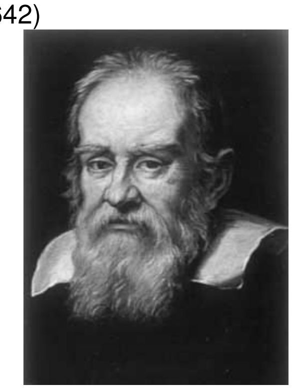
Phy 211: General Physics I

Chapter 2: Motion along a Straight Line
Lecture Notes

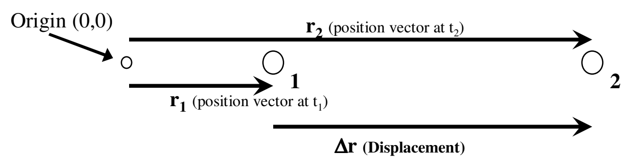
Galileo Galilei (1564-1642)



- Credited with establishing the “scientific method”
- Based his scientific hypotheses on observation and experimentation
- First to use telescope for astronomical observation
  - Observed the following:
    - The craters and features on the Moon
    - The moons of Jupiter & the rings of Saturn
    - Sun spots
  - Based on his observations supported Copernican Theory
- Conclusively refuted Aristotelian ideology (and contradicted Church doctrine)
  - Placed under “house arrest” as punishment
- Studied accelerated motion and established the first equations of kinematics
- Proposed the Law of Inertia, which later became known as Newton’s 1st law of Motion

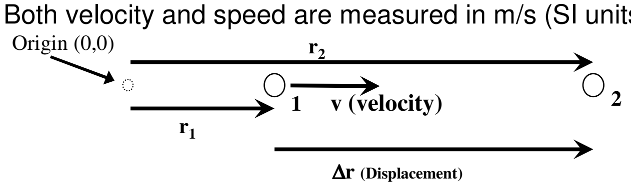
Displacement

- **Motion:** An object is in motion if it changes its position relative to a *reference point (Origin). Motion is relative.*
- **Position ($\vec{r}$)** a vector that connects an object's location to a reference point (origin). As an object moves, the its position changes.
- **Displacement** ($\Delta\vec{r}$) is the difference between the final position and the initial position of a moving object; its magnitude is measured in meters (m)



Speed & Velocity

- **Velocity** is a vector that reflects the time rate of change of displacement. The direction of the velocity vector is the same as the displacement vector ($\Delta\vec{r}$). The magnitude of a velocity vector is the **speed**. Both velocity and speed are measured in m/s (SI units).



- **Average Velocity:** $\vec{v}_{avg} = \dfrac{\vec{displacement}}{time} = \dfrac{\Delta\vec{r}}{\Delta t}$
- **Average Speed:** $v_{avg} = \left|v_{avg}\right| = \dfrac{\text{distance traveled}}{\Delta t}$
- **Instantaneous Velocity:** $\vec{v} = \lim_{\Delta t \to 0}\left(\dfrac{\Delta\vec{r}}{\Delta t}\right) = \dfrac{d\vec{r}}{dt}$
- **Speed:** $v = \left|\vec{v}\right| = \text{"the magnitude of the velocity"}$

Acceleration

- When the velocity vector is changing in time, the **acceleration** vector describes the time rate of change of the velocity vector.
- **Acceleration** (**a**) is a vector with the same direction as the change in velocity (**Δv**) vector. It is measured in m/s^2.
- **Average Acceleration:** $\vec{a}_{avg} = \dfrac{change\_in\_velocity}{time} = \dfrac{\Delta\vec{v}}{\Delta t}$
- **Instantaneous Acceleration:** $\vec{a} = \lim_{\Delta t \to 0}\left(\dfrac{\Delta\vec{v}}{\Delta t}\right) = \dfrac{d\vec{v}}{dt} = \dfrac{d^2\vec{r}}{dt^2}$

Example 1: A car accelerates from 0 to 30 m/s in 6 s. Find the average acceleration.

Since **Δv** = 30 m/s, $\vec{a}_{avg} = \dfrac{\Delta\vec{v}}{\Delta t} = \dfrac{30\ {}^{m}\!/\!_{s}}{6\ \text{s}} = 5\dfrac{m}{s^2}$

Example 2: A ball descends in water with a velocity defined by: v(t) = -1t^2 + 10t + 5 (assume SI units). Find the instantaneous acceleration after 2 seconds.

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(-1t^2 + 10t + 5\right) = -2t + 10 = 6\frac{m}{s^2}$$

Analyzing 1-D Motion

How are the definitions of displacement, velocity and acceleration applied to describe linear motion?

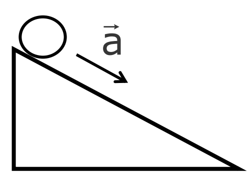
Example 1: A ball rolls down an incline, from rest at t_o=0s, at an acceleration of 2.5 m/s^2.

