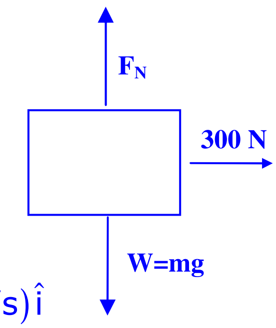
1. A 50 kg box is pulled horizontally a distance of 5 m along a frictionless floor by a 300 N force.



a) How much work is performed on the box?

Ans. $W_{net}=(300\text{ N})(5\text{ m})(\cos 0^o)=1500\text{ J}$

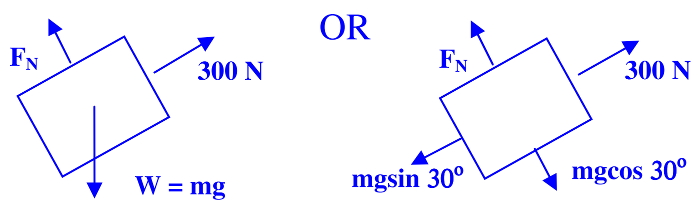
b) What is change in kinetic energy of the box?

Ans. Work-Energy Theorem, $\Delta K = W_{net} = 1500\text{ J}$

c) What is the horizontal velocity of the box at the end of the 5 m?

Ans. assuming $\vec{v}_{xo}= 0\text{ m/s }\hat{i} + 0\text{ m/s }\hat{j}$, $\Delta\vec{v}_x = \vec{v}_{xf} = \sqrt{\frac{2\Delta K}{m}} = (7.75\text{ m/s})\hat{i}$

2. A 50 kg box is pulled a distance of 5 m up a 30^o incline along a frictionless floor by a 300 N force.



a) How much work is performed on the box?

Ans.
$$\vec{F}_{net} = \left(300\text{ N} - mg\sin 30^o\right)\hat{i} + \left(F_N - mg\cos 30^o\right)\hat{j} = 55\text{ N }\hat{i}$$
$$W_{net} = \vec{F}_{net}\cdot\Delta\vec{r} = (55\text{ N})(5\text{ m})\cos 0^o = 275\text{ J}$$

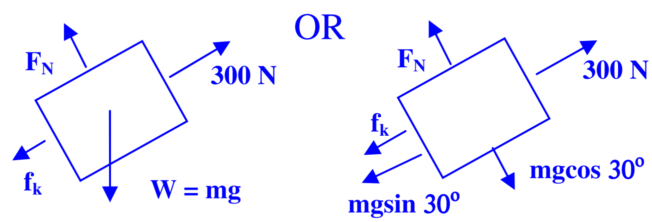
b) What is change in kinetic energy of the box?

Ans. Work-Energy Theorem, $\Delta K = W_{net} = 275\text{ J}$

c) What is the velocity of the box at the end of the 5 m?

Ans. assuming $\vec{v}_{xo}= 0\text{ m/s }\hat{i} + 0\text{ m/s }\hat{j}$, $\Delta\vec{v}_x = \vec{v}_{xf} = \sqrt{\frac{2\Delta K}{m}} = (3.32\text{ m/s})\hat{i}$

d) If there is kinetic friction ($\mu_k=0.1$) between the box and the incline, how much work is performed on the box by the 300 N force?



Ans. $W = \vec{F}\cdot\Delta\vec{r} = (300\text{ N})(5\text{ m})\cos 0^o = 1500\text{ J}$

e) How much work is performed (net) on the box? Is there a difference? Why?

Ans.
$$\vec{F}_{net} = \left(300\text{ N} - f_k - mg\sin 30^o\right)\hat{i} + \left(F_N - mg\cos 30^o\right)\hat{j} = m\vec{a}\text{ (along incline)},$$
since $\vec{f}_k = -\mu_k F_N\hat{i} = 42.4\text{ N }\hat{i}$

$$\vec{F}_{Net} = (300\text{ N} - 42.4\text{ N} - 245\text{ N})\hat{i} = 12.6\text{ N }\hat{i},\quad W_{net} = \vec{F}_{Net}\cdot\Delta\vec{r} = (12.6\text{ N})(5\text{ m}) = 63\text{ J}$$

3. According to the label, a 2 oz. Snickers candy bar contains 266 nutritional calories (or kcal).

a) How much energy in Joules is in one Snickers bar? (1 kcal = 1000 cal = 4186 J)

Ans. $E = 266 \text{ kcal} = 1.11\text{x}10^6 \text{ J}$

b) If an average person (m=60 kg) were eat a Snickers bar and use its energy for mechanical work, how high could this person climb in elevation (ignoring drag and friction)?

