1) The motion of a 0.1 kg ball tossed vertically into the air was recorded using a motion detector. The initial velocity for the ball was 5 m/s (see Graph 1). Analysis of the velocity vs. time graph yielded the acceleration of the ball during 3 phases of the motion: upward, near the top and downward (Graph 2). During the upward movement, the average acceleration (a_{upward}) is -12.3 m/s^2. Near the top of the trajectory, where air drag was minimal, the average acceleration of the ball is -10.1 m/s^2 (why is it not -9.8 m/s^2). During the descent, the average acceleration ($a_{downward}$) is -7.8 m/s^2.

Problems:

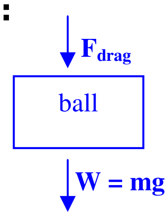
a) Observing Graph 1, how does air drag affect shape of the velocity vs. time graph (i.e. the motion of the ball) compared to no drag?

b) Why is the upward acceleration of the ball greater than at the top or during the descent?

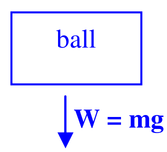
Ans. Air drag force is in the same direction as $m\vec{g}$ on the ascent and opposes $m\vec{g}$ during the descent

c) Sketch a force vector diagram for each phase of the ball's motion:

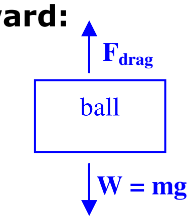
Upward:



At the top:



Downward:



d) Using your force diagrams as reference, apply Newton's 2nd Law of motion to the ball for each phase:

Upward:

$$\vec{F}_{Net} = -(mg + F_{drag})\,\hat{j} = ma_y\,\hat{j} = (0.1\text{ kg})(-12.3\text{ m/s}^2)\,\hat{j} = -1.23\text{ N}\,\hat{j}$$

At the top: $\vec{F}_{Net} = -mg\,\hat{j} = ma_y\,\hat{j} = (0.1\text{ kg})(-9.8\text{ m/s}^2)\,\hat{j} = -0.98\text{ N}\,\hat{j}$

Downward:

$$\vec{F}_{Net} = (-mg + F_{drag})\,\hat{j} = ma_y\,\hat{j} = (0.1\text{ kg})(-7.8\text{ m/s}^2)\,\hat{j} = -0.78\text{ N}\,\hat{j}$$

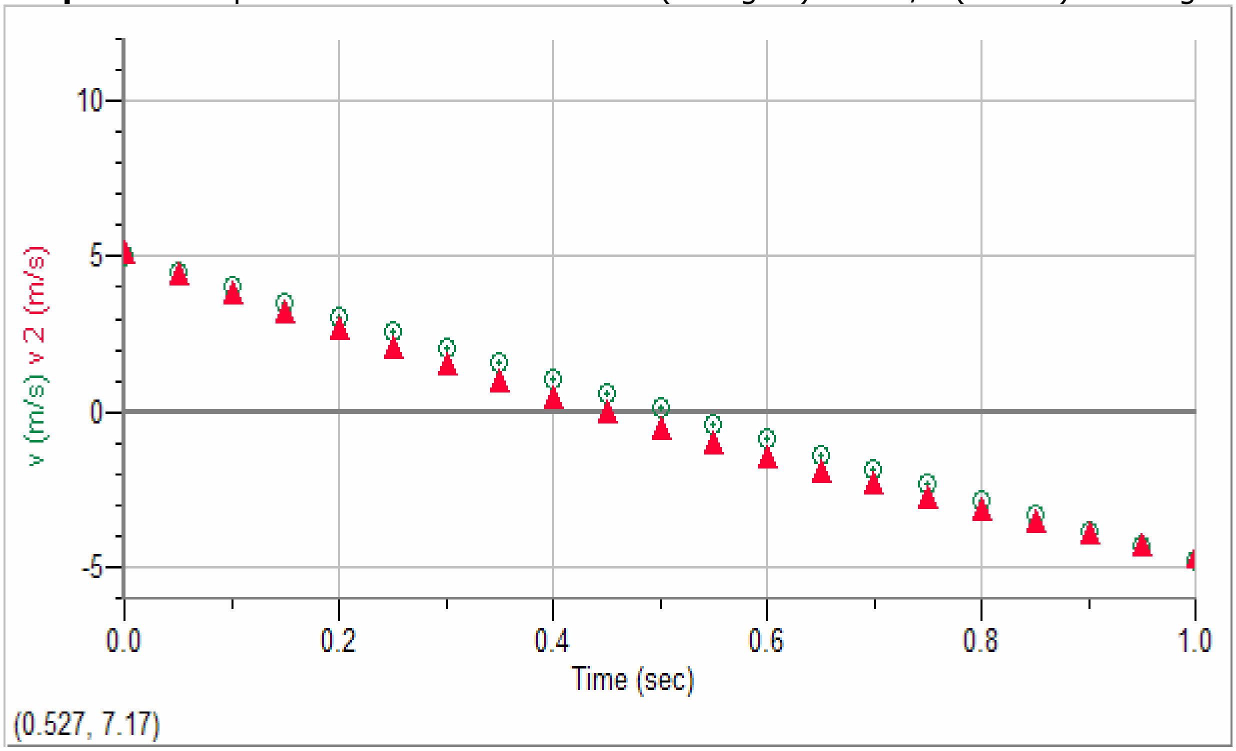
e) Determine the average force of air drag on the ball for each phase:

