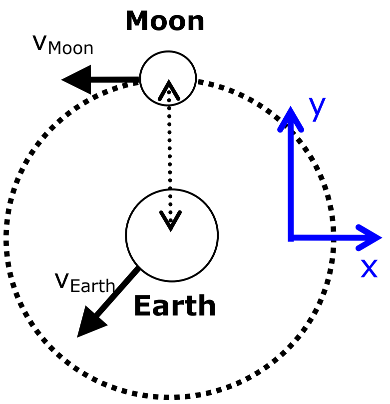
Center of Mass:

1. a) Determine the center of mass for the Earth-Moon system.



Ans. Using the center of mass for the earth as the origin (0,0) & standard Cartesian oriented coordinates:

$$\vec{r}_{com} = \frac{m_{moon}\vec{r}_{moon} + m_{earth}\vec{r}_{earth}}{m_{moon} + m_{earth}} = \frac{m_{moon}\vec{r}_{moon}}{m_{moon} + m_{earth}} \quad \text{where } \vec{r}_{earth}=0$$

$$\vec{r}_{com} = \left(\frac{7.36\text{x}10^{22}\ \text{kg}}{7.36\text{x}10^{22}\ \text{kg} + 5.98\text{x}10^{24}\ \text{kg}}\right)(3.82\text{x}10^{8}\ \text{m})\ \hat{j}$$

$$\vec{r}_{com} = (4.64\text{x}10^{6}\ \text{m})\ \hat{j}$$

b) At a particular moment, the moon is moving with a linear speed (v_{moon}) of 992 m/s (while in a "circular" orbit around the Earth) while the Earth is moving at $2.99\text{x}10^4$ m/s (v_{earth}) at a 45° counter-clockwise angle to v_{moon}. Determine the velocity of the center-of-mass for the Earth-Moon system.

Ans.

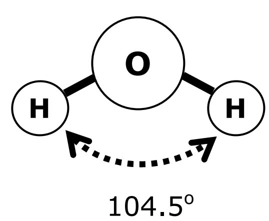
$$\vec{v}_{com} = \frac{m_{moon}\vec{v}_{moon} + m_{earth}\vec{v}_{earth}}{m_{moon} + m_{earth}}$$

$$\vec{v}_{com} = \left(\frac{(7.36\text{x}10^{22}\text{kg})(-992\,\tfrac{m}{s})\hat{i} + (5.98\text{x}10^{24}\text{kg})(2.99\text{x}10^{4}\,\tfrac{m}{s})(\sin 225^{o}\hat{i} + \cos 225^{o}\hat{j})}{7.36\text{x}10^{22}\ \text{kg} + 5.98\text{x}10^{24}\ \text{kg}}\right)$$

$$\vec{v}_{com} = (-2.09\text{x}10^{4}\,\tfrac{m}{s})\ \hat{i} + (-2.09\text{x}10^{4}\,\tfrac{m}{s})\ \hat{j}$$

2. a) The average mass of an oxygen and hydrogen, respectively, is 15.999 amu and 1.0079 amu (1 amu = $1.661\text{x}10^{-27}$ kg). Calculate the mass of the oxygen and hydrogen atom, respectively, in SI units.

The Water Molecule



Ans.

$$m_H = \left(\frac{1.0079\ \text{amu}}{1}\right)\left(\frac{1.661 \text{ x } 10^{-27}\ \text{kg}}{1\ \text{amu}}\right) = 1.674 \text{ x } 10^{-27}\ \text{kg}$$

$$m_O = \left(\frac{15.999\ \text{amu}}{1}\right)\left(\frac{1.661 \text{ x } 10^{-27}\ \text{kg}}{1\ \text{amu}}\right) = 26.574 \text{ x } 10^{-27}\ \text{kg}$$

b) The center-to-center distance between the oxygen and either hydrogen atom is 0.957854 Å (1 Å = 10^{-10} m). Calculate the distance between an oxygen atom and a hydrogen atom, respectively, in SI units.

