1. Ethanol has a density of 659 kg/m^3. If the speed of sound in ethanol is 1162 m/s, what is its adiabatic bulk modulus?

Ans. $B_{ethanol} = \rho_{ethanol} v_{sound}^2 = 8.90\text{x}10^9 \ \frac{N}{m^2}$

2. A physics student measures the length of a long, slender, copper rod. The Young's modulus of copper is 1.1x10^{11} N/m^2; its density is 8890 kg/m^3, the time for the pulse to travel from one end to the other is 1.56 x 10^{-3} s. How long is the rod?

Ans. $v_{sound} = \sqrt{\frac{Y_{copper}}{\rho_{copper}}} = \frac{\Delta x}{\Delta t} \Rightarrow \Delta x = \Delta t \cdot \sqrt{\frac{Y_{copper}}{\rho_{copper}}} = 5.49 \text{ m}$

Sound: Intensity & Decibels

3. A blaring sound system generates 50 W of total power through a single spherical speaker.

a. What is the intensity of sound at a distance of 2.0 m from the speaker?

Ans. $I = \frac{P}{4\pi r^2} = \frac{50\text{W}}{4\pi(2\text{m})^2} = 0.995 \ \frac{W}{m^2}$

b. What is the intensity of sound at a distance of 10.0 m from the speaker?

Ans. $I = \frac{P}{4\pi r^2} = \frac{50\text{W}}{4\pi(10\text{ m})^2} = 0.040 \ \frac{W}{m^2}$

c. What is the intensity level, in decibels at a distance of 2.0 m from the speaker?

Ans. $\beta = (10\text{dB})\log\left(\frac{0.995 \ \frac{W}{m^2}}{10^{-12} \ \frac{W}{m^2}}\right) = 120 \text{ dB}$

d. What is the intensity level, in decibels at a distance of 10.0 m from the speaker?

Ans. $\beta = (10\text{dB})\log\left(\frac{0.040 \ \frac{W}{m^2}}{10^{-12} \ \frac{W}{m^2}}\right) = 106 \text{ dB}$

e. What is the difference in intensity levels in decibels at 2.0 m and 10.0 m from the speakers?

Ans. $\Delta\beta = (10\text{dB})\log\left(\frac{0.995 \ \frac{W}{m^2}}{0.040 \ \frac{W}{m^2}}\right) = 14 \text{ dB}$

f. In general, what is the mathematical relationship for the difference in decibel levels between any 2 distances from a sound source?

Ans. $\Delta\beta = \beta_2 - \beta_1 = (10\text{dB})\log(I_2 - I_1) = (10\text{dB})\log\left(\frac{I_2}{I_1}\right)$

g. How would the intensity of sound (W/m^2) compare between a 100 W sound system and a 50 W system (compared at 10.0 m from the speaker)?

Ans. $\frac{I_{100W}}{I_{50W}} = \frac{100\text{W}}{50\text{W}} = 2 \Rightarrow I_{100W} = 2I_{50W}$

h. How much "louder" in decibels would a 100 W sound system be at 10.0 m, compared to a 50 W system?

Ans. $\Delta\beta = (10\text{dB})\log\left(\frac{2I_{50W}}{I_{50W}}\right) = 3\text{ dB}$

4. The intensity of a spherical wave at a distance of 4.0 m from the source is 120 W/m^2. What is the intensity at a point 9.0 m away from the source?

Ans.

$$I_{4m} = \frac{P}{4\pi(4m)^2} = 120\tfrac{W}{m^2} \text{ and } I_{9m} = \frac{P}{4\pi(9m)^2}$$

$$\frac{I_{9m}}{I_{4m}} = \frac{I_{9m}}{120\tfrac{W}{m^2}} = \frac{(4m)^2}{(9m)^2} \Rightarrow I_{9m} = \left(120\tfrac{W}{m^2}\right)\frac{(4m)^2}{(9m)^2} = 23.7\tfrac{W}{m^2}$$

Application of Sound: Ultrasound Imaging

5. A device utilizing ultrasound generates a sound wave with frequency of 5.0x10^6 Hz. The device sends brief sound pulses through the body tissue. Measurements of the resultant echoes are used to produce an image (sonogram) of a person's abdomen. The speed of sound in the body is 1540 m/s.

a. What is the wavelength of the ultrasonic wave used by this device?