Upward: $\vec{F}_{drag} = -(ma_y + mg)\,\hat{j} = (1.23\text{ N} - 0.98\text{ N})\,\hat{j} = -0.25\text{ N}\,\hat{j}$

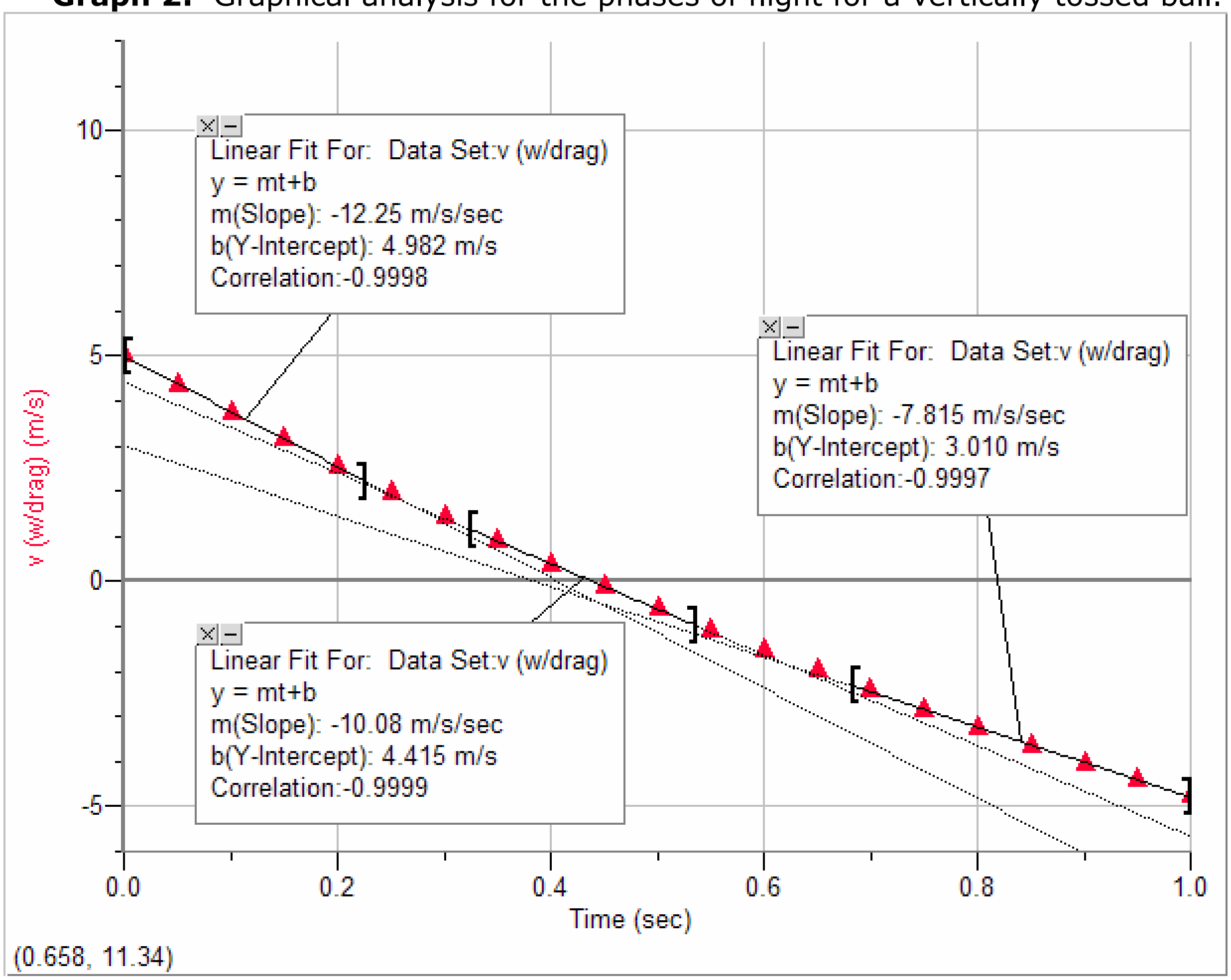
At the top: $\vec{F}_{drag} = 0\text{ N}\,\hat{j}$

