Portland Community College

Phy 212: General Physics II

Chapter 16: Waves I

Lecture Notes

Portland Community College

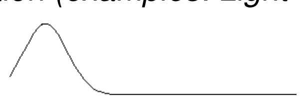
Types of Waves

Types of waves:

a. Mechanical waves
b. Electromagnetic waves
c. Matter waves

Classifying Waves:

1. Transverse Waves: *oscillate perpendicular to the direction of wave propagation (examples: Light & E-M waves)*



2. Longitudinal Waves: *oscillate parallel to the direction of wave propagation (example: Sound)*

Traveling Waves

General form of a traveling wave:

$$y(x,t) = y_m \sin(kx \pm \omega t)$$

Transverse Velocity: $v_{transverse} = \frac{d}{dt}[y(x,t)]$

$$v_{transverse} = \pm\omega y_m \cos(kx \pm \omega t)$$

Velocity Amplitude: $v_m = \pm\omega y_m \Rightarrow |v_m| = \omega y_m$

Acceleration: $a_{transverse} = \frac{d}{dt}[v(x,t)]$

$$a_{transverse} = \mp\omega^2 y_m \sin(kx \pm \omega t)$$

Acceleration Amplitude: $a_m = \mp\omega^2 y_m \Rightarrow |a_m| = \omega^2 y_m$

Wave speed on Stretched String

1. When a transverse disturbance travels along a string, the tension (F_T)in the string functions as the elastic (restoring) force resulting in a traveling wave.
2. The linear mass density (μ) of the string acts as the inertial element for the string which restricts the wave propagation.
3. The wave speed along a string is therefore expressed as:

$$v_{wave} = \sqrt{\frac{F_T}{\mu}}$$

Energy Transmission (Power) in Waves

As a wave travels along a string, the total energy (dE) for a mass element of the string (dx), is the combination of both its KE and PE:

$$dE_{tot} = dK(t) + dU(t)$$

Where: $dK(t) = \frac{1}{2}mv(t)^2 = \frac{1}{2}(\mu dx)\omega^2 x_m^2 \cos^2(kx - \omega t)$

The average energy is: $(dE_{tot})_{avg} = dK(t)_{avg} + dU(t)_{avg}$

$$(dE_{tot})_{avg} = \frac{1}{2}\mu dx \omega^2 x_m^2$$

where: $dK(t)_{avg} = dU(t)_{avg}$

The average power is:

$$P_{avg} = \left(\frac{dE_{tot}}{dt}\right)_{avg} = \frac{1}{2}\mu \frac{dx}{dt}\omega^2 x_m^2 = \frac{1}{2}\mu v \omega^2 x_m^2$$

The Wave Equation

- The motion of a traveling wave is dependent on both time and position:

$$y(x,t) = y_m \sin(kx \pm \omega t)$$

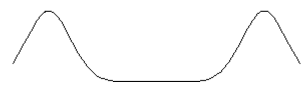
- This equation (and all equations) that describe traveling wave motion, must satisfy the wave equation:

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \cdot \frac{\partial^2 y}{\partial t^2}$$

Superposition of Waves & Interference

Waves of the same type that cross paths do not alter the other.

- They do appear to combine to produce a resulting wave that is a superposition of each wave.
- For 2 identical waves (y_1 and y_2) that are out of phase, the resultant wave is given by:



$$y'(t) = y_1(t) + y_2(t) = y_m\sin(kx-\omega t+\phi) + y_m\sin(kx-\omega t)$$

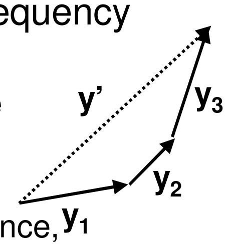
$$y'(t) = 2y_m\left[\sin\left(\frac{(kx-\omega t+\phi)+(kx-\omega t)}{2}\right)\cos\left(\frac{(kx-\omega t+\phi)-(kx-\omega t)}{2}\right)\right]$$

$$y'(t) = \left[2y_m\cos\left(\frac{\phi}{2}\right)\right]\sin\left(kx-\omega t+\frac{\phi}{2}\right) = y'_m\sin\left(kx-\omega t+\frac{\phi}{2}\right)$$

- The resultant wave is itself sinusoidal and its magnitude is depends on the phase difference between y_1 and y_2

Phasors

Phasors are analogous to vectors and are useful for analyzing the superposition of same frequency traveling waves:



1. A phasor is a vector representation of the magnitude (length) and phase (angular direction) for a wave
2. When 2 or more waves undergo interference, the magnitude and phase of the resultant (superimposed) wave can be determined by the vector addition of the corresponding phasors
    a. The resultant phasor magnitude is the magnitude of resultant wave
    b. The resultant phasor angle is the phase angle of resultant wave

Standing Waves in a String

1. Standing waves are self-reinforcing waves produced by the constructive interference of a wave with its reflection.
2. Consider a wave traveling along a string (fixed at each end). When the wave reaches the end of the string it reflects back in the opposite direction (180^o out of phase).
3. The wave will produce a standing wave if the nodes for the resultant wave are stationary

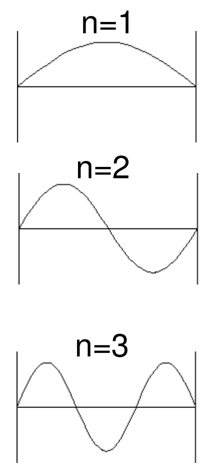
Condition for a standing wave:

$$L = n\left(\frac{\lambda}{2}\right)$$

where:

L is length of string

n is the number of ½ wavelengths (λ)



Standing Wave Superposition