Momentum & Impulse:

Brad Pitt (80 kg) and Angelina Jolie (50 kg) enjoy a relaxing drive across the south of France in their red sports car (1400 kg). Starting at rest (at a stop sign), they accelerate to a cruising speed of 35 m/s then slow back down to rest as they approach an upcoming intersection.

a) What is the combined linear momentum of Brad and Angelina's (including car), as a system, at the stop first sign?

Ans. $\vec{v}_o = 0 \Rightarrow \vec{p}_o = 0$

b) What is the combined linear momentum of this system at cruising speed?

Ans. $\vec{v}_o = \left(35\ \tfrac{m}{s}\right)\hat{i} \Rightarrow \vec{p}_f = (80\text{ kg} + 50\text{ kg} + 1400\text{ kg})\left(35\ \tfrac{m}{s}\right)\hat{i} = \left(53{,}550\text{ kg}\tfrac{m}{s}\right)\hat{i}$

c) What are Brad and Angelina's linear momentum, respectively, at cruising speed?

Ans.
$$\vec{p}_{Brad} = (80\text{ kg})\left(35\ \tfrac{m}{s}\right)\hat{i} = \left(2800\text{ kg}\tfrac{m}{s}\right)\hat{i}$$
$$\vec{p}_{Angelina} = (50\text{ kg})\left(35\ \tfrac{m}{s}\right)\hat{i} = \left(1750\text{ kg}\tfrac{m}{s}\right)\hat{i}$$

d) Calculate individual and combined impulse vectors for the Brad-Angelina-car system as they go from rest to cruising speed.

Ans.
$$\vec{J}_{Brad} = \Delta\vec{p}_{Brad} = (2800\text{ N}\cdot\text{s})\hat{i}$$
$$\vec{J}_{Angelina} = \Delta\vec{p}_{Angelina} = (1750\text{ N}\cdot\text{s})\hat{i}$$
$$\vec{J}_{car} = \Delta\vec{p}_{car} = \vec{J}_{system} - \vec{J}_{Brad} - \vec{J}_{Angelina} = (49000\text{ N}\cdot\text{s})\hat{i}$$

e) It takes 8 seconds for the car to go from rest to cruising speed, what is the average net force exerted on Brad, Angelina and the car?

Ans. $\vec{F}_{Net\ System} = \dfrac{\Delta\vec{p}_{System}}{\Delta t} = (6694\text{ N})\hat{i}$

f) What is the net impulse vector for the road as the car goes from rest to cruising speed? Which of Newton's Laws explains this phenomenon?

Ans. $\Delta\vec{p}_{Road} = -\Delta\vec{p}_{System} = (-53{,}550\text{ N}\cdot\text{s})\hat{i}$ This is Newton's 3rd Law.

g) Determine the impulse vector for the car seat due, to Angelina, as the car accelerates to cruising speed.

Ans. $\vec{J}_{carseat} = -\vec{J}_{Angelina} = (-1750\text{ N}\cdot\text{s})\hat{i}$

Scenario 1: Inelastic Collision

During impact, Jennifer's convertible collides inelastically with Brad and Angelina's car and both vehicles are thrust into the intersection.

1. What is the linear momentum of Jennifer (and her car) just before impact?

   Ans. $\vec{p}_{J_o} = (45\text{ kg} + 1300\text{ kg})(40\ \tfrac{m}{s})\hat{j} = (53{,}800\text{ kg}\tfrac{m}{s})\hat{j}$

2. What is the momentum of Brad, Angelina (and their car) just before impact?

   Ans. $\vec{p}_{B\text{-}A\text{-}car_o} = (80\text{ kg} + 50\text{ kg} + 1400\text{ kg})(25\ \tfrac{m}{s})\hat{i} = (38{,}250\text{ kg}\tfrac{m}{s})\hat{i}$

3. What is the combined momentum of this system immediately after impact?

   7Ans. $\vec{p}_{system_o} = \vec{p}_{J_o} + \vec{p}_{B\text{-}A\text{-}car_o} = (38{,}250\text{ kg}\tfrac{m}{s})\hat{i} + (53{,}800\text{ kg}\tfrac{m}{s})\hat{j} = \vec{p}_{System_f}$

4. What is the velocity vector for each vehicle <u>immediately</u> following impact?

   Ans. $\vec{v}_{system_f} = \dfrac{\vec{p}_{system_f}}{m_{system}} = \dfrac{(38{,}250\text{ kg}\frac{m}{s})\hat{i} + (53{,}550\text{ kg}\frac{m}{s})\hat{j}}{2870\text{ kg}} = (13.3\tfrac{m}{s})\hat{i} + (18.7\tfrac{m}{s})\hat{j}$