Downward: $\vec{F}_{drag} = (ma_y - mg)\,\hat{j} = (-0.78\text{ N} + 0.98\text{ N})\,\hat{j} = 0.2\text{ N}\,\hat{j}$

Graph 1: Comparison of a tossed ball with (triangles) and w/o (circles) air drag.

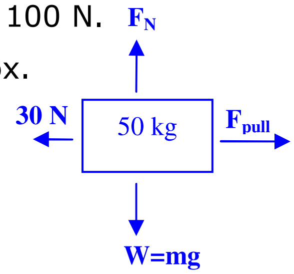


Graph 2: Graphical analysis for the phases of flight for a vertically tossed ball.



2) A 50 kg box is pulled horizontally along a floor against a 30 N kinetic friction force. The observed net force acting on the box is 100 N.

a) Draw a free-body diagram for the box.



b) What is the magnitude and direction of the net force (ΣF) acting on the box?

Ans. $\vec{F}_{Net} = ma_x\ \hat{i} = (F_{pull} - 30\text{ N})\hat{i} + (F_N - mg)\ \hat{j} = 100\text{ N}\ \hat{i} + 0\text{N}\ \hat{j} = 100\text{ N}\ \hat{i}$

c) What is the coefficient of kinetic friction (μ_k)?

Ans. Since the kinetic friction vector is: $\vec{f}_k = \mu_k \vec{F}_N = -\mu_k mg\ \hat{i} = -30\text{N}\ \hat{i}$

The magnitude of the kinetic friction is: $\left|\vec{f}_k\right| = 30\text{N}$

$$\mu_k = \frac{|f_k|}{|F_N|} = \frac{30\text{ N}}{mg} = \frac{30\text{ N}}{490\text{ N}} = 0.06$$

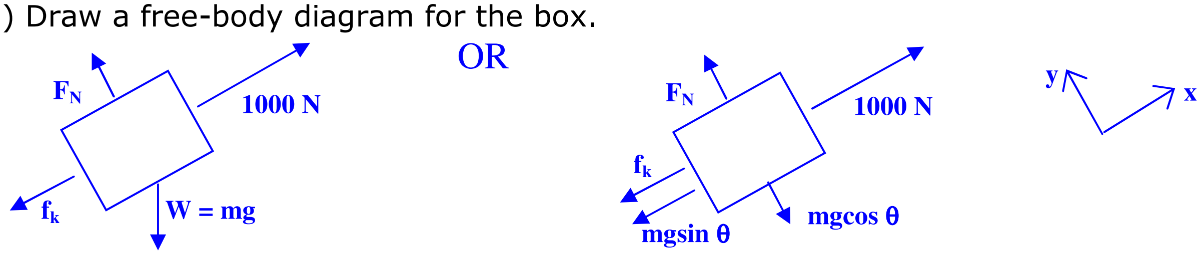
d) What is the horizontal acceleration of the box and how far does the box travel in 2.5 seconds?

