Phy 212: General Physics II

Chapter 20: Entropy & Heat Engines
Lecture Notes

Entropy

1. A measure of the disorder (or randomness) of a system
2. For a reversible process, the change in entropy is measured as the ratio of heat energy gained to the temperature:

$$dS = (dQ/T)_R$$

$$\Delta S = (Q/T)_R = S_{final} - S_{initial}$$

   a. When heat energy is gained by a system, entropy is gained by the system (and lost by the surrounding environment)
   b. When heat is lost by a system, entropy is lost by the system (and gained by the surrounding environment)
3. The net entropy change by the universe due to a thermodynamic process:

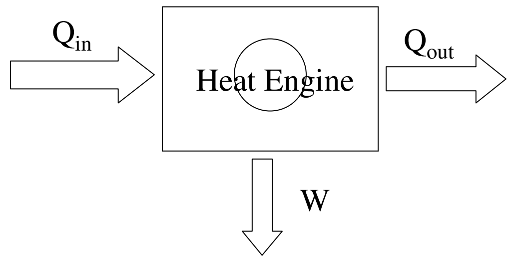
$$\Delta S_{universe} = S_{gained} - S_{lost} = Q_{cold}/T_{cold} - Q_{hot}/T_{hot}$$

4. The total entropy of the universe ($S_{universe}$) will never decrease, it will either
   a. Remain unchanged (for a reversible process)
   b. Increase (for an irreversible process)
5. Change in entropy is related to the amount of energy that is lost irretrievably by a thermodynamic process:

$$dW_{unavailable} = T_{cold}dS_{universe} \text{ or } W_{unavailable} = T_{cold}\Delta S_{universe}$$

Heat Engines

1. A cyclical process that utilizes heat energy input (Q_{in}) to enable a working substance perform work output



2. Heat energy input equals the work performed plus the heat energy discarded:

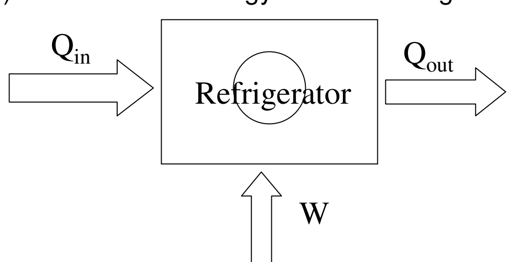
$$Q_{in} + Q_{out} = W \text{ \{Note: } Q_{out} \text{ is negative\}}$$

Or:

$$|Q_{in}| = W - |Q_{out}|$$

Refrigerators

1. A cyclical process that performs negative work, work is performed on system to absorb heat at low temperature then release heat energy at high temperature
2. Refrigerators operate as a heat energy in reverse and require input work (energy) to “drive” heat energy from low to high temperature



3. Heat energy discarded equals the work performed plus the heat energy input:

$$Q_{in} + Q_{out} = W \text{ \{Note: } Q_{out} \text{ is negative\}}$$

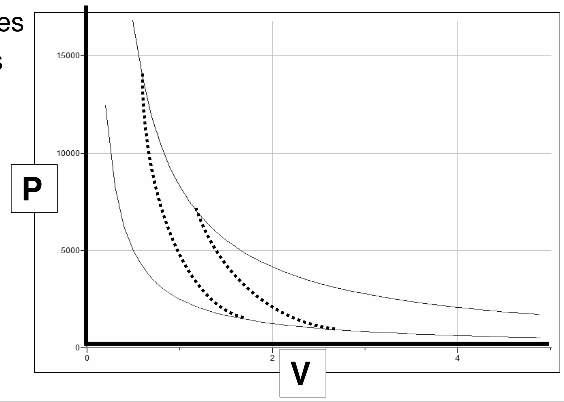
Or:

$$|Q_{out}| = W + |Q_{in}|$$

The Carnot Engine

The Carnot Cycle (Engine)

1. An "ideal" reversible heat engine (no heat engine can be more efficient than a Carnot engine)
2. A 4 stage engine that operates between 2 temperature reservoirs (T_{hot} and T_{cold}) consisting of
   a. 2 isothermic phases
   b. 2 adabatic phases



Efficiency of a Heat Engine

1. A measure of the effectiveness of a heat engine is its thermodynamic efficiency:

$$\text{Efficiency} = e = \frac{W_{performed}}{Q_{in}} = \frac{Q_{in} - Q_{out}}{Q_{in}}$$

or

$$\%\ \text{Efficiency} = \%e = \left(\frac{W_{performed}}{Q_{in}}\right) \times 100\%$$

2. For the Carnot ("ideal") engine:

$$\text{Carnot Efficiency} = e_{Carnot} = 1 - \frac{T_C}{T_H}$$

where $\frac{Q_{in}}{Q_{out}} = \frac{T_H}{T_C}$

Statistical Interpretation of Entropy:

1. For a system of N bodies and i possible states (n), the entropy associated with a particular state (in J/K) is:

$$S = k \ln W = k \ln\left(\frac{N!}{n_1! n_2! \ldots n_i!}\right)$$

   – where W is the # of microstates, or ways that the state can occur (called the "multiplicity" of configurations)

2. For large N, Stirling's approximation is useful:

$$\ln N! \approx N(\ln N) - N$$

Note: The entropy of a state increases with the # of possible microstates will produce it

