^{5}\ J + 1.04\times10^{5}\ J = 2.94\times10^{5}\ J$$

d. The piece of copper in (a) is then placed in a thermally isolated container, called a calorimeter, containing 1.00 kg of water initially at 20.0°C. What is the equilibrium temperature of the copper/water system?

Ans.

$$Q_{H_2O} = -Q_{Cu} \Rightarrow c_{H_2O}m_{H_2O}\Delta T_{H_2O} = -c_{Cu}m_{Cu}\Delta T_{Cu} \Rightarrow c_{H_2O}m_{H_2O}(T-20.0^{o}C) = c_{Cu}m_{Cu}(100.0^{o}C - T)$$

$$T = \frac{\left(\frac{c_{Cu}m_{Cu}}{c_{H_2O}m_{H_2O}}(100.0^{o}C) + 20.0^{o}C\right)}{\left(1+\frac{c_{Cu}m_{Cu}}{c_{H_2O}m_{H_2O}}\right)} = 23.5^{\circ}C$$

e. Suppose the piece of copper from (a) were placed in a calorimeter containing 0.500 kg of an unknown liquid initially at 20.0°C. The equilibrium temperature of the copper/liquid system is 30.9°C. What is the specific heat capacity of the unknown liquid?

Ans. $c_{?} = \frac{c_{Cu}m_{Cu}\Delta T_{Cu}}{m_{?}\Delta T_{?}} = \frac{\left(386\,\frac{J}{kg^{\circ}C}\right)(0.500kg)(69.1^{\circ}C)}{(0.500kg)(10.9^{\circ}C)} = 2450\,\frac{J}{kg^{\circ}C}$

f. What is the identity of the unknown liquid?

Ans. Probably ethyl alcohol…(verify using your textbook)

4. Suppose a 0.70 kg piece of iron and a 0.50 kg piece of copper (both at initial temperature of 100.0°C) were placed together in a calorimeter containing 1.00 kg water (initially at 20.0°C). What is the final temperature of the water in the calorimeter?

Ans.

$$Q_{H_2O} = -Q_{Al} - Q_{Cu} \Rightarrow c_{H_2O}m_{H_2O}\Delta T_{H_2O} = -c_{Al}m_{Al}\Delta T_{Al} - c_{Cu}m_{Cu}\Delta T_{Cu}$$

$$\Rightarrow c_{H_2O}m_{H_2O}\left(T-20.0^{o}C\right) = \left(c_{Al}m_{Al}+c_{Cu}m_{Cu}\right)\left(100.0^{o}C - T\right)$$

$$T = \frac{\left(\frac{c_{Al}m_{Al}+c_{Cu}m_{Cu}}{c_{H_2O}m_{H_2O}}\left(100.0^{o}C\right)+20.0^{o}C\right)}{\left(1+\frac{c_{Al}m_{Al}+c_{Cu}m_{Cu}}{c_{H_2O}m_{H_2O}}\right)}$$

$$T = \frac{\left(\frac{\left(900\,\frac{J}{kg\cdot{}^{o}C}\right)(0.70kg)+\left(386\,\frac{J}{kg\cdot{}^{o}C}\right)(0.50kg)}{\left(4186\,\frac{J}{kg\cdot{}^{o}C}\right)(1.00kg)}\right)\left(100.0^{o}C\right)+20.0^{o}C}{1+\left(\frac{\left(900\,\frac{J}{kg\cdot{}^{o}C}\right)(0.70kg)+\left(386\,\frac{J}{kg\cdot{}^{o}C}\right)(0.50kg)}{\left(4186\,\frac{J}{kg\cdot{}^{o}C}\right)(1.00kg)}\right)} = 33.1^{o}C$$

5. A 0.500 kg glass (c=840 J/kg°C) containing 1.00 L of water (at 20.0°C) is filled with 0.100 kg of ice (at -5.0°C).

a. What is the mass of the liquid water initially in the glass?