Ans. $$\left|\vec{r}_{H\text{ to }O}\right| = \left(\frac{0.957854\ \text{Å}}{1}\right)\left(\frac{1 \text{ x } 10^{-10}\ \text{m}}{1\ \text{Å}}\right) = 9.57854 \text{ x } 10^{-11}\ \text{m}$$

c) Determine the center of mass for a water molecule.

Ans. Using the center of mass for the oxygen atom as the origin:

$$\vec{r}_{com} = \left(\frac{m_H\vec{r}_{H_1} + m_O\vec{r}_O + m_H\vec{r}_{H_2}}{2m_H + m_O}\right) = (0\ \text{m})\ \hat{i} - (6.685\text{x}10^{-12}\ \text{m})\ \hat{j}$$

Impulse and Momentum:

1.
a) What is the change in momentum for this object?

Ans. $\Delta\vec{p} = (50\text{kg})(10\tfrac{\text{m}}{\text{s}} - 30\tfrac{\text{m}}{\text{s}})\hat{i} = -1000\ \tfrac{\text{kg}\cdot\text{m}}{\text{s}}\ \hat{i}$

b) Calculate the impulse received by the object using the force equation above.

Ans. $\vec{J} = \int_{0s}^{0.5s} \vec{F}dt = \int_{0s}^{0.5s} \left(-(4000\tfrac{\text{N}}{\text{s}})t - 1000\text{N}\right)\cdot dt\ \hat{i}$

$\vec{J} = \left(-(2000\tfrac{\text{N}}{\text{s}})t^2 - (500\text{N})t\right)\Big|_{0s}^{0.5s}\ \hat{i} = -1000\text{N}\cdot\text{s}\ \hat{i}$

c) What is the average force exerted on the object?

Ans. $\vec{F}_{avg} = \dfrac{\Delta\vec{p}}{\Delta t} = \dfrac{-1000\ \tfrac{\text{kg}\cdot\text{m}}{\text{s}}}{0.5\ \text{s}}\ \hat{i} = -2000\ \text{N}\ \hat{i}$

d) Calculate the impulse received by the object using the average force calculated in (c).

Ans. $\vec{J} = \vec{F}_{avg}\Delta t = (-2000\ \text{N})(0.5\ \text{s})\ \hat{i} = -1000\text{N}\cdot\text{s}\ \hat{i}$

e) If the deceleration from 30 m/s to 10 m/s had occurred over a time interval of 5 sec (obviously subject to a different force equation), what is the impulse received by the object?

Ans. Since the initial and final momentum values are the same as above,

$\vec{J} = \Delta\vec{p} = (50\text{kg})(10\tfrac{\text{m}}{\text{s}} - 30\tfrac{\text{m}}{\text{s}})\hat{i} = -1000\ \tfrac{\text{kg}\cdot\text{m}}{\text{s}}\ \hat{i}$

f) Calculate the average force exerted on the object in (e).

Ans. $\vec{F}_{avg} = \dfrac{\Delta\vec{p}}{\Delta t} = \dfrac{-1000\ \tfrac{\text{kg}\cdot\text{m}}{\text{s}}}{5\ \text{s}}\ \hat{i} = -200\ \text{N}\ \hat{i}$

2. a) What is the initial and final linear momentum of the vehicle for this road test (in SI units)?

Ans. $\vec{p}_i = 0\tfrac{\text{kg}\cdot\text{m}}{\text{s}}\ \hat{i}$ & $\vec{p}_f = (1581.8\ \text{kg})(51.2\tfrac{\text{m}}{\text{s}})\hat{i} = 8.10\text{x}10^4\ \tfrac{\text{kg}\cdot\text{m}}{\text{s}}\ \hat{i}$

b) What is the change in momentum for the Corvette during this trial?

Ans. $\Delta\vec{p} = \vec{p}_f - \vec{p}_i = 8.10\text{x}10^4\ \tfrac{\text{kg}\cdot\text{m}}{\text{s}}\ \hat{i}$

c) What is the rate of momentum change for the Corvette during this test? How does this value compare the average force vector exerted by the road on the Corvette during this trial?