5. Calculate the impulse vector for each vehicle (including drivers) due to the collision?

   $$\vec{J}_J = \Delta\vec{p}_J = (45\text{kg}+1300\text{kg})\left((13.3\tfrac{m}{s})\hat{i} + (18.7\tfrac{m}{s} - 40.0\tfrac{m}{s})\hat{j}\right)$$

   Ans. $$\vec{J}_J = (1345\text{kg})\left((13.3\tfrac{m}{s})\hat{i} + (-21.3\tfrac{m}{s})\hat{j}\right) = (17{,}889\text{ kg}\tfrac{m}{s})\hat{i} + (-28{,}649\text{ kg}\tfrac{m}{s})\hat{j}$$

   $$\vec{J}_{BA} = -\vec{J}_J = (-17{,}889\text{ kg}\tfrac{m}{s})\hat{i} + (28{,}649\text{ kg}\tfrac{m}{s})\hat{j}$$

6. Suppose the impact lasts for 0.5 sec. What is average impact force exerted on each vehicle?

   $$\vec{F}_J = \frac{\vec{J}_J}{\Delta t} = \frac{(17{,}889\text{ kg}\frac{m}{s})\hat{i} + (-28{,}649\text{ kg}\frac{m}{s})\hat{j}}{0.5\text{ s}}$$

   Ans. $$\vec{F}_J = (35{,}777)\hat{i} + (-57298\text{ N})\hat{j}$$

   $$\vec{F}_{BA} = -\vec{F}_J = (-35{,}777)\hat{i} + (57298\text{ N})\hat{j}$$

7. How does the impact force received by Jennifer's vehicle compare with the force received by Brad and Angelina's?

   Ans. The impact forces are equal magnitude but opposite direction.

8. How does the acceleration (due to impact) of Jennifer's vehicle compare with the acceleration of Brad and Angelina's? Calculate the respective accelerations.

   Ans.
   $$\vec{a}_J = \frac{\vec{F}_J}{m_J} = \frac{(35{,}777)\hat{i} + (-57298\text{ N})\hat{j}}{1345\text{ kg}} = (26.6\tfrac{m}{s^2})\hat{i} + (-42.6\tfrac{m}{s^2})\hat{j}$$

   $$\vec{a}_{BA} = \frac{\vec{F}_{BA}}{m_{BA}} = \frac{(-35{,}777)\hat{i} + (57298\text{ N})\hat{j}}{1530\text{ kg}} = (-23.4\tfrac{m}{s^2})\hat{i} + (-37.4\tfrac{m}{s^2})\hat{j}$$

9. If Jennifer were driving a 2500 kg SUV instead of a small car, would her vehicle receive more, less, or the same impact force as Brad and Angelina's car? Explain.

   Ans. The impact forces would still bee equal magnitude but opposite direction.

Scenario 2: Elastic Collision

Alternatively, as Jennifer's car collides into the side of Brad and Angelina's, the impact knocks Brad and Angelina's car diagonally into the intersection but the vehicles do not become attached and remain independent of each other, a perfectly elastic collision.

1. What is the combined momentum of this system immediately following impact?

Ans. $\vec{p}_{system_o} = \vec{p}_{J_o} + \vec{p}_{B\text{-}A\text{-}car_o} = (38{,}250\ kg\tfrac{m}{s})\hat{i} + (53{,}800\ kg\tfrac{m}{s})\hat{j} = \vec{p}_{System_f}$

2. Calculate the velocity for the system pre-impact.

Ans. $$\vec{v}_{system_o} = \frac{\vec{p}_{system_o}}{m_{system}} = \frac{(38{,}250\ kg\tfrac{m}{s})\hat{i} + (53{,}550\ kg\tfrac{m}{s})\hat{j}}{2870\ kg} = (13.3\tfrac{m}{s})\hat{i} + (18.7\tfrac{m}{s})\hat{j}$$

3. What is the KE of each vehicle (w/passengers), respectively, prior to impact?

Ans.
$$K_J = \tfrac{1}{2}m_J v_J^2 = \tfrac{1}{2}(1345kg)(40\tfrac{m}{s})^2 = 1.076x10^6\ J$$
$$K_{BA} = \tfrac{1}{2}m_{BA} v_{BA}^2 = \tfrac{1}{2}(1530kg)(25\tfrac{m}{s})^2 = 4.78x10^5\ J$$

4. Calculate the total KE for the 2 vehicle system, just after impact.

Ans.
$$K_{system_f} = K_{system_o} = K_J + K_{BA}$$
$$K_{system_f} = 1.08x10^6\ J + 4.78x10^5\ J = 1.55x10^6 J$$

