Traveling Waves

1. A particular wave traveling along a string is described by:

$$y(x,t) = (0.0060\text{m})\sin\left(\left(0.006\tfrac{\text{rad}}{\text{m}}\right)x + \left(600\tfrac{\text{rad}}{\text{s}}\right)t\right)$$

a. How much time does any particular point in the string take to move between the displacements y=+2 mm and y=-2 mm?

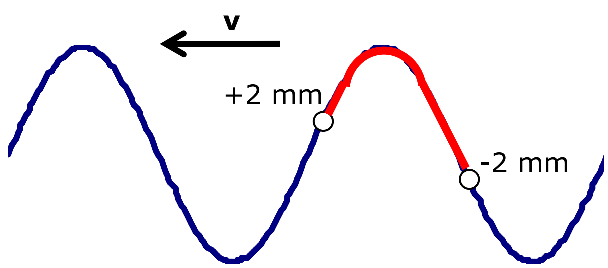
Ans. There 2 interpretations to this problem.

Interpretation 1:

The period, T, for this wave is: $T=\dfrac{2\pi}{\omega}=0.0105$ s

The displacement from +2mm to -2mm is ½ λ and therefore the time elapsed is:

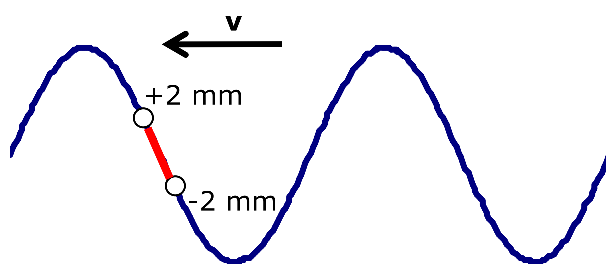
$$t = \frac{T}{2} = 0.00525 \text{ s}.$$



Interpretation 2:

This one is more complex. The phase difference between +2mm to -2mm is:

$$\phi = \pi\left(\frac{0.0040\text{m}}{0.0120\text{m}}\right) = \frac{\pi}{3}$$



The time elapsed is related to period, T, by:

$$\frac{t}{T}=\frac{\phi}{2\pi}=\frac{\left(\frac{\pi}{3}\right)}{2\pi} \Rightarrow t = \frac{T}{6} = 0.0018 \text{ s}$$

b. Determine the amplitude, frequency, period and wave speed for this traveling wave.

Ans. $y_m=0.0060$ m, $f=\dfrac{\omega}{2\pi}=95.5$ Hz, $T=\dfrac{1}{f}=0.0105$ s, and $v=\dfrac{\omega}{k} = 1.05\text{x}10^5\,\tfrac{\text{m}}{\text{s}}$

c. What is the equation for the transverse velocity of any particular point along the string?

Ans. $v = \dfrac{dy}{dt} = \left(3.6\tfrac{\text{m}}{\text{s}}\right)\sin\left[\left(0.006\tfrac{\text{rad}}{\text{m}}\right)x + \left(600\tfrac{\text{rad}}{\text{s}}\right)t\right]$

d. What is the maximum transverse velocity of any particular point along the string?

Ans. $v_m = \omega y_m = 3.6$ m/s

2. A sinusoidal wave of frequency 500 Hz has a speed of 350 m/s.

a. How far apart are 2 points that differ in phase by π/3 radians?

Ans. $\lambda = \dfrac{350\frac{\text{m}}{\text{s}}}{500\text{ Hz}} = 0.7 \text{ m} \Rightarrow \dfrac{0.7\text{ m}}{\Delta x} = \dfrac{2\pi}{\left(\frac{\pi}{3}\right)} \Rightarrow \Delta x = 0.117$ m

b. What is the phase difference between 2 displacements at a certain point at times 1.000 ms apart?

Ans. $T = \dfrac{1}{500\text{ Hz}} = 0.002\text{s} \Rightarrow \dfrac{\Delta t}{T} = \dfrac{0.001\text{s}}{0.002\text{ s}} = \dfrac{\phi}{2\pi} \Rightarrow \phi = \pi$ rad

Waves on a Stretched String

3. The linear density of a string is $1.5\text{x}10^{-4}$ kg/m. The equation of a transverse wave on a string is:

$$y(x,t) = (0.025\text{m})\sin\left((2.0\tfrac{\text{rad}}{\text{m}})x - (30\tfrac{\text{rad}}{\text{s}})t\right)$$

a. What is the wave speed?

Ans. $v = \dfrac{\omega}{k} = \dfrac{30\frac{\text{rad}}{\text{s}}}{2.0\frac{\text{rad}}{\text{m}}} = 15\tfrac{\text{m}}{\text{s}}$

b. What is the tension in the string?

Ans. $F_T = \mu v^2 = \left(1.5\text{x}10^{-4}\tfrac{\text{kg}}{\text{m}}\right)\left(15\tfrac{\text{m}}{\text{s}}\right)^2 = 0.0338\text{ N}$

c. What is the average power for this wave?

Ans. $P = (½)\mu v\omega^2 y_m{}^2 = 0.00064\text{ W}$

4. The equation of a transverse wave on a string is:

$$y(x,t) = (2.0\text{mm})\sin\left((20\tfrac{\text{rad}}{\text{m}})x - (600\tfrac{\text{rad}}{\text{s}})t\right)$$

The tension in the string is 15 N.

a. What is the wave speed?