$\dfrac{\Delta\vec{p}}{\Delta t} = \dfrac{8.09\text{x}10^4\ \tfrac{\text{kg}\cdot\text{m}}{\text{s}}}{12.8\ \text{s}}\ \hat{i} = 6.33\text{x}10^3\text{N}\ \hat{i} = \vec{F}_{Net}$

Ans. $\vec{F}_{Net} = \vec{f}_{s_{by\ road}} + \vec{F}_{drag} = \left(f_{s_{by\ road}} - F_{drag}\right)\hat{i}$.

$\Rightarrow \vec{f}_{s_{by\ road}} = \left(F_{Net} + F_{drag}\right)\hat{i}$

Thus the rate of momentum change is less than the (static friction) force by the road exerted on the vehicle.

d) How much impulse (change in momentum) does the earth receive during this trial?

Ans. $\vec{J}_{earth} = -\vec{J}_{Corvette} = \Delta\vec{p}_{Corvette} = -8.10\text{x}10^4\ \tfrac{\text{kg}\cdot\text{m}}{\text{s}}\ \hat{i}$

e) Estimate the change in velocity ($\Delta\vec{v}_{earth}$) for the earth during this same road test. The mass of the earth is $5.98\text{x}10^{24}$ kg.

Ans. $\Delta\vec{v}_{earth}=\dfrac{\vec{J}_{earth}}{m_{earth}}=\left(\dfrac{-8.10\text{x}10^{4}\ \frac{kg\cdot m}{s}}{5.98\text{x}10^{24}kg}\right)\hat{i} = \left(-1.35\text{x}10^{-20}\ \frac{m}{s}\right)\hat{i}$

Conservation of Linear Momentum:

1. An 100 kg object traveling at 50 m/s collides (perfectly inelastic) with a 50 kg object initially at rest.

a. Determine the linear momentum vector for each object prior to the collision?

Ans. $\vec{p}_{1i} = (100\text{ kg})(50\tfrac{m}{s})\hat{i}= 5000\tfrac{kg\cdot m}{s}\hat{i}$ & $\vec{p}_{2i} = 0\tfrac{kg\cdot m}{s}\hat{i}$

b. If the collision is perfectly inelastic, what is the total momentum vector for the 2-object system following the collision?

Ans. $\vec{p}_{system_{before}} = \vec{p}_{system_{after}} = 5000\tfrac{kg\cdot m}{s}\hat{i}$

c. What is the center-of-mass velocity vector for the two combined objects following the collision?

Ans. $\vec{v}_{system}= \dfrac{\vec{p}_{system_{after}}}{m_{system}} = \left(\dfrac{5000\ \frac{kg\cdot m}{s}}{150\text{ kg}}\right)\hat{i}= 33.3\tfrac{m}{s}\hat{i}$

d. Calculate the impulse vector exerted on each object.

$$\vec{J}_1=\Delta\vec{p}_1=(100\text{ kg})(33.3\tfrac{m}{s}-50\tfrac{m}{s})\hat{i}=-1.67\text{x}10^{3}N\cdot s$$

Ans. and

$$\vec{J}_2=\Delta\vec{p}_2=(50\text{ kg})(33.3\tfrac{m}{s})\hat{i}=1.67\text{x}10^{3}N\cdot s$$

e. What is the kinetic energy of each object following the collision?

Ans.
$$K_1=\tfrac{1}{2}m_1v_1^2=\tfrac{1}{2}(100\text{ kg})(33.3\tfrac{m}{s})^2=5.54\text{x}10^4J$$
$$K_2=\tfrac{1}{2}m_2v_2^2=\tfrac{1}{2}(50\text{ kg})(33.3\tfrac{m}{s})^2=2.77\text{x}10^4J$$

f. Calculate the non-conservative work (W_{nc}) performed on the 2 object system.

