Phy 212: General Physics II

Chapter 20: Entropy & Heat Engines
Lecture Notes

Entropy

1. Entropy is a state variable (P, V and V are also state variables)
   a. State variables describe the state of a system
   b. Changes in state variables depend only on the initial and final states of a system and do not dependent on the specific path between the states.
2. Entropy is measure of the disorder (or randomness) of a system
3. Entropy change reflects the changes in disorder associated with a thermodynamic process.

   For reversible processes, entropy change (dS) is measured as the ratio of heat energy gained to the state temperature:

$$dS_{rev} = \left(\frac{dQ}{T}\right)_{rev} \quad \text{or} \quad \Delta S_{rev} = \int_i^f dS = S_f - S_i = \int_i^f \frac{dQ}{T}$$

   a. When net heat flow is positive for a system, the system entropy increases (and lost by the surrounding environment)
   b. When net heat flow is negative, system entropy decreases (and gained by the surrounding environment)

Entropy (cont.)

NOTE: In a closed system

The entropy change for an irreversible process cannot be calculated, but IS equal to the entropy change for an equivalent reversible process:

$$\Delta S_{irrev} = \Delta S_{rev} = \left(S_f - S_i\right)_{rev}$$

Examples of Entropy Change for Reversible Processes:

a. For a reversible isothermal process:

$$\Delta S_{system} = \frac{1}{T}\int_i^f dQ = \frac{Q}{T}$$

b. For an approximately isothermal process ($\Delta T << T_{avg}$):

$$\Delta S_{system} \approx \frac{Q}{T_{avg}}$$

c. For an ideal gas (when P, V and T can vary):

$$\Delta S_{system} = S_f - S_i = nR \cdot \ln\left(\frac{V_f}{V_i}\right) + nC_V \cdot \ln\left(\frac{T_f}{T_i}\right)$$

2nd Law of Thermodynamics Revisited

1. The total entropy of the universe ($S_{universe}$) will never decrease, it will either:
   a. Remain unchanged (for a reversible process) $\Delta S_{rev} = 0\,\frac{J}{K}$
   b. Increase (for an irreversible process) $\Delta S_{irrev} > 0\,\frac{J}{K}$
2. The net entropy change by the universe due to a complete thermodynamic process:

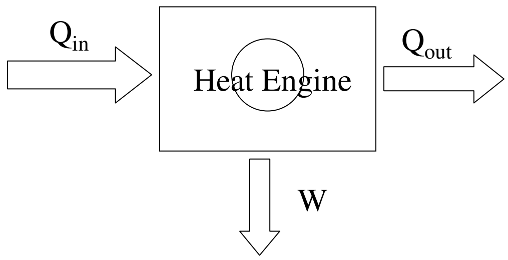
$$\Delta S_{universe} \geq 0$$

3. This interpretation of the 2nd Law is consistent with the previously described heat and temperature definition:

"In nature, heat only flows spontaneously in the direction of high T to cold T"

Heat Engines

1. A cyclical process that utilizes thermal energy input (Q_{in}) to enable a working substance perform work:



2. Heat absorbed equals the work performed plus the heat discarded:

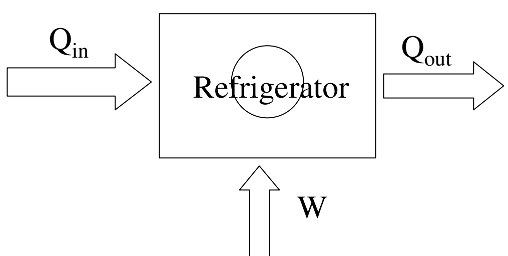
$$Q_{in} + Q_{out} = W \text{ \{Note: } Q_{out} \text{ is negative\}}$$

Or:

$$|Q_{in}| = W - |Q_{out}|$$

Refrigerators

1. A cyclical process that performs negative work, work is performed on system to absorb heat at low temperature then release heat energy at high temperature
2. Refrigerators operate as a heat energy in reverse and require input work (energy) to “drive” heat energy from low to high temperature