Ans. $\lambda = \frac{v_{sound}}{f} = \frac{1540\tfrac{m}{s}}{5.0\text{x}10^6\text{Hz}} = 3.08\text{x}10^{-4}\text{ m}$

b. Following a single pulse, an echo is detected by the monitor 9.5 x 10^{-6} seconds later. How deep in the tissue is the object/interface that produced the echo located?

Ans. $\Delta x = \frac{v_{sound}\Delta t}{2} = \frac{(1540\text{ m/s})(9.5\text{ x }10^{-6}\text{ s})}{2} = 0.0073\text{ m}$

c. How much time would elapse between echoes produced by the front and back of an internal organ that is 0.030 m in thickness, as measured from the front of the ultrasound transducer?

Ans. $\Delta t = \frac{\Delta x}{v_{sound}} = \frac{2(0.030\text{m})}{1540\tfrac{m}{s}} = 3.90\text{x}10^{-5}\text{s}$

d. Using the speed of sound in human tissue and assuming that the tissue is essentially water, determine the adiabatic bulk modulus in the body.

Ans. $B_{body} = \rho_{body}v_{sound}^2 = \left(998\tfrac{kg}{m^3}\right)\left(1540\tfrac{m}{s}\right)^2 = 2.37\text{x}10^9\tfrac{N}{m^2}$

e. Would it be possible to observe an object of size 0.1 mm using this ultrasound wave frequency? If not, what would be the minimum frequency necessary to observe such an object?

Ans. No, the wavelength of the ultrasound is 3.08x10^{-4}m. In order to resolve an object, the wavelength must be smaller than the object. Therefore, the minimum frequency needed to image a 0.1 mm object would be slight greater than 1.54x10^7 Hz, probably more like 2x10^7 Hz.

The Doppler Effect (moving source):

6. A sound wave (f =300 Hz) produced by a moving source (v_{source} = 40 m/s) is observed by a stationary observer. The speed of sound (v_{sound}) in ambient air is 345 m/s.

a. As the source moves toward the observer, how far does the source travel between subsequent wave pulses?

Ans. One period following the first wave pulse, the source will emit a second wave pulse. Therefore, the distance traveled by the source will be:

$$\Delta x = v_{source}\Delta t = \left(40\tfrac{m}{s}\right)\left(\frac{1}{300Hz}\right) = 0.132\text{ m}$$

b. What is the apparent wavelength of the sound wave as perceived by the observer?

Ans. $$\lambda' = \lambda - \Delta x = \left(\frac{345\tfrac{m}{s}}{300Hz}\right) - 0.321m = 1.02\text{ m}$$

c. What is the frequency (f') of sound perceived by the observer? *Note, when the source is in motion, the speed of the sound waves remains the same to both the observer and the source.*

Ans. $$f' = \frac{v_{sound}}{\lambda'} = 339Hz \quad \text{or} \quad f' = \frac{f}{\left(1-\frac{v_{source}}{v_{sound}}\right)} = 339\text{ Hz}$$

d. Does the frequency of the perceived sound waves increase or decrease when the source is moving toward the observer? When the source is moving away from the observer?

Ans. f' increases as the source approaches the observer & decreases as the source moves away from the observer.

e. What is the mathematical relationship between the frequencies f (actual) and f' (perceived) in terms of v_{source} and v_{sound} when the source is moving toward the observer?

Ans. $$f' = \frac{f}{\left(1 - \frac{v_{source}}{v_{sound}}\right)}$$

f. What is the mathematical relationship between the frequencies f (actual) and f' (perceived) in terms of v_{source} and v_{sound} when the source is moving away from the observer?

Ans. $$f' = \frac{f}{\left(1 + \frac{v_{source}}{v_{sound}}\right)}$$

The Doppler Effect (moving observer):

7. A sound wave (f =350 Hz) produced by a stationary source is observed by a moving observer (v_{source} = 35 m/s). Assume the speed of sound (v_{sound}) in ambient air is 345 m/s.

a. What is the wavelength of the sound wave?

Ans. $\lambda = \left(\frac{345\frac{m}{s}}{350Hz}\right) = 0.986 \text{ m}$

b. According to the observer, as he/she moves toward the source, how fast does the sound wave appear to travel toward her/him?

Ans. $\Delta v = v' - v_{sound} = v_{obs} \Rightarrow v' = v_{sound} + v_{obs} = 380\frac{m}{s}$

c. What is the apparent frequency (f') of the sound wave as perceived by the observer? *Note, when the observer is in motion, the wavelength of the sound remains the same to both the observer and the source.*

Ans. $f' = \frac{v'}{\lambda} = \frac{380\frac{m}{s}}{0.986 \text{ m}} = 386Hz$ or $f' = f\left(1 + \frac{v_{obs}}{v_{sound}}\right) = 386Hz$

d. What is the mathematical relationship between the frequencies f (actual) and f' (perceived) in terms of $v_{observer}$ and v_{sound} when the observer is moving toward the source?