Ans. $v = \dfrac{\omega}{k} = \dfrac{600\frac{\text{rad}}{\text{s}}}{20\frac{\text{rad}}{\text{m}}} = 30\tfrac{\text{m}}{\text{s}}$

b. What is the linear mass density of the string, in grams per meter?

Ans. $\mu = \dfrac{F_T}{v^2} = \dfrac{15\text{N}}{\left(30\frac{\text{m}}{\text{s}}\right)^2} = 1.67\text{x}10^{-2}\tfrac{\text{kg}}{\text{m}}$

c. Determine the average power transmitted by this wave.

Ans. $P = (½)\mu v\omega^2 y_m{}^2 = 0.36\text{ W}$

d. Demonstrate that this transverse wave satisfies the wave equation: $\dfrac{\partial^2 y}{\partial x^2} = \dfrac{1}{v^2}\dfrac{\partial^2 y}{\partial t^2}$.

The Wave Equation:

5. Consider the following wave: $y(x,t) = (0.05\text{m})\sin\left((5.0\tfrac{\text{rad}}{\text{m}})x - (10\tfrac{\text{rad}}{\text{s}})t\right)$

a. Verify that this wave satisfies the wave equation: $\dfrac{\partial^2 y}{\partial x^2} = \dfrac{1}{v^2}\dfrac{\partial^2 y}{\partial t^2}$

Ans. $\dfrac{\partial^2 y}{\partial x^2} = -\left(5.0\tfrac{\text{rad}}{\text{m}}\right)^2(0.05\text{m})\sin\left((5.0\tfrac{\text{rad}}{\text{m}})x-(10\tfrac{\text{rad}}{\text{s}})t\right) = -\left(1.25\text{m}^{-1}\right)\sin\left((5.0\tfrac{\text{rad}}{\text{m}})x-(10\tfrac{\text{rad}}{\text{s}})t\right)$

$\dfrac{1}{v^2}\dfrac{\partial^2 y}{\partial t^2} = -\dfrac{\left(10\frac{\text{rad}}{\text{s}}\right)^2(0.05\text{m})}{\left(2\frac{\text{m}}{\text{s}}\right)^2}\sin\left((5.0\tfrac{\text{rad}}{\text{m}})x-(10\tfrac{\text{rad}}{\text{s}})t\right) = -\left(1.25\text{m}^{-1}\right)\sin\left((5.0\tfrac{\text{rad}}{\text{m}})x-(10\tfrac{\text{rad}}{\text{s}})t\right)$

b. Use the wave equation to determine the speed of the wave.

Ans. Setting the above equations together, and not assuming a priori that $v = \omega/k$, leads to:

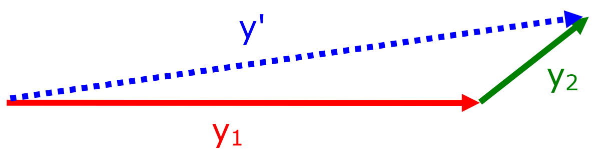
$$\frac{\left(10\,\frac{rad}{s}\right)^2(0.05m)}{v^2} = \left(5.0\,\frac{rad}{m}\right)^2(0.05m) \Rightarrow v=\sqrt{v^2}=\frac{10\,\frac{rad}{s}}{5.0\,\frac{rad}{m}} = 2.0\,\tfrac{m}{s}$$

c. Calculate the frequency, wavelength and period for this wave.

Ans. $y_m=0.05$ m, $f=\frac{\omega}{2\pi}=1.59$ Hz, $\lambda = \frac{k}{2\pi} = 0.796$ m, and $T=\frac{1}{f}=0.629$ s

d. Consider a 2nd wave (with same frequency and wavelength) traveling in the same direction, where $y_{2max} = 0.01$ m and $\phi_2 = \pi/3$. Construct a phasor diagram for the combined waves.

Ans.



e. What is the amplitude of the resultant wave?

Ans. $y' = \sqrt{y_1^2 + y_2^2 - 2y_1y_2\cos\left(\pi-\frac{\pi}{3}\right)} = 0.0557$ m

Standing Waves

7. A string is attached to a string vibrator (f=120 Hz) at one end and fixed at the other. The length of the string is 1.2 m.

a. What wavelengths will satisfy the standing wave condition for the string?

Ans. $L = n\left(\frac{\lambda}{2}\right)$ {where n=1,2,3,... etc.}

b. What is the maximum wave speed that will produce a standing wave in this string?

Ans. For the same frequency, the greatest wavelength will produce the maximum speed:

when n=1, $\lambda = \frac{2L}{n} = 2.4$ m and $v = \lambda\cdot f = (2.4\text{ m})(120\text{ Hz}) = 288\,\tfrac{m}{s}$

c. What string tension will correspond to the wave in (b) is the linear density of the string is $2.5\text{x}10^{-4}$ kg/m?

Ans. $F_T = \mu v^2 = \left(2.5\text{x}10^{-4}\,\frac{kg}{m}\right)\left(288\,\frac{m}{s}\right)^2 = 20.7$ N

d. What string tension would correspond to a standing wave that produces 6 nodes in the string?

Ans. For 6 nodes, n=5, $\lambda = \frac{2L}{n} = 0.48$ m and $v = \lambda\cdot f = (0.48\text{ m})(120\text{ Hz}) = 57.6\,\tfrac{m}{s}$

The tension in the string would be: $F_T = \mu v^2 = \left(2.5\text{x}10^{-4}\,\frac{kg}{m}\right)\left(57.6\,\frac{m}{s}\right)^2 = 0.829$ N