5. Express the post-impact total momentum and KE by components.

Ans.
$$\vec{p}_{system_f} = \vec{p}_{system_o} = (38{,}250\ kg\tfrac{m}{s})\hat{i} + (53{,}800\ kg\tfrac{m}{s})\hat{j}$$
$$K_{system\ x} = 4.78x10^5\ J$$
$$K_{system\ y} = 1.08x10^6\ J$$

6. Calculate the system velocity post-impact.

Ans. $$\vec{v}_{system_f} = \frac{\vec{p}_{system_f}}{m_{system}} = \frac{(38{,}250\ kg\tfrac{m}{s})\hat{i} + (53{,}550\ kg\tfrac{m}{s})\hat{j}}{2870\ kg} = (13.3\tfrac{m}{s})\hat{i} + (18.7\tfrac{m}{s})\hat{j}$$

7. What is the velocity of each vehicle (w/passengers), respectively, immediately following impact?

Ans. It is best to assume that the collision is a side-swipe, so that all impact forces are in the y-direction.

Step 1. Analyze the y-direction components, since all of the "action" occurs along this direction:

$$p_{system_y} = 53{,}800\ kg\tfrac{m}{s} = m_J v_{Jy} + m_{BA} v_{BAy}$$

$$K_{system_y} = 1.08x10^6 J = \tfrac{1}{2}m_J v_{Jy}^2 + \tfrac{1}{2}m_{BA} v_{BAy}^2$$

Step 2. Solve for a velocity in the momentum equation the substitute this value into the KE equation. Then solve for the velocities:

$$v_{Jy} = \frac{53{,}800\ \text{kg}\frac{m}{s} - (1530\ \text{kg})v_{BAy}}{(1345\ \text{kg})} = 40.0\tfrac{m}{s} - 1.14v_{BAy}$$

$$K_{system_y} = \tfrac{1}{2}m_J\left(40.0\tfrac{m}{s} - 1.14v_{BAy}\right)^2 + \tfrac{1}{2}m_{BA}v_{BAy}^2 = 1.08\text{x}10^6\text{J}$$

$$\Rightarrow \tfrac{1}{2}(1345\ \text{kg})\left(40.0\tfrac{m}{s} - 1.14v_{BAy}\right)^2 + \tfrac{1}{2}(1530\ \text{kg})v_{BAy}^2 - 1.076\text{x}10^6\text{J} = 0$$

$$(1345\ \text{kg})\left(40.0\tfrac{m}{s} - 1.14v_{BAy}\right)^2 + (1530\ \text{kg})v_{BAy}^2 - 2.16\text{x}10^6\text{J} = 0$$

$$1600\tfrac{m^2}{s^2} - (91.2\tfrac{m}{s})v_{BAy} + 1.30v_{BAy}^2 + 1.14v_{BAy}^2 - 1606\tfrac{m^2}{s^2} = 0$$

$$2.44v_{BAy}^2 - (91.2\tfrac{m}{s})v_{BAy} - 6\tfrac{m^2}{s^2} = 0$$

$$v_{BAy}^2 - (37.4\tfrac{m}{s})v_{BAy} - 2.46\tfrac{m^2}{s^2} = 0$$

$$v_{BAy} = 37.5\tfrac{m}{s}\ \ (\text{or } -0.07\tfrac{m}{s})$$

$$v_{Jy} = 40.0\tfrac{m}{s} - 1.14(37.5\tfrac{m}{s}) = -2.75\tfrac{m}{s}$$

8. What is the linear momentum of each vehicle (w/passengers), respectively, immediately after impact?

Ans.

$$\vec{p}_{J_f} = (1345\ \text{kg})(-2.75\tfrac{m}{s})\hat{j} = -(3814\ \text{kg}\tfrac{m}{s})\hat{j}$$

$$\vec{p}_{B\text{-}A\text{-}car_f} = (1530\ \text{kg})\left[(25\tfrac{m}{s})\hat{i} + (37.5\tfrac{m}{s})\hat{j}\right] = \left[(38{,}250\ \text{kg}\tfrac{m}{s})\hat{i} + (57375\ \text{kg}\tfrac{m}{s})\hat{j}\right]$$

9. What is the KE of each vehicle (w/passengers), respectively, immediately after impact?

Ans.

$$K_J = \tfrac{1}{2}m_Jv_J^2 = \tfrac{1}{2}(1345\text{kg})(-2.75\tfrac{m}{s})^2 = 5086\ \text{J}$$

$$K_{BA} = \tfrac{1}{2}m_{BA}v_{BA}^2 = \tfrac{1}{2}(1530\text{kg})\left[(25\tfrac{m}{s})^2 + (37.5\tfrac{m}{s})^2\right] = 1.55\text{x}10^6\ \text{J}$$