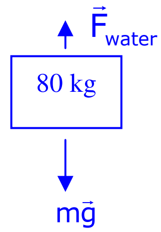
Chapter 5 Worksheet: Force & Motion I

1) An 80kg anchor is dropped in the water and plummets to the ocean floor with an observed acceleration of 3.0 m/s^2.

a) Draw a free-body diagram for the anchor.



b) What is the magnitude of the water resistance acting on the anchor?

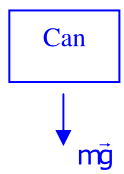
Ans. F_{water} = 544 N (upward)

c) How long does it take the anchor to drop 100m?

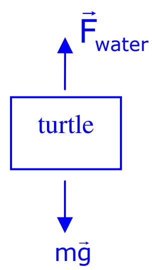
Ans. $\Delta t = (2 \cdot \Delta y / a_y)^{1/2} = 8.2$ s

2) Draw free body diagrams for the following:

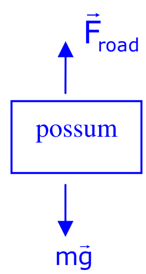
a) a soda can tossed into the air



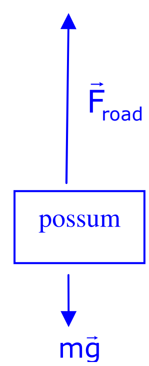
b) a sea turtle floating in the ocean



c) a possum standing by the side of the road

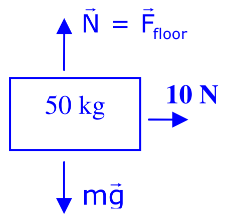


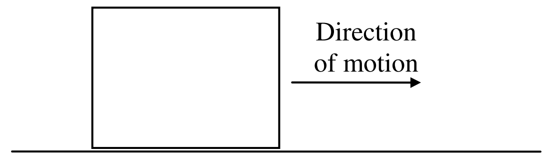
d) a possum jumping vertically (prior to its feet leaving the ground)



3) A 50 kg box is pulled horizontally along a frictionless floor by a 100N force.

a) Draw a free-body diagram for the box.





b) What is the magnitude and direction of the net force ($\vec{F}_{net}$) acting on the box?

Ans. $\vec{F}_{net} = 100\ \text{N}\ \hat{i}$

$\Sigma F_y = F_{floor} - W = 0\ \text{N}$

c) What is the horizontal acceleration of the box?

Ans. $a_x\ \hat{i} = (F_x/m)\ \hat{i} = (100\ \text{N})/(50\ \text{kg})\ \hat{i} = (2.0\ \text{m/s}^2)\ \hat{i}$

d) Express the net force and acceleration vectors in component form.

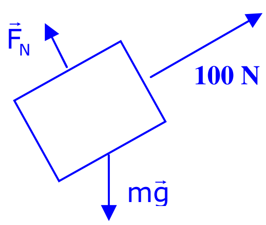
Ans. $\vec{F}_{net} = (100\text{N})\hat{i} + (0\text{N})\hat{j}$ and $\vec{a} = \left(2.0\frac{\text{m}}{\text{s}^2}\right)\hat{i} + \left(0\frac{\text{m}}{\text{s}^2}\right)\hat{j}$

e) How far does the box travel in 2.5 seconds?

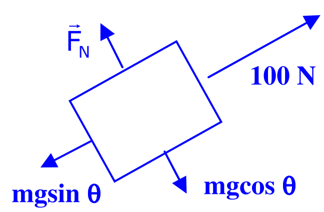
Ans. $\Delta x = ½ \cdot a_x \cdot \Delta t^2 = ½\ (2.0\ \text{m/s}^2)(2.5\ \text{s})^2 = 6.25\ \text{m}$ (to the right)

4) A 50 kg box is pulled up a 30° incline along a frictionless floor by a 100 N force.

a) Draw a free-body diagram for the box.

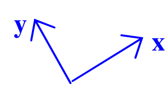


OR



b) What is the magnitude and direction of the net force ($\vec{F}_{net}$) acting on the box?

Ans. *Choose x- and y- axes along the direction of the incline*



$\left|\vec{F}_{net}\right| = 145\ \text{N}$

$\theta = 180^\circ$ (i.e. down the incline!) or

$\theta = 210^\circ$ in traditional horizontal/vertical coordinates)

c) What is the magnitude and direction of the acceleration of the box?

$$|\vec{a}| = \frac{145\ \text{N}}{50\ \text{kg}} = 2.9\ \tfrac{\text{m}}{\text{s}^2}$$

$\theta = 180^\circ$ (i.e. down the incline!) or

$\theta = 210^\circ$ in traditional horizontal/vertical coordinates)

e) Express the $\vec{F}_{net}$ and $\vec{a}$ vectors in component form.

$$\vec{F}_{net} = (-145\ \text{N})\hat{i} + (0\ \text{N})\hat{j}$$

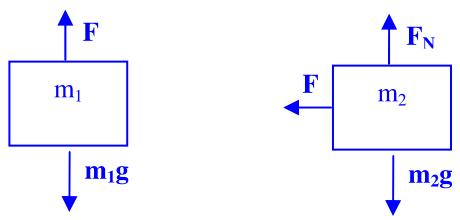
$$\vec{a} = \left(-2.9\ \tfrac{\text{m}}{\text{s}^2}\right)\hat{i} + \left(0\ \tfrac{\text{m}}{\text{s}^2}\right)\hat{j}$$