Examples (Stirling's Approx):

1) $N=60: \ln 60! = 188.6 \approx 60(\ln 60) - 60 = 185.7$

2) $N=10^{23}: \ln\left(10^{23}!\right) \approx 10^{23}\left(\ln 10^{23}\right) - 10^{23} = 5.20\times10^{24}$

A 2 Coin System

A 2 identical coin system (N+1=3 possible configurations)

– 2 possible states for each coin, n_1 = heads & n_2 = "tails"

1) The entropy associated with both coins initially "heads" is:

$$W = \left(\frac{2!}{2! \cdot 0!}\right) = 1$$

$$S = k \ln W = \left(1.38\times10^{-23}\,\tfrac{J}{K}\right)\ln(1) = 0\,\tfrac{J}{K}$$

2) When they are flipped, they land with 1 "head" & 1 "tail". The entropy of this (most probable) state is:

$$W = \left(\frac{2!}{1! \cdot 1!}\right) = 2$$

$$S = k \ln W = \left(1.38\times10^{-23}\,\tfrac{J}{K}\right)\ln(2) = 9.57\times10^{-24}\,\tfrac{J}{K}$$

The change in entropy from state (1) to (2) is:

$$\Delta S = S_f - S_i = +9.57\times10^{-24}\,\tfrac{J}{K}$$

The higher entropy state is the most likely to occur…

A 4 Coin System

A 4 identical coin system (N+1=5 possible configurations)

1. The entropy associated with flipping them and obtaining all “heads” (or “tails”) is: $W = \left(\frac{4!}{4!\cdot 0!}\right) = 1$

$$S = k \ln W = \left(1.38\times10^{-23}\,\tfrac{J}{K}\right)\ln(1) = 0\,\tfrac{J}{K}$$

2. The entropy associated with flipping 3 “heads” & 1 “tail” is:

$$W = \left(\frac{4!}{3!\cdot 1!}\right) = 4$$

$$S = k \ln W = \left(1.38\times10^{-23}\,\tfrac{J}{K}\right)\ln(4) = 1.91\times10^{-23}\,\tfrac{J}{K}$$

3. The entropy associated with flipping 3 “heads” & 1 “tail” is:

$$W = \left(\frac{4!}{2!\cdot 2!}\right) = 6$$

$$S = k \ln W = \left(1.38\times10^{-23}\,\tfrac{J}{K}\right)\ln(6) = 2.47\times10^{-23}\,\tfrac{J}{K}$$

A 1000 Coin System

A 1000 coin system (N+1=1001 possible configurations)

1) The entropy associated with flipping them and obtaining all “heads” (or “tails”) is: $W = \left(\frac{1000!}{1000!\cdot 0!}\right) = 1$

$$S = k \ln W = \left(1.38\times10^{-23}\,\tfrac{J}{K}\right)\ln(1) = 0\,\tfrac{J}{K}$$

2) The entropy associated with flipping 500 “heads” & 500 “tails” is:

$$W = \left(\frac{1000!}{500!\cdot 500!}\right) \Rightarrow S = k \ln W = \left(1.38\times10^{-23}\,\tfrac{J}{K}\right)\ln\left(\frac{1000!}{500!\cdot 500!}\right)$$

$$S = \left(1.38\times10^{-23}\,\tfrac{J}{K}\right)\left[\ln(1000!)-2\ln(500!)\right]$$

$$S = \left(1.38\times10^{-23}\,\tfrac{J}{K}\right)\left[1000\cdot\ln1000 - 1000 - 2(500\cdot\ln500 - 500)\right]$$

$$S = \left(1.38\times10^{-23}\,\tfrac{J}{K}\right)(1000)\ln2 = 9.57\times10^{-21}\,\tfrac{J}{K}$$

A 2 Dice System

N = 2 dice

Each die has 6 possible states

1) The entropy associated with 2 "ones" is:

$$W = \left(\frac{2!}{2!\cdot 0!\cdot 0!\cdot 0!\cdot 0!\cdot 0!}\right) = 1$$

$$S = k\ \ln W = \left(1.38\times 10^{-23}\,\tfrac{J}{K}\right)\ln(1) = 0\,\tfrac{J}{K}$$

2) The entropy associated with any non-paired combination is:

$$W = \left(\frac{2!}{1!\cdot 1!\cdot 0!\cdot 0!\cdot 0!\cdot 0!}\right) = 2$$

$$S = k\ \ln W = \left(1.38\times 10^{-23}\,\tfrac{J}{K}\right)\ln(2) = 9.57\times 10^{-24}\,\tfrac{J}{K}$$

Entropy & Information

1. In information theory, the entropy of a system represents the lower boundary of bits needed to represent information:

$$S\ (\text{in bits}) = -\sum_{i=1}^{N} p_i \cdot \log_2 p_i$$

Example: A 26 character alphabet

Assuming that each letter has an equal probability of occurrence, a minimum of 5 bits are required to uniquely express all letters (say, using a keyboard):

$$S = -\sum_{i=1}^{26}\left(\frac{1}{26}\right)\cdot\log_2\left(\frac{1}{26}\right) = \log_2(26) \approx 5 \text{ bits}$$