Rene Descartes (1596-1650)

- Prominent French mathematician & philosopher
- Active toward end of Galileo’s career
- Studied the nature of collisions between objects
- First introduced the concept of momentum
  - he defined momentum ("vis-à-vis") as the product of weight times speed
- Demonstrated the Law of Conservation of Momentum
Linear Momentum

- Linear momentum ($\vec{p}$) represents inertia in motion (Newton momentum as the “quantity of motion”)
  - Conceptually, the effort required to bring a moving object to rest depends not only on its mass (inertia) but also on how fast it is moving

**Definition:** $\vec{p} = m\vec{v}$

- Momentum is a vector quantity with the same direction as the object’s velocity
- SI units are kg m/s

**Newton’s 1st Law revisited:**

*The momentum of an object will remain constant unless it is acted upon by a net force (or impulse)*

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Center of Mass

Center of Mass ($\vec{r}_{cm}$) refers to the average location of mass for a defined mass.

- To determine the center of mass, take the sum of each mass multiplied by its position vector and divide by the total mass of the system or

$$\vec{r}_{cm} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + m_3 \vec{r}_3 + \ldots + m_n \vec{r}_n}{m_1 + m_2 + m_3 + \ldots + m_n} = \frac{\sum_{i=1}^{n} m_i \vec{r}_i}{m_{sys}}$$

- Note, if the objects in the system are in motion, the velocity of the system (center of mass) is:

$$\vec{v}_{cm} = \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2 + \ldots + m_n \vec{v}_n}{m_1 + m_2 + \ldots + m_n} = \frac{\sum_{i=1}^{n} m_i \vec{v}_i}{m_{sys}}$$

- When $\Delta \vec{p}_{\text{system}} = 0$ (i.e. $\vec{F}_{\text{ext}} = 0$) then $\vec{v}_{cm} = \text{constant}$
  - The motion of all bodies even if they are changing individually will always have values such that $\vec{v}_{cm} = \text{constant}$
Impulse-Momentum Theorem

Newton’s 2nd Law, can be rewritten as

\[ \vec{F}_{\text{net}} = m \vec{a} = m \frac{d\vec{v}}{dt} = \frac{d(m\vec{v})}{dt} = \frac{d\vec{p}}{dt} \]

Rearranging terms:

\[ d\vec{p} = \vec{F}_{\text{net}} dt \Rightarrow \int_{\vec{p}_i}^{\vec{p}_f} d\vec{p} = \int_{t_i}^{t_f} \vec{F}_{\text{net}} dt \]

\[ \Delta \vec{p} = \int_{t_i}^{t_f} \vec{F}_{\text{net}} dt \ 	ext{this is the Net Impulse!!} \]

Definition of Impulse associated with an applied force:

\[ J = \Delta \vec{p} = \int_{t_i}^{t_f} \vec{F} dt \]

- The SI units for impulse are N\,s
- For a constant force (or average force), impulse simplifies to:

\[ J = \Delta \vec{p} = \vec{F}_{\text{avg}} \Delta t \]

- Therefore, Impulse represents simultaneously:
  1. The product of the force times the time: \( \vec{F}_{\text{avg}} \Delta t \)
  2. The change in linear momentum of the object: \( \Delta \vec{p} = m\vec{v}_f - m\vec{v}_i \)

Notes on Impulse

- Impulses always occur as action-reaction pairs
- The force-time relationship is observed in many “real world” examples:
  - Automobile safety:
    - Dashboards
    - Airbags
    - Crumple zones
  - Product packaging
    - Styrofoam spacers
  - Sports
    - Tennis: racket string tension
    - Baseball: “juiced” baseballs & baseball bats (corked & aluminum vs. wood)
    - Golf: the “spring-like” effect of golf club heads
    - Boxing gloves: (lower impulsive forces in the hands)
A Superman Problem

It is well known that bullets and missiles bounce off Superman's chest. Suppose a bad guy sprays Superman's chest with 0.003 kg bullets traveling at a speed of 300 m/s (fired from a machine gun at a rate of 100 rounds/min). Each bullet bounces straight back with no loss in speed.

