Phy 212: General Physics II

Chapter 15: Oscillations
Lecture Notes

Simple Harmonic Motion (SHM)

The “typical” form of an SHM equation for oscillation in the x direction: $\vec{x}(t) = x_m\cos(\omega t + \phi)\hat{i}$

The Physical Quantities used to describe SHM:

– **Frequency** (f): the rate of oscillation cycles per unit time
– **Period** (T): the time interval during which a complete oscillation cycle occurs

Relation between frequency & period : $T = \frac{1}{f}$

– **Amplitude** (x_m): the peak magnitude of the oscillation
– **Phase Constant** (ϕ): the initial shift in the oscillation at t=0
– **Angular Frequency** (ω): the time rate of oscillations in radians per unit time

Relation between frequency & period : $\omega = 2\pi f$

Velocity & Acceleration for SHM

- Velocity: $\vec{v}(t) = \dfrac{d\vec{x}(t)}{dt} = \dfrac{d}{dt}[x_m \cos(\omega t + \phi)]\hat{i}$

$$\vec{v}(t) = -\omega x_m \sin(\omega t + \phi)\hat{i}$$

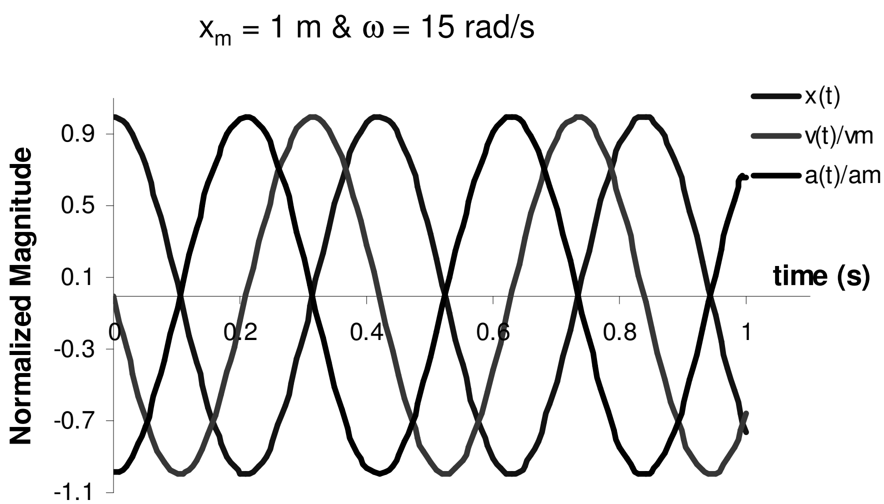
- Velocity Amplitude: $v_m = -\omega x_m$

- Acceleration: $\vec{a}(t) = \dfrac{d\vec{v}(t)}{dt} = \dfrac{d}{dt}[-\omega x_m \sin(\omega t + \phi)]\hat{i}$

$$\vec{a}(t) = -\omega^2 x_m \cos(\omega t + \phi)\hat{i}$$

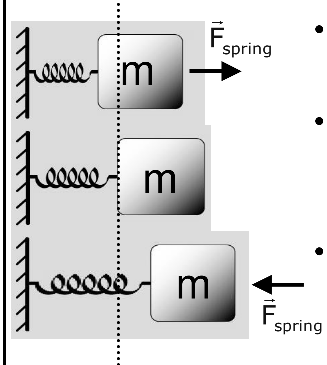
- Acceleration Amplitude: $a_m = -\omega^2 x_m$

Graph of x(t), v(t) and a(t) vs t (for SHM)



Force Law for SHM

- Hooke's Law for elastic restoring force leads to SHM. Consider an ideal spring initially stretched then released (neglect friction and all other forces):



Equilibrium Position

- According to Newton's 2nd Law:

$$\vec{F}_{Net}(t) = \vec{F}_{spring}(t) = -k\vec{x}(t) = m\vec{a}(t)$$

- Since acceleration is related to displacement:

$$\vec{a}(t) = \frac{d^2x(t)}{dt^2} = -\left(\frac{k}{m}\right)\vec{x}(t)$$

- Note that the form of this equation is the same as for SHM:

$$\vec{a}(t) = -\left(\frac{k}{m}\right)\vec{x}(t) = -\omega^2\vec{x}$$

Where: $\omega = \sqrt{\frac{k}{m}} = 2\pi f$ and $T = 2\pi\sqrt{\frac{m}{k}}$

Energy in SHM

For a mass (m) subject to a restoring spring force, the $U_{elastic}$ associated with the mass is:

$$U_{elastic} = \tfrac{1}{2}kx^2$$

As the mass oscillates in SHM, $U_{elastic}$ varies with time:

$$U_{elastic}(t) = \tfrac{1}{2}kx(t)^2 = \tfrac{1}{2}kx_m^2\cos^2(\omega t + \phi)$$