Ans. The acceleration is: $\vec{a} = \vec{a}_x = \frac{F_{Net}}{m}\ \hat{i} = \frac{100\text{ N}}{50\text{ kg}}\ \hat{i} = 2\text{ m/s}^2\ \hat{i}$

The displacement is: $\Delta\vec{r} = \Delta\vec{x} = \left(\frac{a_x t^2}{2}\right)\hat{i} = \left(\frac{(2\frac{m}{s^2})(2.5\text{ s})^2}{2}\right)\hat{i} = 6.25\text{ m}\ \hat{i}$

3) A 50 kg box is pulled up a 30° incline by a 1000 N force. The coefficient of kinetic friction between the box and the inclined plane is $\mu_k = 0.25$.

a) Draw a free-body diagram for the box.



b) What is the frictional force acting on the box?

Ans. $\vec{f}_k = \mu_k \vec{F}_N = -\mu_k mg\cos 30°\ \hat{i} = -106.9\text{ N}\ \hat{i}$

c) What is the magnitude and direction of the acceleration of the box?

Ans. $\vec{a} = \dfrac{\vec{F}_{Net}}{m} = \dfrac{648.9\text{ N}}{50\text{ kg}}\,\hat{i} = 13.0\,\frac{m}{s^2}\,\hat{i}$

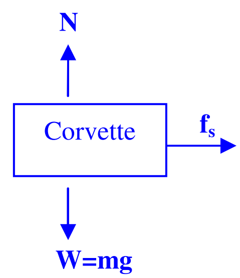
or $|\vec{a}| = 13.0\,\frac{m}{s^2}$ and $\theta = 0^\circ$ (up incline)

d) How long does it take to pull the box 2.0 m up the plane?

Ans. $\Delta t = \sqrt{\dfrac{2\Delta x}{a_x}} = \sqrt{\dfrac{2(2.0\text{ m})}{13.0\,\frac{m}{s^2}}} = 0.55\text{ s}$

4) A Corvette convertible (mass=1581.8 kg) accelerates in a straight line from 0 to 60 mph with an elapsed time of 5.5 s on a flat level track. The tires never lose traction with the road surface.

a) Draw a free body diagram for the Corvette during this road test.



b) What is the weight of the Corvette?

Ans. $\vec{W} = -\,mg\,\hat{j} = -\left((1581.5\text{ kg})(9.8\tfrac{m}{s^2})\right)\hat{j} = -15{,}500\text{ N}\,\hat{j}$

c) Apply Newton's 2nd Law to the Corvette.

Ans.

$\vec{F}_{Net} = \vec{f}_s + \vec{F}_{drag} + \vec{F}_N + \vec{W} = \left(f_s - F_{drag}\right)\hat{i} + \left(F_N - mg\right)\hat{j} = ma_x\,\hat{i}\ \left\{\text{where } \left|\vec{F}_{drag}\right| = 0\text{N}\right\}$

d) What is the average net force exerted on the Corvette?

Ans. *Convert v from mph to m/s:* v = 60 mi/hr = 26.81 m/s

$\vec{a} = \vec{a}_x = \dfrac{\Delta\vec{v}}{\Delta t} = \dfrac{(26.81\text{ m/s} - 0\text{ m/s})}{5.5\text{ s}}\,\hat{i} = 4.88\text{ m/s}^2\,\hat{i}$

$\vec{F}_{Net} = ma_x\,\hat{i} = \left((1581.8\text{ kg})(4.88\text{ m/s}^2)\right)\hat{i} = 7720\text{ N}\,\hat{i}$

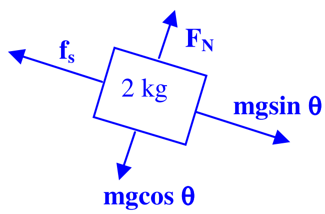
e) What is the minimum coefficient of static friction required to keep the Corvette tires "gripped" to the track surface?

