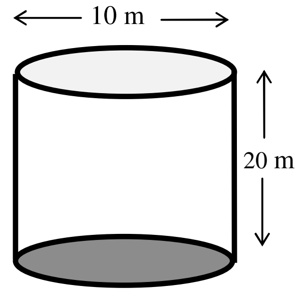
Static & Dynamic Fluids:



1. An open cylindrical container, 20 m high and 10 m diameter, is completely filled with water (ρ_{water} = 998 kg/m^3). The container is standing at ground level and the atmospheric pressure (p_{atm}) is 1.01 x 10^5 Pa.

a. What is the mass of a sphere of water with radius 0.5 m?

Ans. $$m = \left(998\tfrac{kg}{m^3}\right)\left(\frac{4}{3}\right)\pi(0.5m)^2 = 524\ kg$$

b. What is the total pressure at the bottom of the container?

Ans. $$P = 1.01x10^5 Pa + \left(998\tfrac{kg}{m^3}\right)\left(9.8\tfrac{m}{s^2}\right)(20m) = 2.97x10^5 Pa$$

c. What is the total force on the bottom of the container?

Ans. $$F = PA = \left(2.97x10^5 Pa\right)\pi(5m)^2 = 2.33x10^7 N$$

d. If the container were filled with oil (ρ_{oil} = 925 kg/m^3) instead of water, what would the pressure at the bottom be?

Ans. $$P = 1.01x10^5 Pa + \left(925\tfrac{kg}{m^3}\right)\left(9.8\tfrac{m}{s^2}\right)(20m) = 2.83x10^5 Pa$$

Pascal's Principle:

2. The largest barometer ever built was an oil-filled barometer constructed in Leicester, England in 1991. At an atmospheric pressure of 1.01 x 10^5 Pa, height of the oil column was 12.2 m.

a. What was the density of the oil used in the barometer?

Ans. $$\rho = \frac{P_{atm}}{gh} = \frac{1.01x10^5 Pa}{\left(9.8\tfrac{m}{s^2}\right)(12.2m)} = 845\tfrac{kg}{m^3}$$

b. Were the barometer filled with water instead of oil, what would be the height of the water at the same atmospheric pressure?

Ans. $$h = \frac{P_{atm}}{\rho g} = \frac{1.01x10^5 Pa}{\left(998\tfrac{kg}{m^3}\right)\left(9.8\tfrac{m}{s^2}\right)} = 10.3m$$

3. A U-shaped container has a diameter of 0.25 m at one end and 1.0 m at the other with plungers at each end and an applied force of 1000 N is applied to the narrow end.

a. What is the resultant force generated at the wide end?

Ans. $$F_2 = \left(\frac{A_2}{A_1}\right)F_1 = \left(\frac{0.5m}{0.125m}\right)^2(1000N) = 1.60x10^4 N$$

b. When 2000 J of work are performed by the 1000 N applied force, how high does the fluid level rise (on the wide end of the container). Assume the fluid rise occurs at constant velocity.

Ans. $$W_2 = F_2h_2 = 2000J \Rightarrow h_2 = \frac{2000J}{1.60x10^4 N} = 0.125\ m$$

Archimedes' Principle:

4. A turtle with volume 0.25 m^3 is floating in the ocean, 80% of the turtle's volume is submerged. Assume the density of the ocean salt water ($\rho_{sea\ water}$) is 1.024 x 10^3 kg/m^3.

a. What is the buoyant force exerted on the turtle?

Ans. $V_{disp} = (0.80)(0.25\ m^3) = 0.20\ m^3$

$$F_B = \rho g V_{disp} = \left(1.024\text{x}10^3\ \tfrac{kg}{m^3}\right)\left(9.8\tfrac{m}{s^2}\right)\left(0.20\ m^3\right) = 2007\ N$$

b. What is the mass of the turtle?

Ans. $F_{net} = F_B - mg = 0 \rightarrow m = \dfrac{F_B}{g} = \dfrac{2007\ N}{9.8\ \frac{m}{s^2}} = 204.8\ kg$

c. If the fluid were pure water (ρ_{water} = 998 kg/m^3) instead of salt water, how much of the turtle's volume would be submerged in the water?

Ans. $V_{disp} = \dfrac{F_B}{\rho g} = \dfrac{2007\ N}{\left(998\frac{kg}{m^3}\right)\left(9.8\ \frac{m}{s^2}\right)} = 0.205\ kg$

5. A spherical mass (14.7) kg is attached by a mass-less string to a force sensor then submerged into a container of water.

a. What is the force sensor reading (i.e. the tension force in the mass-less string) before the mass is submerged in the fluid?