3. Heat loss equals the work performed plus the heat gain:

$$Q_{in} + Q_{out} = W \text{ \{Note: } Q_{out} \text{ is negative\}}$$

Or:

$$|Q_{out}| = W + |Q_{in}|$$

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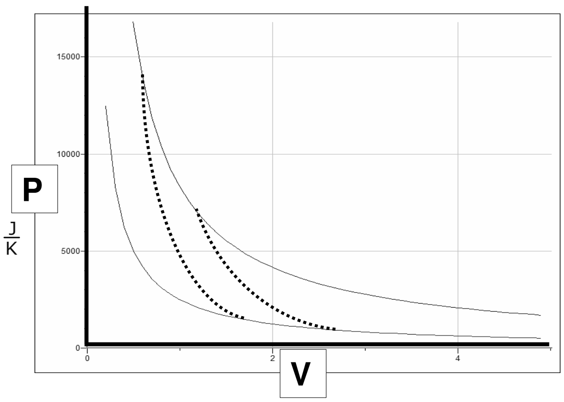
The Carnot Engine

The Carnot Cycle (Engine)

1. An "ideal" reversible heat engine (no heat engine can be more efficient than a Carnot engine)
2. A 4 stage engine that operates between 2 temperature reservoirs (T_{hot} and T_{cold}) consisting of
   a. 2 isothermal phases
   b. 2 adiabatic phases



3. Entropy change:

$$\Delta S = \frac{Q_{hot}}{T_{hot}} - \frac{Q_{cold}}{T_{cold}} = 0\,\frac{J}{K}$$

$$\frac{Q_{hot}}{T_{hot}} = \frac{Q_{cold}}{T_{cold}}$$

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Efficiency of a Heat Engine

1. A measure of the effectiveness of a heat engine is its thermodynamic efficiency:

$$\text{Efficiency} = e = \frac{W_{performed}}{Q_{in}} = \frac{Q_{in} - Q_{out}}{Q_{in}}$$

or

$$\%\ \text{Efficiency} = \%e = \left(\frac{W_{performed}}{Q_{in}}\right)\times 100\%$$

2. For the Carnot ("ideal") engine:

$$\text{Carnot Efficiency} = e_{Carnot} = 1 - \frac{T_C}{T_H}$$

where $\frac{Q_{in}}{Q_{out}} = \frac{T_H}{T_C}$

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Statistical Interpretation of Entropy:

1. For a system of N bodies and i possible states (n), the entropy associated with a particular state (in J/K) is:

$$S = k \ln W = k \ln\left(\frac{N!}{n_1!n_2!\ldots n_i!}\right)$$

   – where W is the # of microstates, or ways that the state can occur (called the "multiplicity" of configurations)

2. For large N, Stirling's approximation is useful:

$$\ln N! \approx N(\ln N) - N$$

Note: The entropy of a state increases with the # of possible microstates will produce it

Examples (Stirling's Approx):

1) $N=60$: $\ln 60! = 188.6 \approx 60(\ln 60) - 60 = 185.7$

2) $N=10^{23}$: $\ln\left(10^{23}!\right) \approx 10^{23}\left(\ln 10^{23}\right) - 10^{23} = 5.20\times10^{24}$

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A 2 Coin System

A 2 identical coin system (N+1=3 possible configurations)

– 2 possible states for each coin, n_1 = heads & n_2 = "tails"

1) The entropy associated with both coins initially "heads" is:

$$W = \left(\frac{2!}{2!\cdot 0!}\right) = 1$$

$$S = k \ln W = \left(1.38\times10^{-23}\,\tfrac{J}{K}\right)\ln(1) = 0\,\tfrac{J}{K}$$

2) When they are flipped, they land with 1 "head" & 1 "tail". The entropy of this (most probable) state is:

$$W = \left(\frac{2!}{1!\cdot 1!}\right) = 2$$

$$S = k \ln W = \left(1.38\times10^{-23}\,\tfrac{J}{K}\right)\ln(2) = 9.57\times10^{-24}\,\tfrac{J}{K}$$