Ans. $m_{H_2O}=\rho_{H_2O}V_{H_2O}= \left(1000\,\frac{kg}{m^3}\right)\left(1.0\text{x}10^{-3}m^3\right)=1.0\text{ kg}$

b. What is the equilibrium temperature of the water and glass when all of the ice has melted? Ignore any heat gained from or lost to the surroundings.

Ans.

$$Q_{ice}+Q_{melt}+Q_{\substack{melted\\ice}} = -Q_{H_2O}-Q_{glass}$$

$$m_{ice}L_{ice}+c_{ice}m_{ice}\left(0.0^{o}C + 5.0^{o}C\right)+c_{H_2O}m_{ice}\left(T-0.0^{o}C\right)=\left(c_{H_2O}m_{H_2O}+c_{glass}m_{glass}\right)\left(20.0^{o}C - T\right)$$

$$\left(c_{H_2O}m_{ice}+c_{H_2O}m_{H_2O}+c_{glass}m_{glass}\right)T = \left(c_{H_2O}m_{H_2O}+c_{glass}m_{glass}\right)\left(20.0^{o}C\right)-m_{ice}L_{ice}-c_{ice}m_{ice}\left(5.0^{o}C\right)$$

$$\Rightarrow T = \frac{\left(c_{H_2O}m_{H_2O}+c_{glass}m_{glass}\right)\left(20.0^{o}C\right)-m_{ice}L_{ice}-c_{ice}m_{ice}\left(5.0^{o}C\right)}{\left(c_{H_2O}m_{ice}+c_{H_2O}m_{H_2O}+c_{glass}m_{glass}\right)} = 11.5^{o}C$$

c. A person then drinks all of the water in the glass. How much heat does the water gain as it is warmed up to 37.0°C in the digestive tract of the person?

Ans. $Q_{H_2O}= c_{H_2O}m_{H_2O}\Delta T_{H_2O} = \left(4186\frac{J}{kg^{o}C}\right)(1.1kg)\left(37.0^{o}C - T\right)=1.17\times10^{5}J$

1st Law of Thermodynamics:

6. An enclosed gas performs the following 3 step closed cycle.

a) Calculate the work performed by the system for:

i) A to B

Ans. Since P is linear from A to B:

$$W_{A\to B}=P_{avg}\Delta V$$

$$W_{A\to B}=\left(\frac{1.20x10^5Pa+1.02x10^5Pa}{2}\right)(1.0m^3)$$

$$W_{A\to B}=1.11x10^5N\cdot m$$

ii) B to C

Ans. P is linear from B to C:

$$W_{B\to C}=P_{avg}\Delta V$$

$$W_{B\to C}=(1.20x10^5Pa)(-1.0m^3)$$

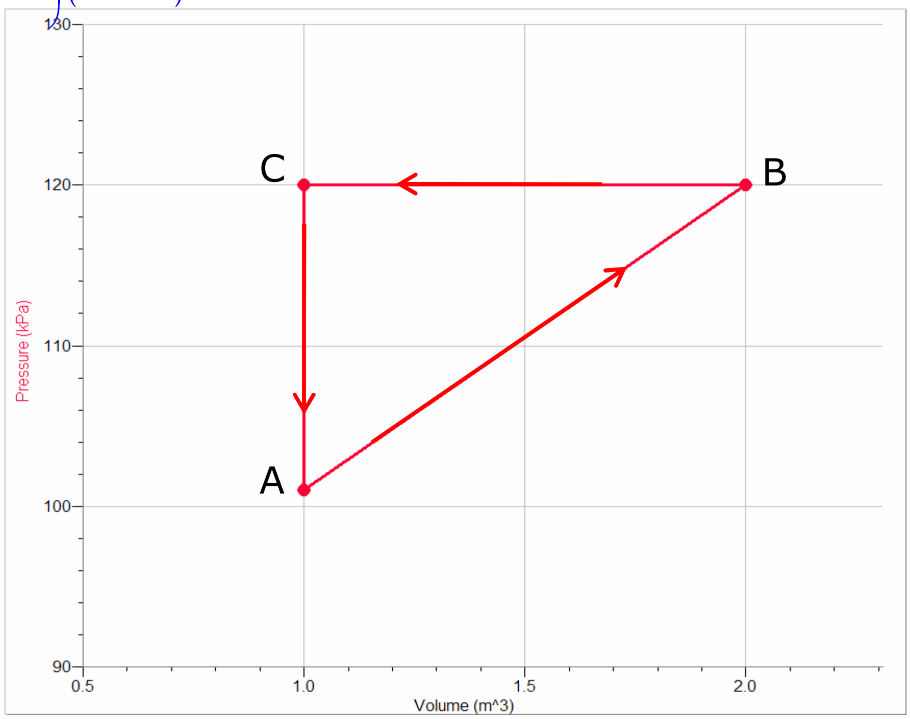
$$W_{B\to C}=-1.20x10^5N\cdot m$$

iii) C to A

Ans. P is linear from C to A:

$$W_{C\to A}=P_{avg}\Delta V$$

$$W_{C\to A}=0\ N\cdot m$$



b) What is the total work performed by the closed cycle?

Ans. W_{net} is the area enclosed in the PV graph or alternatively:

$$W_{Net}=W_{A\to B}+W_{B\to C}+W_{C\to A}$$

$$W_{Net}=-9.x10^3N\cdot m$$

c) Is the net work performed by this system over a complete cycle positive or negative? Explain.

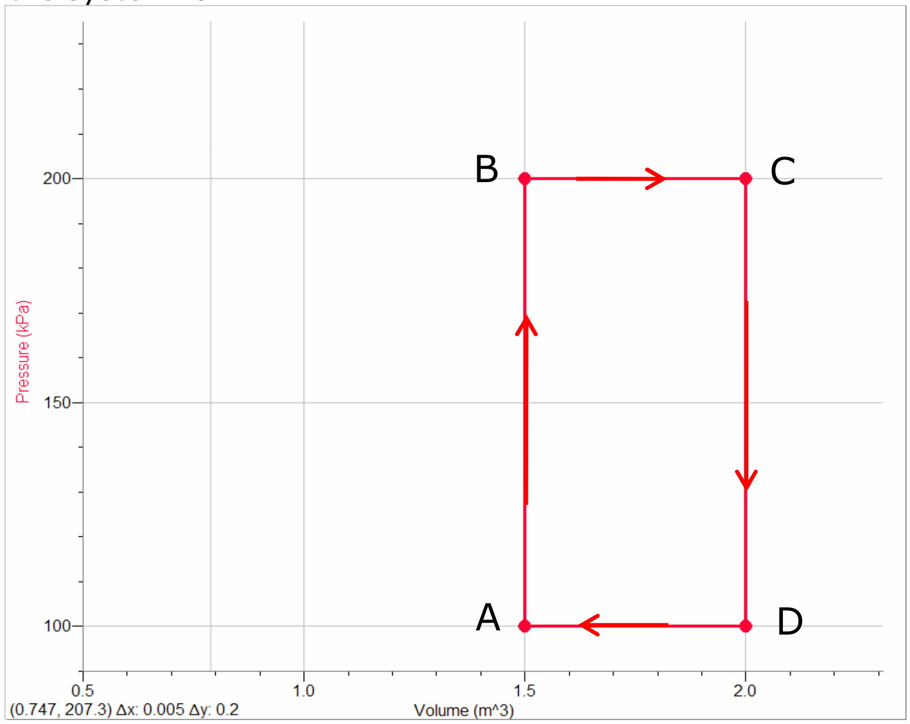
Ans. W_{net} is negative. Net work is performed ON the system, since the closed cycle operates counter-clockwise.

d) Calculate the total heat absorbed by the system during 1 cycle.

Ans. Since this is a closed cycle, ΔE_{int} =0J, therefore Q = W_{net}=-9x10^{3}J. Heat is released(lost) by the system during one cycle.