The kinetic energy of the oscillator also varies with time:

$$K(t) = \tfrac{1}{2}mv(t)^2 = \tfrac{1}{2}m\omega^2x_m^2\sin^2(\omega t + \phi)$$

The total mechanical energy, E_{system}, is

$$E_{system} = U_{elastic}(t) + K(t)$$

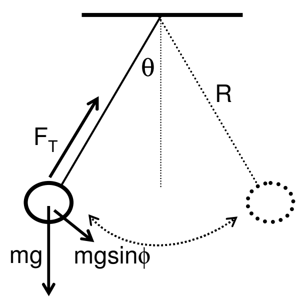
$$E_{system} = \tfrac{1}{2}kx_m^2\cos^2(\omega t + \phi) + \tfrac{1}{2}m\omega^2x_m^2\sin^2(\omega t + \phi)$$

since: $\cos^2(\omega t+\phi)+\sin^2(\omega t+\phi)=1$ & $\omega^2 = \frac{k}{m}$

$$E_{system} = \tfrac{1}{2}kx_m{}^2$$

The Simple Pendulum

A simple pendulum is a moveable “point mass” body, fixed about a single pivot point:



Note: $I_{pendulum} = mR^2$

The gravitational torque exerted on the pendulum is:

$$\tau(t) = -(mg\sin\theta(t))R = I\alpha$$

The angular acceleration is:

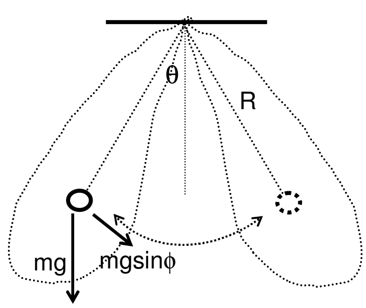
$$\alpha(t) = \left(\frac{mgR}{I}\right)\sin\,\theta(t) \approx \left(\frac{mgR}{I}\right)\theta(t)$$

The period, T, of the motion is:

$$T = 2\pi\sqrt{\frac{I}{mgR}} = 2\pi\sqrt{\frac{R}{g}} \quad \{\text{for small amplitude}\}$$

The Physical Pendulum

A physical pendulum is a more complex body, fixed about a single pivot point:



The gravitational torque exerted the *center of mass* of the pendulum is:

$$\tau(t) = -(mg\sin\theta(t))R = I\alpha$$

The angular acceleration is:

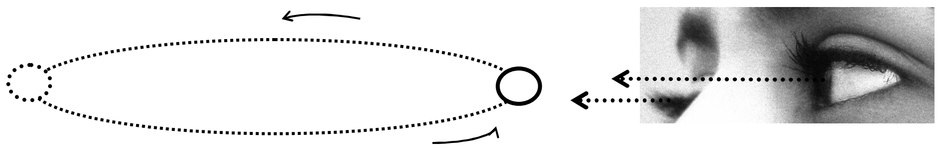
$$\alpha(t) = \left(\frac{mgR}{I}\right)\sin\,\theta(t) \approx \left(\frac{mgR}{I}\right)\theta(t)$$

The period, T, of the motion is:

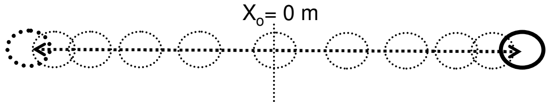
$$T = 2\pi\sqrt{\frac{I}{mgR}} \quad \{\text{for small amplitude}\}$$

SHM & Uniform Circular Motion

1. Consider an object in uniform circular motion



2. Observing the object along the plane of its circular motion, *the object motion appears to be in simple harmonic motion*



$\vec{x}(t) = x_m \cos(\omega t + \phi)\hat{i}$

$\vec{v}(t) = -\omega x_m \sin(\omega t + \phi)\hat{i}$

$\vec{a}(t) = -\omega^2 x_m \cos(\omega t + \phi)\hat{i}$