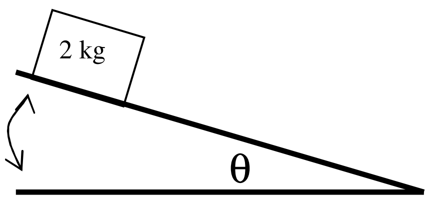
Ans. Since $\vec{F}_{Net} = \vec{f}_s = ma_x\ \hat{i}$ where the traction (or static friction) force is $\vec{f}_s = \mu_s mg\ \hat{i}$

$$\mu_s = \frac{\left|\vec{F}_{Net}\right|}{\left|m\vec{g}\right|} = \frac{7720\text{ N}}{15{,}500\text{ N}} = 0.498$$

5) A 2 kg block is supported by a flat wooden platform. The platform has an adjustable incline that can be raised from 0° to 90° (with respect to the horizontal direction). The coefficient of static friction between the block and the platform surface is 0.5.

a) As the platform is raised to some angle, θ, draw the free body diagram for the block as it sits at rest on the incline.





b) Apply Newton's 2nd Law to the block described in part a. What is the direction and magnitude of the net force exerted on the block as a function of the incline angle?

Ans. $\vec{F}_{Net} = \left(mg\sin 30^\circ - f_s\right)\hat{i} + \left(F_N - mg\cos 30^\circ\right)\hat{j} = 0\text{ N}$

c) Determine the maximum angle, θ, that the incline can be raised before the block will begin to slide.

Ans. The maximum angle (θ_{max}) occurs when f_s is maximum:

From part (b), $f_s = \mu_s F_N = \mu_s mg\cos\theta_{max} = mg\sin\theta_{max}$

Solving for the coefficient of static friction: $\mu_s = \tan\theta_{max} = 0.5$

$\rightarrow \theta_{max} = \tan^{-1}(0.5) = 26.6^\circ$

d) What is the magnitude of the normal force at the angle determined in part c?

Ans. $F_N = mg\cos\theta_{max} = mg\cos 26.6^\circ = 17.5\text{ N}$

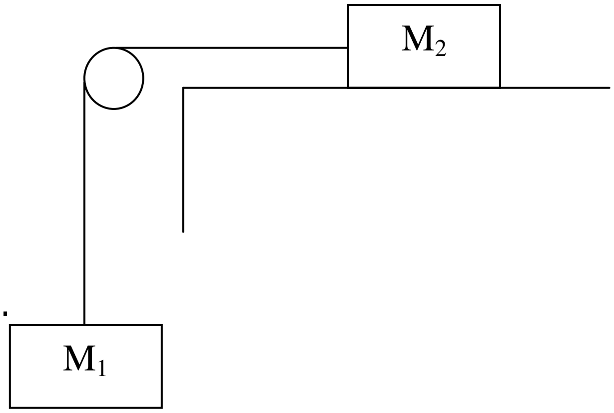
e) What is the magnitude of the (maximum) static friction force acting on the block in part c?

Ans. $f_s^{max} = \mu_s F_N = \mu_s mg\cos\theta_{max} = 8.76\text{ N}$

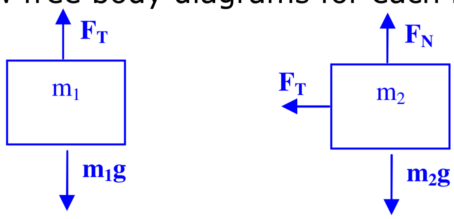
Or

$f_s^{max} = mg\sin\theta_{max} = 8.76\text{ N}$

6. Consider the following pulley system where there <u>is</u> friction (static and/or kinetic) between M_2 and the horizontal surface.



a. Draw free body diagrams for each mass.



b. Apply Newton's 2nd law in vector form to each mass.