7. An enclosed gas performs the following 3 step closed cycle.

a) Calculate the work performed by the system for:



i) A to B

Ans. $W_{A\to B}=P_{avg}\Delta V=0\ N\cdot m$

ii) B to C

Ans.

$$W_{B\to C}=P_{avg}\Delta V$$
$$W_{B\to C}=\left(2.00x10^5Pa\right)\left(0.5m^3\right)$$
$$W_{B\to C}=1.00x10^5N\cdot m$$

iii) C to D

Ans. $W_{C\to D}=P_{avg}\Delta V=0\ N\cdot m$

ii) D to A

Ans.

$$W_{D\to A}=P_{avg}\Delta V$$
$$W_{D\to A}=\left(1.00x10^5Pa\right)\left(0.5m^3\right)$$
$$W_{D\to A}=-5.0x10^4N\cdot m$$

b) What is the total work performed by the closed cycle?

Ans. W_{net} is the area enclosed in the PV graph or alternatively:

$$W_{Net}=W_{A\to B}+W_{B\to C}+W_{C\to D}+W_{D\to A}$$
$$W_{Net}=5.0x10^4N\cdot m$$

c) Is the net work performed by this system over a complete cycle positive or negative? Explain.

Ans. W_{net} is positive, net work is performed BY the system (the closed cycle operates clockwise).

d) Calculate the total heat absorbed by the system during 1 cycle.

Ans. Since this is a closed cycle, ΔE_{int} =0J, therefore Q = W_{net}=+5.x10^{4}J. Heat is gained (absorbed) by the system during one cycle.

Heat Transfer & Cooking:

8. A 0.2 cm iron pan is used to fry a cylindrical piece of meat (mass = 0.12 kg, 1.5 cm thickness and 10.0 cm diameter). The temperature of the pan is heated to a temperature of 350 °F. The thermal conductivity of the meat is 0.40 W/(m·K) and the emissivity of the meat is 0.9. Assume the specific heat capacity of the meat is 3500 J/(kg·K).

a) What is the temperature of the pan in Celsius & Kelvin?

Ans. $$T_{°C} = \left(\frac{5°C}{9°F}\right)\left(T_{°F} - 32°F\right) = \left(\frac{5°C}{9°F}\right)\left(350°F - 32°F\right) = 177.°C$$

$$T_K = \left(\frac{1K}{1°C}\right)\left(177°C\right) + 273.15K = 450K$$

b) What is the area of the frying surface of the meat in m^2?

Ans. $$A = \pi r^2 = \pi\left(\frac{0.10m}{2}\right)^2 = 0.0079\ m^2$$

c) When the meat placed on the pan its temperature is raised to a constant temperature of 100°C and the opposite face of the meat initially is 5°C. What is the rate of conductive heat flow through the meat?

Ans. $$\frac{dQ}{dt} = k\left(\frac{A}{\Delta y}\right)dT = 19.9\,\tfrac{J}{s}$$

d) Why does the heat of the "hot surface" of the meat never get warmer than 100°C when the meat is moist?

Ans. The water on the surface limits the surface temperature until it has all evaporated.

e) When enough heat has passed through the meat, the opposite face of the meat will reach the temperature of the room (25°C), what is the temperature at the center of the meat?

Ans. Assuming the temperature profile through the meat is linear:

$$T_{middle} = \frac{T_{bottom} + T_{top}}{2} = 37.5°C$$

f) What is the rate of conductive heat flow through the meat in (e)?

Ans. $$\frac{dQ}{dt} = k\left(\frac{A}{\Delta y}\right)dT = 15.8\,\tfrac{J}{s}$$

g) How much total heat (energy) has the meat absorbed (compared to when the whole piece was at 5°C) when the outer surface is 25°C?

Ans. Assuming the temperature profile through the meat is linear:

$$Q = c_{meat} m_{meat}\left(T_{middle} - 5°C\right) = 1.37 \times 10^4 J$$

h) What is the net rate of radiative heat flow from the meat when the outer surface is at 5°C and 25°C respectively?