The change in entropy from state (1) to (2) is:

$$\Delta S = S_f - S_i = +9.57\times10^{-24}\,\tfrac{J}{K}$$

The higher entropy state is the most likely to occur…

A 4 Coin System

A 4 identical coin system (N+1=5 possible configurations)

1. The entropy associated with flipping them and obtaining all "heads" (or "tails") is: $W = \left(\frac{4!}{4!\cdot 0!}\right) = 1$

$$S = k \ln W = \left(1.38\times10^{-23}\,\tfrac{J}{K}\right)\ln(1) = 0\,\tfrac{J}{K}$$

2. The entropy associated with flipping 3 "heads" & 1 "tail" is:

$$W = \left(\frac{4!}{3!\cdot 1!}\right) = 4$$

$$S = k \ln W = \left(1.38\times10^{-23}\,\tfrac{J}{K}\right)\ln(4) = 1.91\times10^{-23}\,\tfrac{J}{K}$$

3. The entropy associated with flipping 3 "heads" & 1 "tail" is:

$$W = \left(\frac{4!}{2!\cdot 2!}\right) = 6$$

$$S = k \ln W = \left(1.38\times10^{-23}\,\tfrac{J}{K}\right)\ln(6) = 2.47\times10^{-23}\,\tfrac{J}{K}$$

A 1000 Coin System

A 1000 coin system (N+1=1001 possible configurations)

1) The entropy associated with flipping them and obtaining all "heads" (or "tails") is: $W = \left(\frac{1000!}{1000!\cdot 0!}\right) = 1$

$$S = k \ln W = \left(1.38\times10^{-23}\,\tfrac{J}{K}\right)\ln(1) = 0\,\tfrac{J}{K}$$

2) The entropy associated with flipping 500 "heads" & 500 "tails" is:

$$W = \left(\frac{1000!}{500!\cdot 500!}\right) \Rightarrow S = k \ln W = \left(1.38\times10^{-23}\,\tfrac{J}{K}\right)\ln\left(\frac{1000!}{500!\cdot 500!}\right)$$

$$S = \left(1.38\times10^{-23}\,\tfrac{J}{K}\right)\left[\ln(1000!) - 2\ln(500!)\right]$$

$$S = \left(1.38\times10^{-23}\,\tfrac{J}{K}\right)\left[1000\cdot\ln 1000 - 1000 - 2(500\cdot\ln 500 - 500)\right]$$

$$S = \left(1.38\times10^{-23}\,\tfrac{J}{K}\right)(1000)\ln 2 = 9.57\times10^{-21}\,\tfrac{J}{K}$$

A 2 Dice System

For N = 2 dice, each die has 6 possible states

1) The entropy associated with 2 "ones" is:

$$W = \left(\frac{2!}{2!\cdot 0!\cdot 0!\cdot 0!\cdot 0!\cdot 0!}\right) = 1$$

$$S = k\ \ln W = \left(1.38\times 10^{-23}\,\tfrac{J}{K}\right)\ln(1) = 0\,\tfrac{J}{K}$$

2) The entropy associated with any non-paired combination is:

$$W = \left(\frac{2!}{1!\cdot 1!\cdot 0!\cdot 0!\cdot 0!\cdot 0!}\right) = 2$$

$$S = k\ \ln W = \left(1.38\times 10^{-23}\,\tfrac{J}{K}\right)\ln(2) = 9.57\times 10^{-24}\,\tfrac{J}{K}$$

Entropy & Information

1. In information theory, the entropy of a system represents the lower boundary of bits needed to represent information:

$$S\ (\text{in bits}) = -\sum_{i=1}^{N} p_i \cdot \log_2 p_i$$

Example: A 26 character alphabet

Assuming that each letter has an equal probability of occurrence, a minimum of 5 bits are required to uniquely express all letters (say, using a keyboard):

$$S = -\sum_{i=1}^{26}\left(\frac{1}{26}\right)\cdot \log_2\left(\frac{1}{26}\right) = \log_2(26) \approx 5\ \text{bits}$$