Ans. *m₁*: $\vec{F}_{Net\,1} = \vec{F}_T - m_1\vec{g} = (F_T - m_1 g)\,\hat{j} = m_1\vec{a}_1$

m₂: $\vec{F}_{Net\,2} = \vec{F}_T + \vec{f} + \vec{F}_N + m\vec{g} = (f - F_T)\,\hat{i}' + (F_N - m_2 g)\,\hat{j}' = m_2\vec{a}_2$

(note i' direction is to the right and j' direction is upward)

c. Determine an equation for the acceleration vector for each mass in terms of only the masses, μ_k and g.

Ans. It is convenient to recognize that the **j** direction for M_1 corresponds to the **i'** direction for M_2.

Solve for F_T using the M_2 equation: $\vec{F}_{Net\,2} = (f - F_T)\,\hat{i}' = m_2\vec{a}_2 \Rightarrow F_T = f - m_2 a_2$

Substituting F_T into the equation for M_1: $\vec{F}_{Net\,1} = (F_T - m_1 g)\,\hat{j} = m_1\vec{a}_1$

$\Rightarrow F_T - m_1 g = f - m_2 a_2 - m_1 g = m_1 a_1$

Since the masses are attached their accelerations are equal: $a_1 = a_2 = a$

Solving for a: $f - m_2 a - m_1 g = m_1 a$

$$a = \left(\frac{f - m_1 g}{m_1 + m_2}\right)$$

Or $$\vec{a}_1 = \left(\frac{f - m_1 g}{m_1 + m_2}\right)\hat{j} \text{ and } \vec{a}_2 = -\left(\frac{f - m_1 g}{m_1 + m_2}\right)\hat{i}'$$

d. If M_1 = 10 kg and M_2= 20 kg, what is the minimum static friction coefficient that will keep the system from moving?

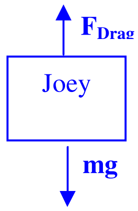
Ans. When system is at rest, a = 0 m/s^2: $a = \left(\frac{f_s - m_1 g}{m_1 + m_2}\right) = 0$

Solving for f_s: $f_s = \mu_s m_2 g = m_1 g$

Solving for μ_s: $\mu_s = m_1/m_2 = (10\text{ kg})/(20\text{ kg}) = 0.5$

7. Joey (60 kg) jumps out of an airplane and plummets to the ground. Descending in spread eagle fashion (area = 0.5 m^2), he reaches a terminal velocity of 120 miles/hour. Assume the air density (ρ_{air}) is 1.10 kg/m^3.

a) Draw a free body diagram for Joey during his fall.



b) What is the terminal velocity in SI units?

$$\text{Ans.}\quad |v_T| = \left(\frac{120\text{ mi}}{1\text{ hr}}\right)\left(\frac{1609\text{ m}}{1\text{ mi}}\right)\left(\frac{1\text{ hr}}{3600\text{ s}}\right) = 53.6\ \tfrac{m}{s}\ \text{and}\ \vec{v}_T = (-\,53.6\ \tfrac{m}{s})\,\hat{j}$$

c) Apply Newton's 2^{nd} Law to our intrepid skydiver, at terminal velocity. What is the coefficient of drag for Joey during the descent?

$$\text{Ans.}\quad \vec{F}_{Net} = \vec{F}_{Drag} - m\vec{g} = \left(\frac{\rho CA}{2}v^2 - mg\right)\hat{j} = m\vec{a}$$

Since at terminal velocity, a = 0,

$$\frac{\rho CA}{2}v_T^2 = mg \;\Rightarrow\; C = \frac{2mg}{\rho A v_T^2} = \frac{2(60\text{ kg})(9.8\,\frac{m}{s^2})}{(1.10\ \frac{kg}{m^3})(0.5\text{ m}^2)(53.6\,\frac{m}{s})^2} = 0.744$$

d) Getting cocky, Joey quickly flips into a nose dive decreasing his effective surface area to 0.2 m^2. What is Joey's new terminal velocity?

$$\text{Ans.}\quad v_T = \sqrt{\frac{2mg}{\rho CA}} = \sqrt{\frac{2(60\text{ kg})(9.8\,\frac{m}{s^2})}{(1.10\ \frac{kg}{m^3})(0.744)(0.2\text{ m}^2)}} = 84.8\ \tfrac{m}{s}\ \text{or}\ \vec{v}_T = (-\,84.8\ \tfrac{m}{s})\hat{j}$$

e) What is the net force exerted on Joey the moment he flips into the nose dive?