Ans. When meat surface temperature is T=5°C:

$$\frac{dQ}{dt} = \varepsilon\sigma A\left(T_{meat}^4 - T_{room}^4\right) = (0.9)\left(5.67\times10^{-8}\,\tfrac{J}{m^2\cdot K^4}\right)\pi\left(0.05m^2\right)\left[(273.15K)^4 - (298.15K)^4\right]$$

$$\frac{dQ}{dt} = -0.94\ W$$

When meat surface temperature is T=25°C:

$$\frac{dQ}{dt} = \varepsilon\sigma A\left(T_{meat}^4 - T_{room}^4\right) = (0.9)\left(5.67\times10^{-8}\,\tfrac{J}{m^2\cdot K^4}\right)\pi(0.05m)^2\left[(298.15K)^4 - (298.15K)^4\right]$$

$$\frac{dQ}{dt} = 0\ W$$

i) If the meat were left on the pan (same side) what would be the final "equilibrium" temperature of the outer surface of the meat? {Hint: Try using the equation solver feature of your (or your neighbor's) TI85/89 calculator}

Ans. This is a tough one!

$$\left(\frac{dQ}{dt}\right)_{cond} = \left(\frac{dQ}{dt}\right)_{net\ rad}$$

$$\left(\frac{kA}{\Delta y}\right)\left(T_{bottom} - T_{top}\right) = \varepsilon\sigma A\left(T_{top}^4 - T_{room}^4\right)$$

$$T_{top}^4 - T_{room}^4 = \left(\frac{k}{\varepsilon\sigma\Delta y}\right)\left(T_{bottom} - T_{top}\right) = \left(\frac{k}{\varepsilon\sigma\Delta y}\right)T_{bottom} - \left(\frac{k}{\varepsilon\sigma\Delta y}\right)T_{top}$$

$$T_{top}^4 = -\left(\frac{k}{\varepsilon\sigma\Delta y}\right)T_{top} + \left(\left(\frac{k}{\varepsilon\sigma\Delta y}\right)T_{bottom} + T_{room}^4\right)$$

$$\Rightarrow T_{top} \approx 357\ K \text{ or } 84^\circ C$$

{I solved this graphically, check it w/ your calculator...}

j) What is the net rate of radiative/conductive heat flow from the meat when the meat is in thermal equilibrium?

Ans. The net rate of thermal transfer into the meat is zero, since it is in thermal equilibrium. This can be checked by calculating the radiative heat loss, when meat surface temperature is T=357 K or 137°C:

$$\frac{dQ}{dt} = \varepsilon\sigma A\left(T_{surface}^4 - T_{room}^4\right) = (0.9)\left(5.67\times10^{-8}\,\tfrac{J}{m^2\cdot K^4}\right)\pi\left(0.05m^2\right)\left[(357K)^4 - (298.15K)^4\right]$$

$$\frac{dQ}{dt} = 3.4\ W$$

Compare this to the conductive heat flow across the meat:

$$\left(\frac{dQ}{dt}\right)_{cond} = \left(\frac{kA}{\Delta y}\right)\left(T_{bottom} - T_{top}\right)$$

$$\left(\frac{dQ}{dt}\right)_{cond} = \left(\frac{\left(0.4\,\frac{W}{m^2\cdot K}\right)\pi(0.05m)^2}{0.015m}\right)(373.15K - 357K) = 3.4W$$

9. The **Greenhouse Effect** can be analyzed using the radiation model of Ch 18. Assume that radiation is the primary mechanism of heat transfer between the earth and the universe, where both the earth and sun are perfect emitters (ε = 1.0). Use the following values (all other values can be found in the back of the textbook):

T_{sun} = 5350 K (the average temperature of the surface of the sun)

T_{earth} = 288 K (the average temperature of the surface of the earth)

a. Determine the rate at which energy is radiated by the sun.