Ans. If all the energy were converted to Potential Energy, $E = PE_{grav} = mg\Delta y$:

$\Delta y = E/mg = (1.11\text{x}10^6 \text{ J})/(60 \text{ kg})(9.8 \text{ m/s}^2) = 1887 \text{ m or } 1.887 \text{ km}$

c) How fast could an average person accelerate to (from rest) using only the energy of this Snickers bar? Apply the Work-Energy Theorem to solve this problem.

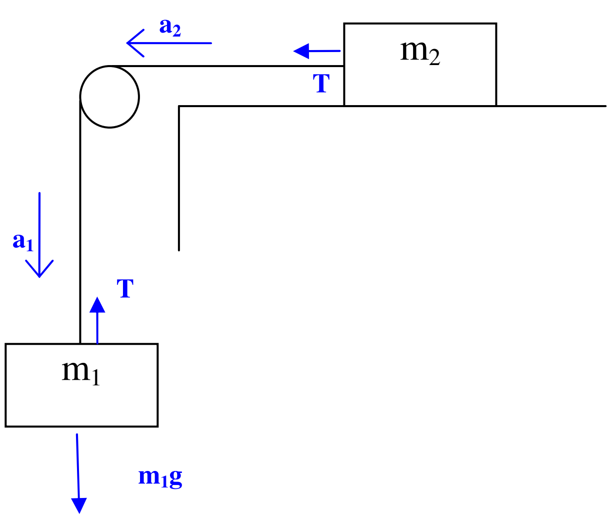
Ans. If all the energy were converted to work, $E = W_{net} = KE = \frac{1}{2} mv^2$:

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{(2)(1.11\text{x}10^6 \text{ J})}{60 \text{ kg}}} = 192 \text{ m/s} \textit{ (clearly, we are drag limited!)}$$

d) The average person expends approximately 1500 kcal per day just to maintain physiological function. What is the power generated by an average person, in Watts?

Ans. $$P = \frac{E}{\Delta t} = \frac{6.28\text{x}10^6 \text{ J}}{86400 \text{ s}} = 72.7 \text{ W}$$

4. Consider the 2 mass pulley system to the right where M_1 = 10 kg and M_2 = 20 kg. Assume the pulley has no mass and there is no friction between (i) M_2 and the surface supporting it and (ii) the pulley itself.



a. If M_1 drops a distance of 1 m, how much work is performed on the block?

Ans. Apply Newton's 2[nd] Law to both blocks:

$$\vec{F}_{Net\ m_1} = (m_1 g - T)\hat{j}_1 = m_1\vec{a}_1$$

$$\& \ \vec{F}_{Net\ m_2} = T\ \hat{i}_2 = m_2\vec{a}_2 \ \left\{\text{where } \hat{j}_1 = \hat{i}_2 \ \& \ |a_1| = |a_2|\right\}$$

Solving for T: $$|\vec{T}| = \frac{m_1 g}{\left(\frac{m_1}{m_2} + 1\right)}$$

$$W_{net\ m_1} = \vec{F}_{net\ m_1} \cdot \Delta\vec{y} = m_1 g\left(1 - \frac{1}{\left(\frac{m_1}{m_2} + 1\right)}\right)\Delta y = 32.7 \text{ J}$$

b. How much work is performed on M_2?

Ans. $$W_{net\ m_2} = \vec{F}_{Net\ m_2} \cdot \Delta\vec{r}_2 = T\ \hat{i}_2 = \frac{m_1 g \Delta y}{\left(\frac{m_1}{m_2} + 1\right)} = 65.3 \text{ J}$$

c. How much work is performed by the tension force on the 2 blocks?

Ans. $W_{T\ on\ m1} = (-65.3 \text{ N})(1 \text{ m}) = -65.3 \text{ J}$ & $W_{T\ on\ m2} = (65.3 \text{ N})(1 \text{ m}) = 65.3 \text{ J}$

Therefore: $W_{net\ by\ T} = W_{T\ on\ m1} + W_{T\ on\ m2} = -65.3\text{ J} + 65.3\text{ J} = 0\text{ J}$ {T does no work}

d. What is the kinetic energy of each block at the end of the drop?

Ans. Apply the Work-Energy Theorem:

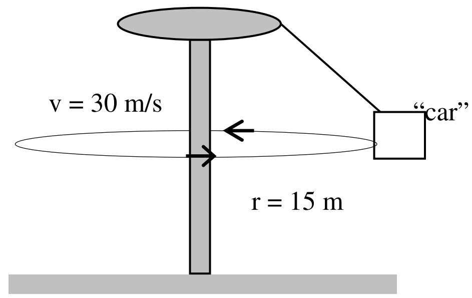
$K_{m1} = \Delta K_{m1} = W_{net\ m1} = 32.7\text{ J}$ & $K_{m2} = \Delta K_{m2} = W_{net\ m2} = 65.3\text{ J}$

e. How fast are the two blocks moving at the end of the drop?