Ans. When Joey flips into the nose dive, he is initially moving at 53.6 m/s, NOT the terminal velocity, 84.8 m/s, calculated in step (d).

$$\vec{F}_{Net} = \left(\frac{\rho CA}{2}v^2 - mg\right)\hat{j}$$

$$\vec{F}_{Net} = \left(\frac{(1.10\ \frac{kg}{m^3})(0.744)(0.2\text{ m}^2)}{2}(53.6\,\tfrac{m}{s})^2 - (60\text{ kg})(9.8\,\tfrac{m}{s^2})\right)\hat{j}$$

$$\vec{F}_{Net} = (-352.9\text{ N})\hat{j} = m\vec{a}$$

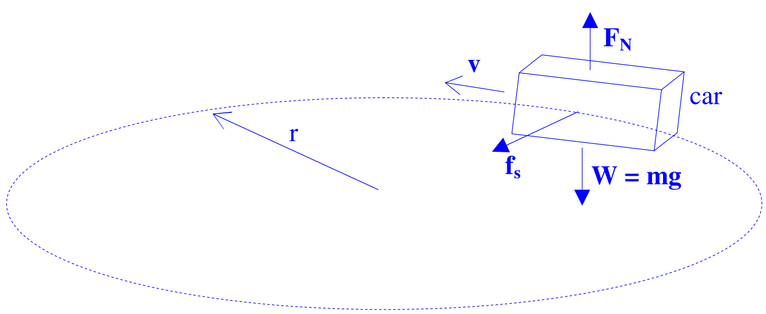
f) Finally, Joey pulls his parachute cord and the parachute rapidly opens (cross-sectional radius = 5 m). What is Joey's terminal velocity with the open 'chute (assuming he has enough time to reach it prior to hitting the ground)?

Ans. $v_T = \sqrt{\dfrac{2mg}{\rho CA}} = \sqrt{\dfrac{2(60\ \text{kg})(9.8\ \frac{m}{s^2})}{(1.10\ \frac{kg}{m^3})(0.744)(78.5\ \text{m}^2)}} = 4.28\ \frac{m}{s}$ or $\vec{v}_T = (-\,4.28\ \frac{m}{s})\hat{j}$

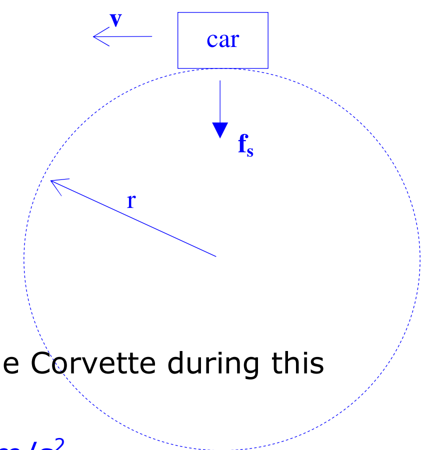
8. **Lateral Acceleration & Centripetal Force.** According to Road & Track magazine, the maximum lateral acceleration of the Corvette Convertible (mass = 1581.8 kg) is 0.98 g ($a_{gravity}$ not grams). To measure lateral acceleration, the Corvette is driven around a flat 200 ft radius track (the "skidpad") at the highest speed possible until the tires lose grip with the road.

a. Draw a simple free body diagram for the car.

Side view



Looking down on car



b. Determine the magnitude of centripetal force vector exerted on the Corvette during this test.

Ans. The lateral (or centripetal acceleration): $|a_c| = 0.98\ g = 9.60\ \text{m/s}^2$

c. What speed would correspond to the acceleration measured above.

Ans. Since the magnitude of the centripetal acceleration is: $a_c = v^2/r = 9.60\ \text{m/s}^2$

and the radius of the track is: $r = (200\ \text{ft})(1\ \text{m}/3.281\ \text{ft}) = 61.0\ \text{m}$

$$v = \sqrt{a_c r} = \sqrt{(9.60\ \text{m/s}^2)(61.0\ \text{m})} = 24.2\ \text{m/s}$$

d. Apply Newton's 2nd law to the car and express $\vec{F}_{net}$ in component form.

