Traveling Waves

1. A particular wave traveling along a string is described by:

\[ y(x,t) = 0.0060\text{m}\sin\left(0.006\text{rad}\right)x + (600\text{rad/s})t \]

a. How much time does any particular point in the string take to move between the displacements \( y=+2 \text{ mm} \) and \( y=-2 \text{ mm} \)?

**Ans.** There are 2 interpretations to this problem.

**Interpretation 1:**

The period, \( T \), for this wave is: \( T = \frac{2\pi}{\omega} = 0.0105 \text{ s} \)

The displacement from \(+2\text{ mm}\) to \(-2\text{ mm}\) is \( \frac{1}{2} \lambda \) and therefore the time elapsed is:

\[ t = \frac{T}{2} = 0.00525 \text{ s} \]

**Interpretation 2:**

This one is more complex. The phase difference between \(+2\text{ mm}\) to \(-2\text{ mm}\) is:

\[ \phi = \pi \left( \frac{0.0040\text{ m}}{0.0120\text{ m}} \right) = \frac{\pi}{3} \]

The time elapsed is related to period, \( T \), by:

\[ t = \frac{T}{2\pi} = \frac{3\pi}{2\pi} \Rightarrow t = \frac{T}{6} = 0.0018 \text{ s} \]

b. Determine the amplitude, frequency, period and wave speed for this traveling wave.

**Ans.** \( y_m = 0.0060 \text{ m}, \ f = \frac{\omega}{2\pi} = 95.5 \text{ Hz}, \ T = \frac{1}{f} = 0.0105 \text{ s}, \) and \( v = \frac{\omega}{k} = 1.05 \times 10^5 \text{ m/s} \)

c. What is the equation for the transverse velocity of any particular point along the string?

**Ans.** \( v = \frac{dy}{dt} = (3.6 \text{ m/s})\sin\left[(0.006\text{ rad/m})x + (600\text{ rad/s})t\right] \)

d. What is the maximum transverse velocity of any particular point along the string?

**Ans.** \( v_m = \omega y_m = 3.6 \text{ m/s} \)

2. A sinusoidal wave of frequency 500 Hz has a speed of 350 m/s.

a. How far apart are 2 points that differ in phase by \( \pi/3 \) radians?

**Ans.** \( \lambda = \frac{350 \text{ m/s}}{500 \text{ Hz}} = 0.7 \text{ m} \Rightarrow \frac{0.7 \text{ m}}{\Delta x} = \frac{2\pi}{(\pi/3)} \Rightarrow \Delta x = 0.117 \text{ m} \)

b. What is the phase difference between 2 displacements at a certain point at times 1.000 ms apart?

**Ans.** \( T = \frac{1}{500 \text{ Hz}} = 0.002 \text{ s} \Rightarrow \frac{\Delta t}{T} = \frac{0.001 \text{ s}}{0.002 \text{ s}} = \frac{\phi}{2\pi} \Rightarrow \phi = \pi \text{ rad} \)
Waves on a Stretched String

3. The linear density of a string is $1.5 \times 10^{-4}$ kg/m. The equation of a transverse wave on a string is:

$$y(x,t) = (0.025\text{m})\sin((2.0 \text{ rad/m})x - (30 \text{ rad/s})t)$$

a. What is the wave speed?

Ans. $v = \frac{\omega}{k} = \frac{30 \text{ rad/s}}{2.0 \text{ rad/m}} = 15 \text{ m/s}$

b. What is the tension in the string?

Ans. $F_T = \mu v^2 = (1.5 \times 10^{-4}\text{kg/m})(15 \text{ m/s})^2 = 0.0338 \text{ N}$

c. What is the average power for this wave?

Ans. $P = \frac{1}{2} \mu v \omega y \frac{y}{m^2} = 0.00064 \text{ W}$

4. The equation of a transverse wave on a string is:

$$y(x,t) = (2.0\text{mm})\sin((20 \text{ rad/m})x - (600 \text{ rad/s})t)$$

The tension in the string is 15 N.

a. What is the wave speed?

Ans. $v = \frac{\omega}{k} = \frac{600 \text{ rad/s}}{20 \text{ rad/m}} = 30 \text{ m/s}$

b. What is the linear mass density of the string, in grams per meter?

Ans. $\mu = \frac{F_T}{v^2} = \frac{15\text{N}}{(30 \text{ m/s})^2} = 1.67 \times 10^{-2} \text{ kg/m}$

c. Determine the average power transmitted by this wave.

