1. A 50 kg painter stands on top of a 3 m platform on the roof of a 20 m building.

a) What is the painter's gravitational potential energy (with respect to the roof surface).

Ans. $U_{platform} = mgy_{platform} = 1470\ J$

b) What is the painter's gravitational potential energy (with respect to the ground).

Ans. $U_{platform+building} = mgy_{platform+building} = 11270\ J$

c) The painter jumps off the platform onto the rooftop. Using the roof as reference, what is the kinetic energy of the painter just before he lands on the roof?

Ans. $K_{roof} = U_{platform} = 1470\ J$

d) Using the ground as a reference, calculate the kinetic energy of the painter just before he hits the roof.

Ans. $K_{roof} = U_{platform+building} - U_{building} = 11270\ J - 9800\ J = 1470\ J$

2. An Olympic high diver (mass = 70 kg) stands on a platform 20 m above the surface of the water.

a) What is the diver's gravitational potential energy?

Ans. $U_{platform} = mgy_{platform} = 13720\ J$

b) The athlete in (a) then dives into the water. If it assumed that all of the diver's potential energy is conserved, what is the diver's kinetic energy at the moment just before entering the water?

Ans. $K_{at\ water} + U_{at\ water} = K_{platform} + U_{platform} = 13720\ J$ or $K_{at\ water} = U_{platform} = 13720\ J$

c) What is the diver's speed just before entering the water?

Ans. $$|\vec{v}| = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(13720\ J)}{70\ kg}} = 19.8\ \tfrac{m}{s}$$

d) Calculate the instantaneous power generated by the gravitational force just before the diver enters the water.

Ans. $$P = m\vec{g}\cdot\vec{v} = (70\ kg)\left(-9.8\tfrac{m}{s^2}\right)\left(-19.8\tfrac{m}{s}\right) = 13583\ W$$

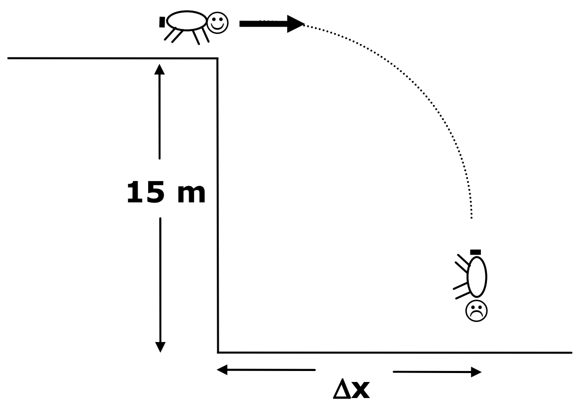
e) Determine the average power generated by the gravitational force to pull the diver to the earth.

Ans. $$\vec{v}_{avg} = \left(\frac{19.8\ \frac{m}{s}}{2}\right)\hat{j} = -9.9\tfrac{m}{s}\hat{j}$$

Therefore, $$P_{avg} = m\vec{g}\cdot\vec{v}_{avg} = (70\ kg)\left(-9.8\tfrac{m}{s^2}\right)\left(-9.9\tfrac{m}{s}\right) = 6791\ W$$

3. A sheep (mass = 40 kg) scampering at 15 m/s jumps off the ledge of a cliff 15 m above the ground below.

a) What is the sheep's gravitational potential energy at the top of the cliff? Use the ground level below the cliff as reference.



Ans. $U = mgy = 5880\ \text{J}$

b) What is the sheep's kinetic energy at the top of the cliff?

Ans. $K = \frac{1}{2}m\vec{v}\cdot\vec{v} = \frac{1}{2}(40\ \text{kg})(15\frac{\text{m}}{\text{s}})^2 = 4500\ \text{J}$

c) Neglecting air drag, how much does the sheep's kinetic energy change during its fall (just before it impacts the ground below)?

Ans. $\Delta K = -\Delta U = 4500\ \text{J}$

d) What is the sheep's speed just before impacting the ground?

Ans. $|\vec{v}_x| = 15\frac{\text{m}}{\text{s}}$ and $|\vec{v}_y| = \sqrt{\frac{2\Delta K}{m}} = \sqrt{\frac{2(5800\ \text{J})}{40\ \text{kg}}} = 17.0\frac{\text{m}}{\text{s}}$

$$|\vec{v}| = \sqrt{(15\tfrac{\text{m}}{\text{s}})^2 + (17.0\tfrac{\text{m}}{\text{s}})^2} = 22.7\ \tfrac{\text{m}}{\text{s}}$$

e) What is the magnitude and direction of sheep's velocity just before impacting the ground?