Problems:

a) What is the impulse exerted on Superman's chest by a single bullet?

b) What is the average force exerted by the stream of bullets on Superman's chest?

Collisions

A specific type of interaction between 2 objects. The basic assumptions of a collision:

1. Interaction is short lived compared to the time of observation
2. A relatively large force acts on each colliding object
3. The motion of one or both objects changes abruptly following collision
4. There is a clean separation between the state of the objects before collision vs. after collision

3 classifications for collisions:

- Perfectly elastic: colliding objects bounce off each other and no energy is lost due to heat formation or deformation ($K_{system}$ is conserved)
- Perfectly inelastic: colliding objects stick together ($K_{system}$ is not conserved)
- Somewhat inelastic (basically all other type of collisions): KE is not conserved
Conservation of Linear Momentum

The total linear momentum of a system will remain constant when no external net force acts upon the system, or

\[(\vec{p}_1 + \vec{p}_2 + \ldots)_{\text{before collision}} = (\vec{p}_1 + \vec{p}_2 + \ldots)_{\text{after collision}}\]

• **Note:** Individual momentum vectors may change due to collisions, etc. but the linear momentum for the system remains constant

• Useful for solving collision problems:
  – Where all information is not known/given
  – To simplify the problem

• Conservation of Momentum is even more fundamental than Newton’s Laws!!

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Conservation of Momentum (Examples)

• The ballistic pendulum

• 2 body collisions (we can’t solve 3-body systems….provider)
  – Perfectly inelastic \( (E_{\text{pre-collision}} \neq E_{\text{post-collision}}) \)
  – Perfectly elastic \( (E_{\text{pre-collision}} = E_{\text{post-collision}}) \)

• Collisions in 2-D or 3-D:
  – Linear momentum is conserved by components:

\[ (\vec{p}_1 + \vec{p}_2 + \ldots)_{\text{before collision}} = (\vec{p}_1 + \vec{p}_2 + \ldots)_{\text{after collision}}\]

By Components:

\[
\begin{align*}
(p_{1x} + p_{2x} + \ldots \hat{i})_{\text{before collision}} &= (p_{1x} + p_{2x} + \ldots \hat{i})_{\text{after collision}} \\
(p_{1y} + p_{2y} + \ldots \hat{j})_{\text{before collision}} &= (p_{1y} + p_{2y} + \ldots \hat{j})_{\text{after collision}}
\end{align*}
\]
Notes on Collisions & Force

• During collisions, the forces generated:
  – Are short in duration
  – Are called impulsive forces (or impact forces or collision forces)
  – Often vary in intensity/magnitude during the event
  – Can be described by an average collision force:

\[ \bar{F}_{\text{net}} = \bar{F}_{\text{avg}} = \frac{\Delta \dot{p}}{\Delta t} \left( \text{i.e., impulse} \right) \text{time} \]

Example: a golf club collides with a 0.1 kg golf ball (initially at rest), \( \Delta t \) 0.01s. The velocity of the ball following the impact is 25 m/s.

The impulse exerted on the ball is:
\[ \Delta p = m \Delta \dot{v} = (0.1 \text{ kg})(25 \text{ m/s} - 0 \text{ m/s}) \hat{i} = 2.5 \text{ N} \cdot \text{s} \hat{i} \]

The average impulsive force exerted on the ball is:
\[ \bar{F}_{\text{avg}} = \frac{\Delta \dot{p}}{\Delta t} = \frac{2.5 \text{ N} \cdot \text{s}}{0.01 \text{ s}} \hat{i} = 250 \text{ N} \hat{i} \]

The average impulsive force exerted on the club is:
\[ \bar{F}_{\text{avg}} = \frac{\Delta \dot{p}}{\Delta t} = \frac{-2.5 \text{ N} \cdot \text{s}}{0.01 \text{ s}} \hat{i} = -250 \text{ N} \hat{i} \]