Ans. $F_{net} = F_T - mg = 0 \rightarrow F_T = mg = (14.7\ kg)(9.8\ m/s^2) = 144\ N$

b. When the mass is submerged, the sensor reading is 92.9 N. What is the buoyant force exerted on the mass?

Ans. $F_{net} = F_{T2} + F_B - mg = F_{T2} + F_B - F_T = 0$

$\rightarrow F_T - F_{T2} = F_B = 144\ N - 92.9\ N = 51.2\ N$

c. What is the density of the hanging mass?

Ans. $V_{disp} = \dfrac{F_B}{\rho g} = \dfrac{51.2\ N}{\left(998\frac{kg}{m^3}\right)\left(9.8\ \frac{m}{s^2}\right)} = 5.22\text{x}10^{-2}\ m^3 \rightarrow \rho_{mass} = \dfrac{m}{V} = \dfrac{14.7\ kg}{5.22\text{x}10^{-2}\ m^3} = 2.81\text{x}10^3\ \frac{kg}{m^3}$

6. A 60 kg person is attached by a mass-less string to a force sensor then submerged into a container of water. *Assume all of the air is out of her lungs when she is submerged.*

a. What is the force sensor reading (i.e. the tension force in the mass-less string) before the mass is submerged in the fluid?

Ans. $F_{net} = F_T - mg = 0 \rightarrow F_T = mg = 588\ N$

b. When the mass is submerged, the sensor reading is 37.2 N. What is the density of the person?

Ans. $F_{net} = F_{T2} + F_B - mg = F_{T2} + F_B - F_T = 0$

$F_T - F_{T2} = F_B = 588\ N - 37.2\ N = 551\ N$

$$\rightarrow \rho = \frac{m}{V} = \frac{m}{\left(\dfrac{F_B}{\rho_w g}\right)} = \frac{m}{\left(\dfrac{F_{T2}\text{-}F_T}{\rho_w g}\right)} = \frac{(60\ kg)\left(998\frac{kg}{m^3}\right)\left(9.8\ \frac{m}{s^2}\right)}{(588N\ -\ 37.2N)} = 1065\ \frac{kg}{m^3}$$

c. The density of body fat ($\rho_{body\ fat}$) is 918 kg/m^3 and the density of "lean body mass" ($\rho_{lean\ mass}$) is 1100 kg/m^3. Determine the amount of body fat, in kg, for this person. Assume that the total body mass is the sum of fat mass plus lean mass: $m_{total} = m_{fat} + m_{lean}$.

Ans. $$V_{tot} = V_{fat} + V_{lean} = \frac{m}{\rho} = \frac{60\text{kg}}{1068\,\frac{\text{kg}}{\text{m}^3}} = 0.0562\ \text{m}^3$$

$$m_{tot} = \rho_{fat}V_{fat} + \rho_{lean}V_{lean} = \rho_{fat}V_{fat} + \rho_{lean}(V_{tot} - V_{fat}) = 60\ \text{kg}$$

$$V_{fat} = \frac{60\text{kg} - \rho_{lean}V_{tot}}{(\rho_{fat} - \rho_{lean})} = \frac{60\text{kg} - 61.8\text{kg}}{918\,\frac{\text{kg}}{\text{m}^3} - 1100\,\frac{\text{kg}}{\text{m}^3}} = 0.00989\ \text{m}^3$$

$$m_{fat} = \rho_{fat}V_{fat} = \left(918\,\tfrac{\text{kg}}{\text{m}^3}\right)\left(0.00989\ \text{m}^3\right) = 9.08\ \text{kg}$$

Equation of Continuity & Bernoulli's Equation:

7. Water flows through a pipe of diameter 8.0 cm with a speed of 10.0 m/s. It then enters a smaller pipe of diameter 3.0 cm.

a. What is the speed of the water as it flows through the smaller pipe?

Ans. $$v_{3.0cm} = v_{8.0cm}\left(\frac{A_{8.0cm}}{A_{3.0cm}}\right) = \left(10.0\,\tfrac{\text{m}}{\text{s}}\right)\frac{\pi(8.0\text{cm})^2}{\pi(3.0\text{cm})^2} = 71.1\,\tfrac{\text{m}}{\text{s}}$$

b. What is the volume flow rate of water through the narrow pipe?

Ans. $$\frac{\Delta V}{\Delta t} = A\frac{\Delta x}{\Delta t} = \pi(0.015\text{m})^2\left(71.1\,\tfrac{\text{m}}{\text{s}}\right) = 0.050\,\tfrac{\text{m}^3}{\text{s}}$$

c. What is the pressure difference between the wide pipe and the narrow pipe?