1. What is the velocity after the rock has fallen 3 s?

$$\vec{a} = \vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t} \Rightarrow \Delta\vec{v} = \vec{v} - \vec{v}_o = \vec{a}t \qquad \text{(eqn. a)}$$

since $\vec{v}_o = 0\frac{m}{s}$ the velocity down the incline is:

$$\vec{v} = \vec{a}t = \left(2.5\tfrac{m}{s^2}\right)(3s) = 7.5\tfrac{m}{s}\ \text{(down the incline)}$$



2. How far does the rock travel during this 3 s?

   There are 2 ways to approach this problem:

   i. Using acceleration:

$$\vec{a} = \frac{d\vec{v}}{dt} \Rightarrow d\vec{v} = \vec{a}dt \Rightarrow \int_{v_o}^{v} d\vec{v} = \int_0^t \vec{a}dt \Rightarrow \vec{v} - \vec{v}_o = \vec{a}t \ \text{ or}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \vec{v}_o + \vec{a}t \quad d\vec{r} = \vec{v}dt \Rightarrow \int_{r_o}^{r} d\vec{r} = \int_0^t \vec{v}dt = \int_0^t \left(\vec{v}_o + \vec{a}t\right)dt$$

$$\Rightarrow \vec{r} - \vec{r}_o = \vec{v}_o t + \tfrac{1}{2}\vec{a}t^2 \qquad \text{(eqn. b)} \Rightarrow \vec{r} - \vec{r}_o = \tfrac{1}{2}\left(2.5\tfrac{m}{s^2}\right)(3s)^2 = 11.25\ \text{m}$$

Analyzing 1-D Motion (cont.)

ii. Using initial and final velocity:

$$\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{v}_o + \vec{v}}{2} \Rightarrow \Delta\vec{r} = \vec{r} - \vec{r}_o = \left(\frac{\vec{v}_o + \vec{v}}{2}\right)t \quad \text{(eqn. c)}$$

$$\Rightarrow \vec{r} - \vec{r}_o = \left(\frac{0\frac{m}{s} + 7.5\frac{m}{s}}{2}\right)(3s) = 11.25\ m$$

3. How fast is the rock moving after it has fallen 10 m?
   In this problem, time is not explicitly given so we have to identify a workaround:

$$v - v_o = at \Rightarrow t = \frac{v - v_o}{a} \ \& \ r - r_o = v_o t + \tfrac{1}{2}at^2$$

$$\Rightarrow r - r_o = v_o\left(\frac{v - v_o}{a}\right) + \tfrac{1}{2}a\left(\frac{v - v_o}{a}\right)^2 \Rightarrow r - r_o = v_o\left(\frac{v - v_o}{a}\right) + \tfrac{1}{2}a\left(\frac{v - v_o}{a}\right)^2$$

$$\Rightarrow r - r_o = \frac{vv_o - v_o^2}{a} + \frac{v^2 - v_o^2}{2a} \Rightarrow r - r_o = \frac{vv_o - v_o^2}{a} + \frac{v^2 - 2vv_o + v_o^2}{2a}$$

or $v^2 - v_o^2 = 2a(r - r_o) \quad$ (eqn. d)

$$\Rightarrow \quad v = \pm\sqrt{v_o^2 + 2a(r - r_o)} = \pm\sqrt{2\left(2.5\tfrac{m}{s^2}\right)(10m)} = 7.07\tfrac{m}{s}$$

Equations of Kinematics

To summarize the previous 2 slides, when $t_o = 0$ and $\vec{a}$ is constant), a useful set of equations of motion has been derived for 1-D motion in scalar form:

a. $v_1 - v_o = at$ {Velocity Equation}

b. $r_1 - r_o = v_o t + ½ at^2$ {Displacement Equation}

c. $r_1 - r_o = ½(v_o + v_1)t$ {Average Velocity}

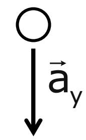
d. $v_1^2 - v_o^2 = 2a(r - r_o)$ {Galileo's Equation}

These equations will allow us to completely describe the motion of a moving object and represent our "toolbox" for studying kinematics!

Free Falling Bodies

1. The 1-D motion of objects subject to gravity
2. The only motion considered is vertical *(use **y** for position)*
3. There is no air resistance
4. Acceleration of falling object is constant:

$$\vec{a}_y = -g\,\hat{y} = -\left(9.8\ \text{m/s}^2\right)\,\hat{y}$$



Example 1: Consider a rock dropped from rest.

1. What is the velocity after the rock has fallen 3 s?
2. How far does the rock travel during this 3 s?
3. How fast is the rock moving after it has fallen 10 m?

Example 2: Consider a rock tossed into the air with an initial upward velocity of 5 m/s.

1. How long would it take for the rock to return to the thrower (initial height)?
2. What is the highest position the rock reaches during its ascent?

Graphical Analysis of Velocity & Acceleration

- position vs. time graph
  - The slope is the velocity
- velocity vs. time graph
  - The slope is the acceleration
  - The area under the curve is the displacement