Ans. from above: $|\vec{v}| = 22.7\ \frac{\text{m}}{\text{s}}$ and $\theta_v = \tan^{-1}\left(\frac{-17.0\frac{\text{m}}{\text{s}}}{15.0\frac{\text{m}}{\text{s}}}\right) = -48.6^\circ$

In component form: $\vec{v} = 15\frac{\text{m}}{\text{s}}\ \hat{i} - 17.0\frac{\text{m}}{\text{s}}\ \hat{j}$

f) Calculate the instantaneous power generated by the gravitational force just before the sheep hits the ground.

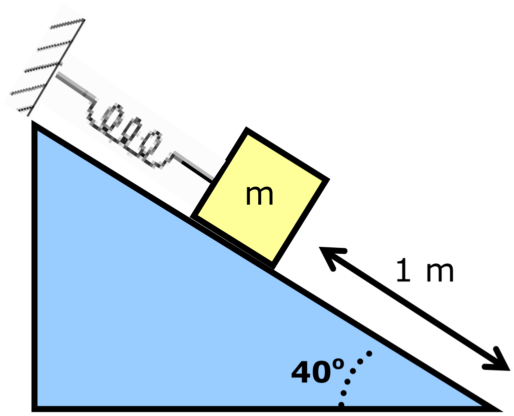
Ans. $P = m\vec{g}\cdot\vec{v} = (40\ \text{kg})(-9.8\frac{\text{m}}{\text{s}^2})(-17\frac{\text{m}}{\text{s}}) = 6664\ \text{W}$

g) Determine the average (time) rate of mechanical energy transfer to the sheep during its fateful descent.

Ans. The average rate of power is: $\vec{v}_{avg} = (15\frac{\text{m}}{\text{s}})\ \hat{i} + \left(\frac{-17\frac{\text{m}}{\text{s}}}{2}\right)\hat{j} = 15\frac{\text{m}}{\text{s}}\hat{i} - 8.5\frac{\text{m}}{\text{s}}\hat{j}$

Therefore, $P_{avg} = m\vec{g}\cdot\vec{v}_{avg} = (40\ \text{kg})(-9.8\frac{\text{m}}{\text{s}^2})(-8.5\frac{\text{m}}{\text{s}}) = 3332\ \text{W}$

4. A mass (2 kg), at rest, is attached to an ideal spring (k=500 N/m) along the surface of a 40° incline. The mass is positioned 1.0 m up the incline (from the base of the incline).



a) What is the total mechanical energy of the mass-spring system? Use the base of the incline as reference.

Ans. $E_{mech} = U_o + K_o = mgy_o \quad (K_o = 0\ J)$

$E_{mech} = (2\ kg)(9.8\ m/s^2)(1m)(\sin 40^o) = 6.30\ J$

b) An external applied force pulls the mass 0.2 m down the incline, stretching the spring, then releases the mass. How much mechanical energy does the external applied force transfer to the mass-spring system?

Ans. $W_{F_{applied}} = \Delta E_{mech} = \Delta K + \Delta U_{spring} + \Delta U_g$

$$W_{F_{applied}} = \tfrac{1}{2}kr^2 + mg\Delta y = \tfrac{1}{2}\left(500\tfrac{N}{m}\right)(0.2\ m)^2 + (2\ kg)\left(9.8\tfrac{m}{s^2}\right)(-0.2\ m)\sin 40^o$$

$$W_{F_{applied}} = 10\ J - 2.52\ J = 7.48\ J = \Delta E_{mech}$$

Alternatively,

$$W_{applied} = \int_{r_o}^{r_f} \vec{F}_{applied} \cdot d\vec{r} = \int_{r_o}^{r_f} (-k\vec{r} + m\vec{g}) \cdot d\vec{r}$$

$$W_{applied} = -\int_{r_o}^{r_f} krdr(\cos 180^o) + \int_{r_o}^{r_f} mgdr(\cos 50^o)$$