Ans. $$\Delta P = P_{8cm} - P_{3cm} = \tfrac{1}{2}\rho\left(v_{3cm}^2 - v_{8cm}^2\right) = 2.48\text{x}10^5\text{Pa}$$

8. Water circulates throughout a house in a hot-water system. The water is pumped at a speed of 0.50 m/s through a 4.0 cm diameter pipe in the basement under a pressure of 3.0 atm.

a. What is the flow speed in a 2.6 cm diameter pipe on the second floor 5.0 m above? Assume the pipes do not divide into branches.

Ans. $$v_{2.6cm} = v_{4.0cm}\left(\frac{A_{4.0cm}}{A_{2.6cm}}\right) = \left(0.50\,\tfrac{\text{m}}{\text{s}}\right)\frac{\pi(4.0\text{cm})^2}{\pi(2.6\text{cm})^2} = 1.18\,\tfrac{\text{m}}{\text{s}}$$

b. What is pressure in the pipe on the second floor?

Ans. $$P_{2.6cm} = P_{4cm} + \tfrac{1}{2}\rho\left(v_{4cm}^2 - v_{2.6cm}^2\right) + \rho g\left(y_{4cm} - y_{2.6cm}\right)$$

$$P_{2.6cm} = 3.03\text{x}10^5\text{Pa} + \left(998\,\tfrac{\text{kg}}{\text{m}^3}\right)\left[\left(\frac{\left(0.50\,\frac{\text{m}}{\text{s}}\right)^2 - \left(1.18\,\frac{\text{m}}{\text{s}}\right)^2}{2}\right) + \left(9.8\,\tfrac{\text{m}}{\text{s}^2}\right)(-5.0\text{m})\right] = 2.53\text{x}10^5\text{Pa}$$

9. Consider the completely full water container described in Problem 1 above.

a. What is the pressure at a depth of 15 m below the surface of the water?

Ans.
$$P = P_{atm} + \rho gh + P_{applied} \approx P_{atm} + \rho gh$$
$$P \approx 1.01\text{x}10^5\text{Pa} + \left(998\tfrac{kg}{m^3}\right)\left(9.8\tfrac{m}{s^2}\right)(15m) \approx 2.48\text{x}10^5\text{Pa}$$

b. A small spout is opened in the container 5 m above the bottom. As the water spews from the hole, what is the speed of the water as it leaves the hole?

Ans.
$$\Delta P = P_{inside} - P_{outside} = P_{atm} + \rho gh - P_{atm} = \tfrac{1}{2}\rho v_{outside}^2$$
$$\Rightarrow v_{outside} = \sqrt{\frac{2\Delta P}{\rho}} = \sqrt{2gh} = 17.1\tfrac{m}{s}$$

c. How far from the container will the water stream hit the ground?

Ans. $$\Delta x = v_{ox}t = v_{ox}\sqrt{\frac{2\Delta y}{g}} = \left(17.1\tfrac{m}{s}\right)\sqrt{\frac{2(5m)}{9.8\frac{m}{s^2}}} = 17.3\text{ m}$$

d. If the container were filled with oil (ρ_{oil} = 925 kg/m^3) instead of water, how fast would the fluid leave the hole in question (c)?

Ans. The same as answer (b).

10. An airplane has a mass of 2.0 x 10^6 kg and the air flows past the lower surface at 100 m/s. The wings have a combined surface area of 1200 m^2.

a. How fast must the air flow past the upper surface of the wings if the plane is to maintain constant elevation while in the air?

Ans. $$F_{net} = F_{below} - F_{above} - mg = 0 \Rightarrow P_{below}A_{wings} - P_{above}A_{wings} = mg \Rightarrow P_{below} - P_{above} = \frac{mg}{A_{wings}}$$

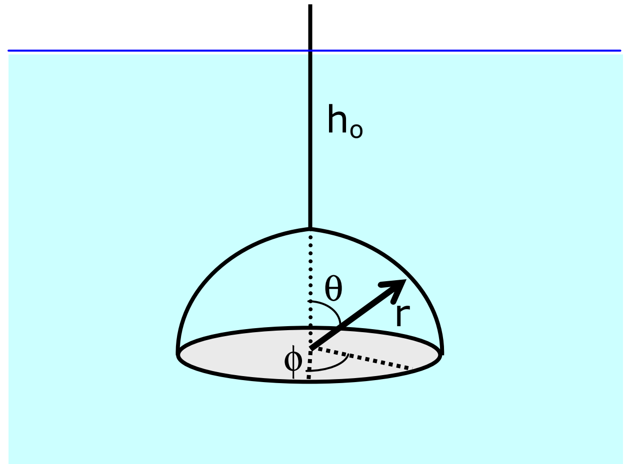
$$P_{below} - P_{above} = \tfrac{1}{2}\rho_{air}\left(v_{above}^2 - v_{below}^2\right) = \frac{mg}{A_{wings}} \Rightarrow v_{above} = \sqrt{\frac{2mg}{\rho_{air}A_{wings}} + v_{below}^2} = 192\ \tfrac{m}{s}$$

b. If the surface area of the wings were increased by 25%, what would be the upward acceleration of the plane?