Ans. $f' = f\left(1 + \frac{v_{obs}}{v_{sound}}\right)$

e. What is the mathematical relationship between the frequencies, f (actual) and f' (perceived) in terms of $v_{observer}$ and v_{sound} when the observer is moving away from the source?

Ans. $f' = f\left(1 - \frac{v_{obs}}{v_{sound}}\right)$

f. Does the frequency of the perceived sound waves increase or decrease when the observer is moving toward the source? When the observer is moving away from the source?

Ans. f' increases as the observer approaches the source & decreases as the observer moves away from the source.

Application of Doppler Effect: Radar Speed Detection

8. A police officer uses a radar gun that generates a microwave with frequency of $36.0\text{x}10^9$ Hz. The device sends brief microwave pulses toward observed moving vehicles. The radar gun then measures the frequency of incoming microwaves reflected off the target vehicle and determines the resultant Doppler frequency shift (Δf). Based on the frequency shift, the radar gun then calculates the oncoming speed of the vehicle. Note: the speed of a microwave ($v_{microwave}$) in air is $3.0\text{x}10^8$ m/s.

a. What is the wavelength of the microwave wave used by this device?

Ans. $\lambda = \left(\dfrac{3.0\text{x}10^8\,\frac{m}{s}}{36\text{x}10^9\text{Hz}}\right) = 0.0083 \text{ m}$

b. The stationary officer targets a car traveling directly toward him and measures a frequency shift of **4000 Hz**. How fast is the car traveling?

Ans. *Note that I changed Δf from 2000Hz to 4000Hz.*

$$v_{car} = c\frac{\Delta f}{2f} = \left(3.0\text{x}10^8\,\tfrac{m}{s}\right)\left(\frac{4000\text{Hz}}{72\text{x}10^9\text{Hz}}\right) = 16.6\,\tfrac{m}{s}$$

c. The posted speed limit is 30 miles/hr. Is the car exceeding the speed limit?

Ans. The car is speeding since: $\left(30\,\frac{mi}{hr}\right)\left(\dfrac{1\text{hr}}{3600\text{ s}}\right)\left(\dfrac{1609\text{ m}}{1\text{ mi}}\right) = 13.4\,\frac{m}{s}$

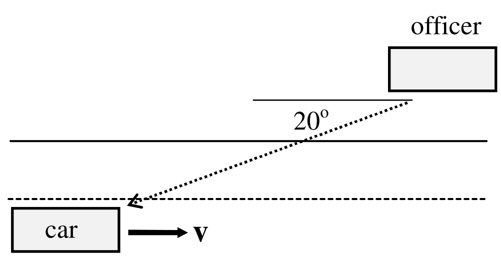
d. Usually, this particular officer will not issue a speeding ticket unless the targeted car is traveling over 5 miles/hr above the posted speed limit. Does the officer write this vehicle a ticket?

Ans. yes, the vehicle is traveling 7.2 mph over the speed limit.

e. What would be the minimum frequency shift that would need to be measured to just exceed the speed limit by 5 miles/hr?

Ans. *Note that I changed Δf from 2000Hz to 4000Hz.*

$$\Delta f = \frac{2v_{car}f}{c} = \frac{2\left(13.4\,\frac{m}{s}\right)\left(36\text{x}10^9\text{Hz}\right)}{\left(3.0\text{x}10^8\,\frac{m}{s}\right)} = 3754 \text{ Hz}$$



f. The same officer targets a car traveling at a 20° angle with respect to the line of sight. A frequency shift of 1500Hz is recorded for the vehicle. How fast is the car traveling toward the officer?

Ans. $$v_{measured} = \frac{c\cdot\Delta f}{2\cdot f} = \frac{\left(3.0\text{x}10^8\,\frac{m}{s}\right)\left(1500\text{Hz}\right)}{\left(72\text{x}10^9\text{Hz}\right)} = 6.25\,\frac{m}{s}$$

h. What is the actual speed of the car?

Ans. $$v_{car} = \frac{v_{measured}}{\cos 20^\circ} = \frac{6.25\,\frac{m}{s}}{\cos 20^\circ} = 6.65\,\frac{m}{s}$$