Ans. If both masses start from rest:

$\vec{v}_{m_1} = 2.6\text{ m/s }\hat{j}_1$ & $\vec{v}_{m_2} = 2.6\text{ m/s }\hat{i}_2$

5. **An Amusement Park Ride.** Consider a dangling car attached to a "massless" rope, as shown below. The car rotates about the center at a speed of 30 m/s rope and has a mass of 150 kg. The radius of the circular path traveled by the car is 15.0 m.



a. How much work is performed by the centripetal force on the car during one complete revolution?

Ans. $W_{Fc} = 0\text{ J}$ since $\vec{F}_c$ is perpendicular to $\vec{v}$

b. How much work is performed by the tension force (in the rope) on the car during one complete revolution?

Ans. Since $\vec{F}_{Net} = \vec{F}_C = \vec{T}$, then $W_{by\ T} = 0\text{ J}$

6. Joey (60 kg) jumps out of a moving airplane ($\vec{v}_{airplane} = 89\text{ m/s }\hat{i}$) and plummets to the ground. During a particular phase of his descent ($v_{oy} = 30$ m/s), he travels downward 100 m against an average air drag force of 310 N.

a. What is the average net force exerted on Joey during this phase of descent?

Ans. $\vec{F}_{Net} = \left(F_{Drag} - mg\right)\hat{j} = (310\text{N} - 588\text{N})\hat{j} = -278\text{N }\hat{j}$

b. How much net work is performed on Joey? *Apply the Work-Energy Theorem.*

Ans. $W_{Net} = \vec{F}_{Net} \cdot \Delta\vec{r} = (-278\text{N})(-100\text{m}) = 27800\text{ J}$

c. How much work is performed by each force exerted on Joey?

Ans.
$$W_{Drag} = \vec{F}_{Drag} \cdot \Delta\vec{r} = (310\text{N})(-100\text{m}) = -31000\text{ J}$$
$$W_{F_g} = m\vec{g} \cdot \Delta\vec{r} = (-588\text{N})(-100\text{m}) = 58800\text{ J}$$

d. Express Joey's velocity vector at the end of this phase?

Ans.

$$W_{Net} = \Delta K_y = 27800\text{ J}$$
$$K_{fy} = K_{oy} + \Delta K_y = 27000\text{ J} + 27800\text{ J} = 54800\text{ J}$$
$$\left|\vec{v}_y\right| = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(54800\text{ J})}{60\text{ kg}}} = 42.7\tfrac{\text{m}}{\text{s}}$$
$$\vec{v} = (89\tfrac{\text{m}}{\text{s}})\hat{i} - (42.7\tfrac{\text{m}}{\text{s}})\hat{j}$$

7. A 2005 Corvette convertible coupe automobile was tested by Road & Track magazine. The following performance data were collected:

Tested Weight (lb)	Mass (kg)	¼ mile time (s)	Final speed (mph)	Final speed (m/s)
3480	1581.8	12.8	114.5	51.2

a) What is the KE at the start of the trial, v = 0 mph?

Ans. $K_o = 0$ J

b) What is the KE at the end of the quarter mile?

Ans. $K_f = ½\, mv^2 = 2.07\text{x}10^6$ J

c) How much work is performed on the Corvette during this trial?

Ans. $W_{net} = \Delta K = K_f - K_o = 2.07\text{x}10^6$ J

d) What is the average net power in (W or J/s) generated by the Corvette during this test?

Ans. $P_{avg} = W_{net}/\Delta t = (2.07\text{x}10^6 \text{ J})/(12.8 \text{ s}) = 1.62\text{x}10^5$ W

e) What is the average net power in (hp) generated by the Corvette during this test? Note: 1 hp = 745.7 W

Ans. $P_{avg} = (1.62\text{x}10^5 \text{ W})(1 \text{ hp}/745.7 \text{ W}) = 217$ hp

f) What is the average speed (in m/s) of the Corvette during this test?

Ans. $$\Delta\vec{r} = (0.25 \text{ miles})\left(\frac{1608 \text{ m}}{1 \text{ mile}}\right)\hat{i} = 402 \text{ m } \hat{i}$$

$$\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t} = \frac{402 \text{ m}}{12.8 \text{ s}}\hat{i} = 31.4 \tfrac{\text{m}}{\text{s}}\ \hat{i}$$

g) What is the average net force exerted on the Corvette during this trial?