Ans. (assigning the **i** direction pointing toward center of the circle and j direction vertical:

$$\vec{F}_{Net} = \vec{f}_s + \vec{F}_N + m\vec{g} = (f_s)\hat{i} + (F_N - mg)\hat{j} = (15{,}185\ \text{N})\hat{i} = (ma_c)\hat{i}$$

e. What is the maximum centripetal force exerted on this car just before the tires lose traction with the road? (Assume μ_s^{max} is 0.88 for dry pavement). Explain the discrepancy between your answer and the answer in (c).

Ans. Since $\mu_s = 0.88$ and the normal force, $F_N = mg = (1581.8\ \text{kg})(9.8\ \text{m/s}^2) = 15502\ \text{N}$

$$\vec{F}_{Net} = \vec{F}_c = m\vec{a}_c = (\mu_s F_N)\ \hat{i} = (0.88)(15502\ \text{N})\hat{i} = (13642\ \text{N})\hat{i}$$

f. What speed would your answer to (f) correspond to?

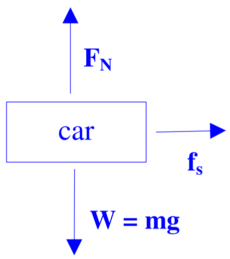
Ans. Since the magnitude of the centripetal acceleration is:

$$|a_c| = \frac{|F_{Net}|}{m} = \frac{13642\text{ N}}{1581.8\text{ kg}} = 8.62\text{ m/s}^2 = \frac{mv^2}{r}$$

$$\Rightarrow |v| = \sqrt{|a_c| r} = \sqrt{(8.62\text{ m/s}^2)(61.0\text{ m})} = 22.9\text{ m/s}$$

8. **Banked Curves.** A traffic engineer has to design a 35 mph (15.6 m/s) road curve with a 30 m radius, keeping in mind safety for both dry and wet road conditions.

a. If the corner is designed with no incline (bank) what is the minimum coefficient of friction between tire and road to hold a vehicle traveling at 15.6 m/s (mass = 680 kg) on the road?

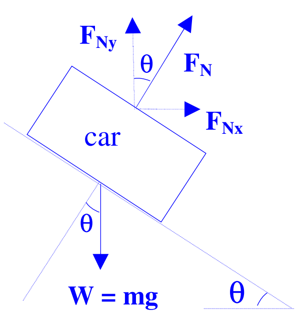


Ans. $\vec{F}_{Net} = f_s\,\hat{i} + (F_N - mg)\,\hat{j} = ma_c\hat{i}$

Therefore: $\mu_s F_N = \mu_s mg = mv^2/r$ or $\mu_s = v^2/gr$

$\mu_s = (15.6\text{ m/s})^2/[(9.8\text{ m/s}^2)(30\text{ m})] = 0.827$

b. If the road design is banked, what is the minimum steepness (angle of incline for the bank) required to keep cars safely on the road at 15.6 m/s? For simplicity, neglect any friction between tire and road.



Ans. $\vec{F}_{Net} = F_N\sin\theta\,\hat{i} + (F_N\cos\theta - mg)\,\hat{j} = \left(\frac{mv^2}{r}\right)\hat{i}$

Separating into x- and y- directions:

(in x) $F_{Net\,x} = F_N\sin\theta = ma_c = mv^2/r$

(in y) $F_{Net\,y} = F_N\cos\theta - mg = 0\text{ N}$ or $F_N = mg/\cos\theta$

Therefore: $F_N\sin\theta = (mg/\cos\theta)\sin\theta = mv^2/r$

Solving for θ: $\tan\theta = v^2/gr$ or

$\theta = \tan^{-1}(v^2/gr) = \tan^{-1}[(15.6\text{ m/s})^2/(9.8\text{ m/s}^2)(30\text{ m})] = 39.6^\circ$

c. **Concept question:** What holds the car to the banked roadway when there is no static friction present between road and tire?

Ans. The normal force between the road and the car generates the centripetal force.