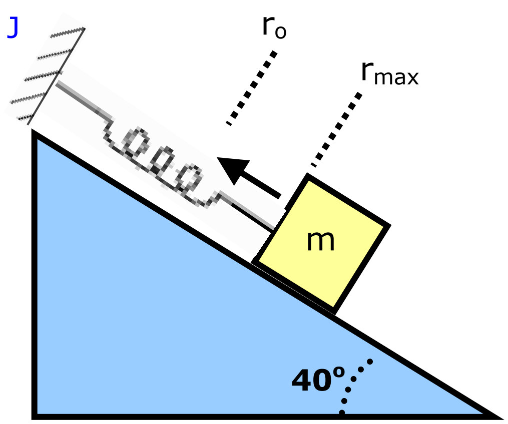
$$W_{applied} = \int_{r_o}^{r_f} krdr + \int_{r_o}^{r_f} mgdr(\cos 50^o)$$

$$W_{applied} = \left(\tfrac{1}{2}kr^2 + mg\cos 50^o r\right)\Big|_{r_o}^{r_f} = \tfrac{1}{2}\left(500\tfrac{N}{m}\right)(-0.2m)^2 - (2kg)\left(9.8\tfrac{m}{s^2}\right)\cos 50^o(-0.2m)$$

$$W_{applied} = 7.48\ J = \Delta E_{mech}$$

c) What is the difference between the gravitational potential energy (ΔU_g) for the mass between its initial position and the stretched position just as it is released?

Ans. $\Delta U_g = mg\Delta y = (2\ kg)(9.8\ m/s^2)(-0.2m)(\sin 40^o) = -2.52\ J$



d) What is the difference between the elastic potential energy (ΔU_{spring}) for the mass between its initial position and the stretched position just as it is released?

Ans. $\Delta U_{spring} = ½\ kr^2 = ½\ (500\ N/m)(-0.2m)^2 = 10\ J$

e) What is the total mechanical energy of the mass-spring system at the stretched position just as it is released?

Ans. $E_{mech} = (E_{mech})_{initial} + \Delta U_g + \Delta U_{spring}$

$E_{mech} = 6.30\ J + (-2.52\ J) + 10\ J = 13.78\ J$

f) What is the speed of the mass just as it reaches its initial (equilibrium) position?

Ans. $$|\vec{v}| = \sqrt{\frac{2\Delta K}{m}} = \sqrt{\frac{2\left(-\Delta U_{spring} - \Delta U_g\right)}{m}} = \sqrt{\frac{2(10\ J - 2.52\ J)}{2kg}} = 2.73\tfrac{m}{s}$$

g) If the kinetic friction coefficient between the mass and the incline is 0.2, how fast is the mass traveling when it reaches the initial (equilibrium) position?

Ans. $\left|\vec{f}_k\right| = \mu_k F_N = \mu_k mg\cos 40^\circ = (0.2)(2\text{ kg})(9.8\,\frac{m}{s^2})\cos 40^\circ = 3.00\text{N}$

$$E_{mech\ final} = E_{mech\ initial} - W_{f_k} = K + U_g$$
$$E_{mech\ final} = 13.78\text{ J} - f_k \Delta r\cos 180^\circ$$
$$E_{mech\ final} = 13.78\text{ J} - 0.6\text{ J} = 13.18\text{ J}$$
$$K = E_{mech\ final} - U_g = 13.18\text{ J} - 6.30\text{ J}$$
$$K = 6.88\text{ J} \Rightarrow |\vec{v}| = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(6.88\text{ J})}{2\text{ kg}}} = 2.62\,\frac{m}{s}$$

Alternatively, since $W_{f_k} = \vec{f}_k \cdot \Delta\vec{r} = (3.00\text{ N})(0.2\text{ m})\cos 180^\circ = -0.60\text{ J}$

$$|\vec{v}| = \sqrt{\frac{2\Delta K}{m}} = \sqrt{\frac{2\left(-\Delta U_{spring} - \Delta U_g + W_{f_k}\right)}{m}} = \sqrt{\frac{2(10\text{ J} - 2.52\text{ J} - 0.60\text{ J})}{2\text{kg}}} = 2.62\,\frac{m}{s}$$

h) Calculate the nonconservative work performed on the mass-spring system (with friction) as the mass travels from its stretched position to the equilibrium position.

Ans. $W_{NC} = W_{f_k} = -0.60\text{ J}$ (all other forces in this system are conservative)