Ans. $$P_{avg} = \vec{F}_{avg}\cdot\vec{v}_{avg} \Rightarrow \left|\vec{F}_{avg}\right| = P_{avg} / \left|\vec{v}_{avg}\right| = \frac{1.62\text{x}10^5 \text{ W}}{31.4 \tfrac{\text{m}}{\text{s}}} = 5159 \text{ N or } \vec{F}_{avg} = 5159 \text{ N } \hat{i}$$

h) The Corvette has a top speed of 83.1 m/s (186 mph). What is the average force exerted by the Corvette when moving at top speed, assuming it is operating a power output determined in (4)?

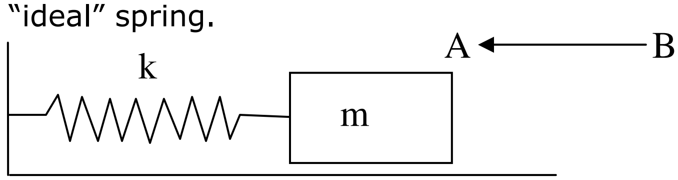
Ans. $$\vec{F}_{\text{Net avg}} = \left(\frac{P_{avg}}{v_{top}}\right)\hat{i} = \left(\frac{1.62\text{x}10^5 \text{ W}}{83.1 \tfrac{\text{m}}{\text{s}}}\right)\hat{i} = 1950 \text{ N } \hat{i}$$

i) This Corvette model has rated maximum power (in hp) of 400 hp. To estimate how this rated value compares to the working power determined in part (4), estimate the % efficiency.

Ans. $$\%\text{ Efficiency} = \left(\frac{P_{rated} - P_{actual}}{P_{rated}}\right) \text{x } 100\% = \left(\frac{400 \text{ hp} - 217 \text{ hp}}{400 \text{ hp}}\right) \text{x } 100\% = 45.6\ \%$$

Energy and Simple (ideal) Springs:

8. A mass (0.5 kg) is attached to a "massless" spring (k=50 N/m), which is attached firmly to a wall at the other end. The mass is initially pulled from position A to position B (a displacement of 0.2 m) then released. Note: there is no friction between the mass and the ground and the spring behaves as an "ideal" spring.



a. What is the net force acting on the mass at position B just as it is released? Apply Newton's 2^{nd} Law.

Ans. $\vec{F}_{Net} = -k\vec{x} = -(50\frac{N}{m})(0.2\text{ m}) = -10N\ \hat{i}$

b. What is the force exerted by the spring on the mass at B?

Ans. $\vec{F}_{Net} = \vec{F}_{spring} = -10N\ \hat{i}$

c. What is the force exerted by the spring on the mass at A?

Ans. $\vec{F}_{spring} = -k\vec{x} = -(50\frac{N}{m})(0\text{ m}) = 0N\ \hat{i}$

d. What is the "average force" applied to the mass as it moves from B to A?

Ans. since the net force is proportional to x: $\vec{F}_{Net\ avg} = \dfrac{-10\text{ N} + 0\text{ N}}{2} = -\ 5N\ \hat{i}$

e. How much work does the spring perform to move the mass from B to A?

Ans. $W_{Net} = \int_{\vec{x}}^{0}\vec{F}_{Net}\cdot d\vec{x} = \frac{1}{2}k\vec{x}\cdot\vec{x} = \frac{1}{2}(50\frac{N}{m})(-0.2\text{ m})^2 = 1\text{ J}$

f. What is the acceleration of the mass (just as it is let go) at B?

Ans. $\vec{F}_{Net} = m\vec{a} = -10N\ \hat{i} \Rightarrow \vec{a} = \dfrac{-10N}{0.5\text{ kg}}\ \hat{i} = -\ 20\frac{m}{s^2}\ \hat{i}$

g. When the mass returns to the initial (rest) position of the spring, at A, what is the acceleration of the mass?

Ans. $\vec{F}_{Net} = m\vec{a} = 0N\ \hat{i} \Rightarrow \vec{a} = \dfrac{0N}{0.5\text{ kg}}\ \hat{i} = 0\frac{m}{s^2}\ \hat{i}$

h. What is the velocity of the spring at position A?

Ans. $W_{Net} = \Delta K = K = 1\text{ J} \Rightarrow \vec{v} = -\left(\sqrt{\dfrac{2(1J)}{0.5\text{ kg}}}\right)\hat{i} = -2\frac{m}{s}\ \hat{i}$

i. As the mass continues moving to the left, how far does the mass overshoot from the rest position? (i.e. what is the displacement). Why?

Ans. Applying the Conservation of Mechanical Energy: $K_1 + U_1 = K_2 + U_2 = 1\text{ J}$

$U_2 = \frac{1}{2} kx_2^2 = 1J \Rightarrow \vec{x}_2 = -\sqrt{\dfrac{2(1J)}{50\frac{N}{m}}}\ \hat{i} = (-0.2\text{ m})\ \hat{i}$