Ans. $$F_{net} = F_{below} - F_{above} - mg = ma \Rightarrow a_{lift} = \frac{(P_{below} - P_{above})A_{wings} - mg}{m} = 2.39\tfrac{m}{s^2}$$

Archimedes' Principle (one more thing…)

11. A face down iron hemisphere (r = 1.0 m) is completely submerged in water, suspended by a cable. The peak of the hemisphere is 0.5 m below the surface of the water.



a. Calculate the gauge pressure on the bottom face, P_{bottom}, of the mass.

Ans.

$$P_{bottom} = \rho g(h_o + r)$$

$$P_{bottom} = \left(998\,\tfrac{kg}{m^3}\right)\left(9.8\,\tfrac{m}{s^2}\right)(1.5\text{ m}) = 1.4671\times10^4\ \tfrac{N}{m^2}$$

b. Calculate the upward force exerted on the lower surface due to P_{bottom}.

Ans.

$$F_{up} = P_{bottom}\left(\pi r^2\right)$$

$$F_{up} = \left(1.4671\times10^4\,\tfrac{N}{m^2}\right)\pi(1\text{m})^2 = 49{,}090\text{N}$$

c. Derive the equation for the vertical gauge pressure, $P_{upper\text{-}y}$ exerted on the upper (spherical) surface.

Ans. $P_{top\text{-}y} = \left[\rho g h_o + \rho g r(1 - \cos\theta)\right]\cos\theta$

d. Calculate the force exerted on the upper surface due to $P_{upper\text{-}y}$.

Ans. The downward fluid force on the a small area dA of the mass is:

$$dF_{down} = P_{top\text{-}y}dA \quad \text{where } dA = (r\cdot d\theta)(r\cdot\sin\theta d\phi)$$

$$F_{down} = \int P_{top\text{-}y}dA = \int\left[\rho g h_o + \rho g r(1 - \cos\theta)\right]\cos\theta(r\cdot d\theta)(r\cdot\sin\theta d\phi)$$

$$F_{down} = \rho g r^2(h_o + r)\int_0^{\frac{\pi}{2}}\cos\theta\sin\theta d\theta\int_0^{2\pi}d\phi - \rho g r^3\int_0^{\frac{\pi}{2}}\cos^2\theta\sin\theta d\theta\int_0^{2\pi}d\phi$$

$$F_{down} = \int P_{top\text{-}y}dA = \int\left[\rho g h_o + \rho g r(1 - \cos\theta)\right]\cos\theta(r\cdot d\theta)(r\cdot\sin\theta d\phi)$$

$$F_{down} = \rho g r^2(h_o + r)\left(\sin^2\theta\Big|_0^{\frac{\pi}{2}}\right)\left(\phi\Big|_0^{2\pi}\right) - \rho g r^3\left(\frac{\cos^3\theta}{3}\Big|_0^{\frac{\pi}{2}}\right)\left(\phi\Big|_0^{2\pi}\right)$$

$$F_{down} = \rho g r^2(h_o + r)\pi - \rho g r^3\left(\frac{1}{3}\right)(2\pi) = \rho g r^2\left(h_o + \frac{r}{3}\right)\pi = 25{,}605\text{ N}$$

e. Calculate the net force exerted on the mass due to the 2 forces above.

Ans. $F_{net} = F_{up} - F_{down} = 49{,}090\text{N} - 25605\text{ N} = 20{,}484\text{ N (upward)}$

f. Apply Archimedes' Principle to the mass and calculate the buoyant force exerted on the mass due to the fluid. Compare this value to the answer calculate in (e).

Ans. $$V_{disp} = \left(\frac{2}{3}\right)\pi(1.0\text{ m})^3 = 2.09\text{ m}^3$$

$$F_B = \rho g V_{disp} = \left(998\,\tfrac{kg}{m^3}\right)\left(9.8\,\tfrac{m}{s^2}\right)\left(2.09\text{ m}^3\right) = 20{,}484\text{ N}$$ {the same value as (e)!}