Ans. Since all of the forces associated with the collision are internal forces,

$$K_{system_{before}}=\tfrac{1}{2}(100\text{ kg})(50\tfrac{m}{s})^2=1.25\text{x}10^5J$$
$$K_{system_{before}}=5.54\text{x}10^4J + 2.77\text{x}10^4J =8.31\text{x}10^4J$$
$$W_{NC}=\Delta K_{system}=K_{system_{after}} - K_{system_{before}}$$
$$W_{NC}=8.31\text{x}10^4J -1.25\text{x}10^5J = -4.90\text{x}10^4J$$

2. A 100 kg object traveling at 50 m/s collides head-on (perfectly elastic) with a 50 kg object initially at rest.

a. What is the kinetic energy of each object prior to the collision?

Ans.
$$K_1=\tfrac{1}{2}m_1v_1^2=\tfrac{1}{2}(100\text{ kg})\left(50\tfrac{m}{s}\right)^2=1.25\text{x}10^5\text{J}$$
$$K_2=\tfrac{1}{2}m_2v_2^2=\tfrac{1}{2}(50\text{ kg})\left(0\tfrac{m}{s}\right)^2=0\text{ J}$$

b. What is the kinetic energy of the two object system immediately following the collision?

Ans. By definition, KE is conserved: $K_{before}=K_{after}=1.25\text{x}10^5\text{J}$

c. Using the laws for conservation of linear momentum and conservation of mechanical energy, determine the velocity vectors for each object following the collision?

Ans. To begin, let's establish that
$$\left|\vec{p}_{system}\right| = m_1v_1 + m_2v_2 = 5000\tfrac{kg\cdot m}{s}$$
$$K_{system} = \tfrac{1}{2}m_1v_1^2 + \tfrac{1}{2}m_2v_1^2 = 1.25\text{x}10^5\text{J}$$

Solve for v1 in the momentum equation the substitute this relation into the energy equation:

$$v_1 = \frac{5000\tfrac{kg\cdot m}{s} - m_2v_2}{m_1} = \frac{5000\tfrac{kg\cdot m}{s} - (50\text{ kg})v_2}{100\text{ kg}} = 50\tfrac{m}{s} - 0.5v_2$$
$$v_1^2 = \left(50\tfrac{m}{s} - 0.5v_2\right) = 2500\tfrac{m^2}{s^2} - \left(50\tfrac{m}{s}\right)v_2 + 0.25v_2^2$$
$$K_{system} = \tfrac{1}{2}m_1v_1^2 + \tfrac{1}{2}m_2v_2^2 = 1.25\text{x}10^5\text{J}$$
$$K_{system} = \tfrac{1}{2}(100\text{kg})\left(2500\tfrac{m^2}{s^2} - \left(50\tfrac{m}{s}\right)v_2 + 0.25v_2^2\right) + \tfrac{1}{2}(50\text{kg})v_2^2 = 1.25\text{x}10^5\text{J}$$
$$K_{system} = 1.25\text{x}10^5\text{J} - \left(2500\tfrac{kg\cdot m}{s}\right)v_2 + (12.5\text{kg})v_2^2 + (25\text{kg})v_2^2 = 1.25\text{x}10^5\text{J}$$

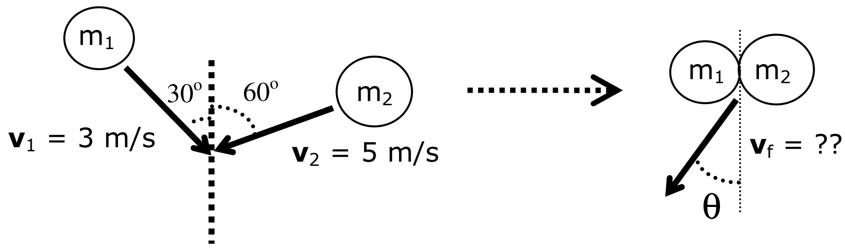
Solving for v_2 and v_1: $\vec{v}_2=\left(\dfrac{2500\tfrac{kg\cdot m}{s}}{37.5\text{kg}}\right)\hat{i}=66.7\tfrac{m}{s}\hat{i}$ and $\vec{v}_1=\left[50\tfrac{m}{s}-0.5\left(66.7\tfrac{m}{s}\right)\right]\hat{i}=16.7\tfrac{m}{s}\hat{i}$

d. What is the impulse received by each object during the collision?