Ans.
$$\frac{dQ}{dt} = \varepsilon\sigma AT^4 = (1.0)\left(5.67\times10^{-8}\,\tfrac{J}{m^2\cdot K^4}\right)4\cdot\pi\left(6.95\times10^8 m\right)^2(5350K)^4$$
$$\frac{dQ}{dt} = 2.58\times10^{26}\ W$$

b. How much of the sun's radiant energy reaches the earth? **Hint:** What is the intensity (power per unit area) at a distance from the sun of $d_{earth-sun}$= $1.50x10^{11}$m?

Ans. The intensity of sun's radiant energy at the earth is:

$$I = \frac{\left(\frac{dQ}{dt}\right)}{A} = \frac{2.58\times10^{26}\ W}{4\cdot\pi\left(1.50\times10^{11}m\right)^2} = 913\,\tfrac{W}{m^2}$$

The incident power is related to intensity x the effective cross sectional area of the earth:

$$\frac{dQ}{dt} = IA_{cross\ section} = \left(913\,\tfrac{W}{m^2}\right)\pi\left(6.37\times10^6 m\right)^2 = 1.16\times10^{17}W$$

c. How much of the sun's power is absorbed by the earth? **Note:** The earth's atmosphere absorbs ~30% of the total radiant energy from the sun.

Ans. $$\left(\frac{dQ}{dt}\right)_{absorbed} = (0.70)\left(1.16\times10^{17}W\right) = 8.15\times10^{16}W$$

d. How much power (at what energy rate) does the earth radiate?

Ans.
$$\left(\frac{dQ}{dt}\right)_{radiated} = \varepsilon\sigma AT^4 = (1.0)\left(5.67\times10^{-8}\,\tfrac{J}{m^2\cdot K^4}\right)4\cdot\pi\left(6.37\times10^6 m\right)^2(295K)^4$$
$$\left(\frac{dQ}{dt}\right)_{radiated} = 1.99\times10^{17}W$$

e. What fraction of the earth's radiant energy is reflected back to the surface and reabsorbed? This is the Greenhouse Effect!

Ans. Assuming the earth is in thermal equilibrium:

$$\left(\frac{dQ}{dt}\right)_{earth} = \left(\frac{dQ}{dt}\right)_{from\ sun} - \left(\frac{dQ}{dt}\right)_{radiated} + \left(\frac{dQ}{dt}\right)_{re\text{-}absorbed} = 0W$$
$$\left(\frac{dQ}{dt}\right)_{re\text{-}absorbed} = \left(\frac{dQ}{dt}\right)_{radiated} - \left(\frac{dQ}{dt}\right)_{from\ sun} = 1.16\times10^{17}W - 8.15\times10^{16}W$$
$$\left(\frac{dQ}{dt}\right)_{re\text{-}absorbed} = 3.48\times10^{16}W$$

f. If there were no reflection and re-absorption of radiant energy by the earth, estimate the average temperature of the earth.

Ans. The temperature (earth surface) for thermal equilibrium:

$$\left(\frac{dQ}{dt}\right)_{earth} = \left(\frac{dQ}{dt}\right)_{from\ sun} - \left(\frac{dQ}{dt}\right)_{radiated} = 0W$$

$$\left(\frac{dQ}{dt}\right)_{radiated} = \varepsilon\sigma AT^4 = (1.0)\left(5.67\times10^{-8}\tfrac{J}{m^2\cdot K^4}\right)4\cdot\pi\left(6.37\times10^6 m\right)^2 T^4 = 8.15\times10^{16}W$$

$$T = 230K$$

g. If the earth had no atmosphere at all, estimate the average temperature of the surface of the earth.

Ans. The temperature (earth surface) for thermal equilibrium:

$$\left(\frac{dQ}{dt}\right)_{radiated} = \varepsilon\sigma AT_{earth}^4 = 1.16\times10^{17}W$$

$$T_{earth} = \left(\frac{1.16\times10^{17}W}{\varepsilon\sigma A}\right)^{\frac{1}{4}} = 252\ K$$