Ans. $P = \frac{1}{2} \mu v \omega y \frac{y}{m^2} = 0.36 \text{ W}$

d. Demonstrate that this transverse wave satisfies the wave equation:

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2}.$$ 

The Wave Equation:

5. Consider the following wave: $y(x,t) = (0.05\text{m})\sin((5.0 \text{ rad/m})x - (10 \text{ rad/s})t)$

a. Verify that this wave satisfies the wave equation:

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2}.$$ 

Ans. $\frac{\partial^2 y}{\partial x^2} = -(5.0 \text{ rad/m})^2(0.05\text{m})\sin((5.0 \text{ rad/m})x - (10 \text{ rad/s})t) = -(1.25\text{m}^{-1})\sin((5.0 \text{ rad/m})x - (10 \text{ rad/s})t)$

$$\frac{1}{v^2} \frac{\partial^2 y}{\partial t^2} = -(10 \text{ rad/s})^2(0.05\text{m}) (2 \text{ m/s})^2 \sin((5.0 \text{ rad/m})x - (10 \text{ rad/s})t) = -(1.25\text{m}^{-1})\sin((5.0 \text{ rad/m})x - (10 \text{ rad/s})t)$
b. Use the wave equation to determine the speed of the wave.

\[ \frac{(10 \text{ rad} \, s^{-2})(0.05 \text{ m})}{v^2} = (5.0 \text{ rad} \, m^{-1})(0.05 \text{ m}) \Rightarrow v^2 = \frac{10 \text{ rad} \, s^{-2}}{5.0 \text{ rad} \, m^{-1}} = 2.0 \text{ m} \, s^{-1} \]

\[ v = \sqrt{v^2} = \sqrt{2.0} \text{ m} \, s^{-1} \]

\[ v = \sqrt{2.0} \text{ m} \, s^{-1} \]

\[ c. \text{ Calculate the frequency, wavelength and period for this wave.} \]

\[ y_m = 0.05 \text{ m}, \quad f = \frac{\omega}{2\pi} = 1.59 \text{ Hz} \]

\[ \lambda = \frac{k}{2\pi} = 0.796 \text{ m} \]

\[ T = \frac{1}{f} = 0.629 \text{ s} \]

d. Consider a 2\textsuperscript{nd} wave (with same frequency and wavelength) traveling in the same direction, where \( y_{2\text{max}} = 0.01 \text{ m} \) and \( \phi_2 = \pi/3 \). Construct a phasor diagram for the combined waves.

\[ \text{Ans.} \]

\[ e. \text{ What is the amplitude of the resultant wave?} \]

\[ y' = \sqrt{y_1^2 + y_2^2 - 2y_1y_2\cos\left(\pi - \frac{\pi}{3}\right)} = 0.0557 \text{ m} \]

\textbf{Standing Waves}

7. A string is attached to a string vibrator (\( f = 120 \text{ Hz} \)) at one end and fixed at the other. The length of the string is 1.2 m.

\[ a. \text{ What wavelengths will satisfy the standing wave condition for the string?} \]

\[ \text{Ans.} \quad L = n\left(\frac{\lambda}{2}\right) \quad \text{where } n=1,2,3,\ldots \text{ etc.} \]

\[ b. \text{ What is the maximum wave speed that will produce a standing wave in this string?} \]

\[ \text{Ans. For the same frequency, the greatest wavelength will produce the maximum speed:} \]

\[ \text{when } n=1, \quad \lambda = \frac{2L}{n} = 2.4 \text{ m} \quad \text{and} \quad v = \frac{\lambda}{f} = (2.4 \text{ m})(120 \text{ Hz}) = 288 \text{ m} \, s^{-1} \]

\[ c. \text{ What string tension will correspond to the wave in (b) is the linear density of the string is } 2.5 \times 10^{-4} \text{ kg/m?} \]

\[ \text{Ans.} \quad F_T = \mu v^2 = (2.5 \times 10^{-4} \text{ kg/m})(288 \text{ m} \, s^{-1})^2 = 20.7 \text{ N} \]

\[ d. \text{ What string tension would correspond to a standing wave that produces 6 nodes in the string?} \]

\[ \text{Ans. For 6 nodes, } n=5, \quad \lambda = \frac{2L}{n} = 0.48 \text{ m} \quad \text{and} \quad v = \frac{\lambda}{f} = (0.48 \text{ m})(120 \text{ Hz}) = 57.6 \text{ m} \, s^{-1} \]

\[ \text{The tension in the string would be: } F_T = \mu v^2 = (2.5 \times 10^{-4} \text{ kg/m})(57.6 \text{ m} \, s^{-1})^2 = 0.829 \text{ N} \]