Ans.
$$\vec{J}_1 = \Delta\vec{p}_1 = (100\text{kg})\left(16.7\tfrac{m}{s}-50\tfrac{m}{s}\right)\hat{i}=-3330\text{ N}\cdot\text{s }\hat{i}$$
$$\vec{J}_2 = \Delta\vec{p}_2 = (50\text{kg})\left(66.7\tfrac{m}{s}-0\tfrac{m}{s}\right)\hat{i}=3330\text{ N}\cdot\text{s }\hat{i}$$

2-D Collisions:

Two objects (m_1=100 kg and m_2 = 150 kg) collide as shown in the diagram (there are no external forces acting on either object):



a) What is the magnitude and direction of the linear momentum vector (prior to the collision) for each object, respectively?

Ans.
$$|\vec{p}_{1i}| = (100\text{ kg})(3\tfrac{m}{s}) = 300\tfrac{kg\cdot m}{s} \Rightarrow \theta_{\vec{p}_1} = 300^o$$
$$|\vec{p}_{2i}| = (150\text{ kg})(5\tfrac{m}{s}) = 750\tfrac{kg\cdot m}{s} \Rightarrow \theta_{\vec{p}_2} = 210^o$$

b) Express the linear momentum vector (prior to the collision) for each object in component form.

Ans.
$$\vec{p}_{1i} = (100\text{ kg})(3\tfrac{m}{s})(\sin30^o\hat{i} - \cos30^o\hat{j}) = 150\tfrac{kg\cdot m}{s}\hat{i} - 260\tfrac{kg\cdot m}{s}\hat{j}$$
$$\vec{p}_{2i} = (150\text{ kg})(5\tfrac{m}{s})(-\sin60^o\hat{i} - \cos60^o\hat{j}) = -650\tfrac{kg\cdot m}{s}\hat{i} - 375\tfrac{kg\cdot m}{s}\hat{j}$$

c) Determine the linear momentum vector for the 2 object system (prior to the collision), magnitude and direction.

Ans.
$$\vec{p}_{system_i} = \vec{p}_{1i} + \vec{p}_{2i} = \left(150\tfrac{kg\cdot m}{s} - 650\tfrac{kg\cdot m}{s}\right)\hat{i} + \left(-260\tfrac{kg\cdot m}{s} - 375\tfrac{kg\cdot m}{s}\right)\hat{j}$$
$$\vec{p}_{system_i} = \left(-500\tfrac{kg\cdot m}{s}\right)\hat{i} + \left(-635\tfrac{kg\cdot m}{s}\right)\hat{j}$$

d) Considering the collision to be perfectly inelastic, what is the linear momentum vector (in component form) for the 2 object system following the collision?

Ans. $\vec{p}_{system_f} = \vec{p}_{system_i} = \left(-500\tfrac{kg\cdot m}{s}\right)\hat{i} + \left(-635\tfrac{kg\cdot m}{s}\right)\hat{j}$

e) Determine the velocity vector, in component form, for the 2 object system following the collision?

Ans.
$$\vec{v}_{system_f} = \frac{\vec{p}_{system_f}}{m_{system}}$$
$$\vec{v}_{system_f} = \frac{\left(-500\tfrac{kg\cdot m}{s}\right)\hat{i} + \left(-635\tfrac{kg\cdot m}{s}\right)\hat{j}}{250\text{ kg}} = \left(-2\tfrac{m}{s}\right)\hat{i} + \left(-2.54\tfrac{m}{s}\right)\hat{j}$$

f) What is the magnitude and direction for the velocity for the 2 object system following the collision?

Ans.
$$\vec{v}_{system_f} = \sqrt{\left(-2\tfrac{m}{s}\right)^2 + \left(-2.54\tfrac{m}{s}\right)^2} = 3.23\tfrac{m}{s}$$
$$\theta_{\vec{v}_{system_f}} = \tan^{-1}\left(\frac{2\tfrac{m}{s}}{2.54\tfrac{m}{s}}\right) = 38.2^o \text{ (according to diagram)}$$