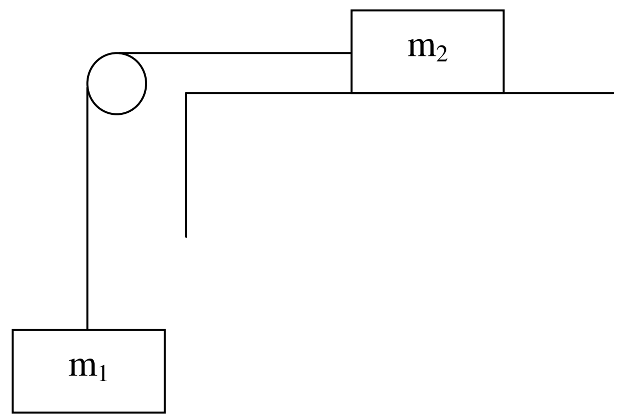
f) How long does it take the box to travel 2.0 m down the hill?

Ans. $\Delta t = ((2 \cdot \Delta x)/a_x)^{1/2} = \mathbf{(}(2 \cdot (2.0\ \text{m}))/(2.9\ \text{m/s}^2)\mathbf{)}^{1/2} = 1.17\ \text{s}$

5. Consider the 2 mass pulley system (see diagram). *Assume the pulley has no mass and there is no friction between (i) M_2 and the surface supporting it and (ii) the pulley.*

a. Draw free body diagrams for each mass.





b. Apply Newton's 2nd law to each mass.

Ans. *{mass 1}* $\vec{F}_{net1} = \vec{F}_{net\,1\,y} = (F_T - m_1g)\,\hat{j} = m_1a_{1y}\,\hat{j}$ (note +y direction is upward)

{mass 2} $\vec{F}_{net2} = \vec{F}_{net\,2\,x} = -\,F_T\ \hat{i} = m_2a_{2x}\ \hat{i}$ (note +x direction is to the right)

c. Determine an equation for the acceleration of the 2 mass system in terms of only the masses and g.

Ans. The easiest way to solve for F_T is to use equation for $\vec{F}_{net\,2\,x}$: $F_T = -\,m_2a_{2x}$

Substituting F_T into the equation for $\vec{F}_{net\,1\,y}$ (mass 1): $-\,m_2a_{2x} - m_1g = m_1a_{1y}$

Since the masses are attached their accelerations are equal: $a_{1y} = a_{2x} = a_{system}$

Solving for a_{system}: $-\,m_2a_{system} - m_1g = m_1a_{system}$

$$a_{system} = (-\,m_1g)/(m_1 + m_2)$$

d. If M_1 = 10 kg and M_2 = 20 kg, what is the acceleration of the system? Express the acceleration vectors for M_1 and M_2 in component form.

Ans. $a_{system} = -(10\text{ kg})(9.8\text{ m/s}^2)/(10\text{ kg} + 20\text{ kg}) = -3.27\text{ m/s}^2$

$$\vec{a}_1 = \left(-3.27\ \tfrac{m}{s^2}\right)\hat{i}$$
$$\vec{a}_2 = \left(-3.27\ \tfrac{m}{s^2}\right)\hat{j}$$

e. What are the tension forces acting on each mass? Express the tension vectors in component form.

Ans. The magnitude of the tension force, $F_T = -m_2 a_{2x} = -(20\text{ kg})(-3.27\text{ m/s}^2) = 65.4\text{ N}$.

The tension force vector, $\vec{F}_{T\text{ on }m_2} = (-F_T)\ \hat{i} = -65.4\text{ N}\ \hat{i}$ and $\vec{F}_{T\text{ on }m_1} = (F_T)\ \hat{j} = 65.4\text{ N}\ \hat{j}$

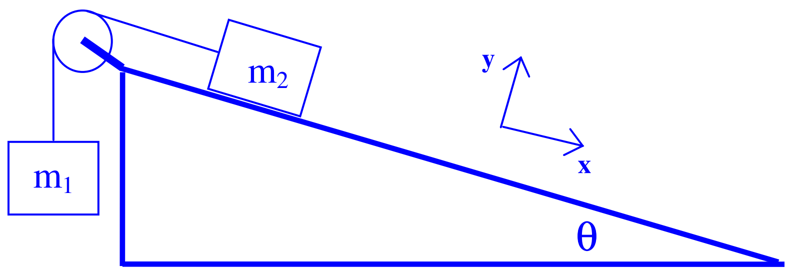
f. Identify as many "Action-Reaction" Force Pairs as you can in this example.

Ans.

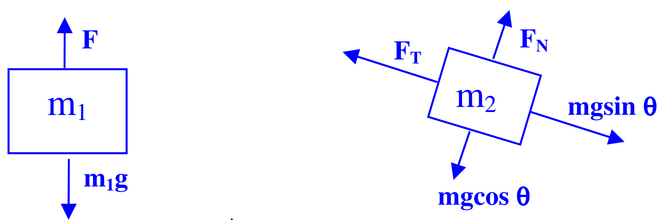
$$\left|\vec{F}_{G\text{ Earth on }m_1}\right| = \left|\vec{F}_{G\ m_1\text{ on Earth}}\right|$$
$$\left|\vec{F}_{G\text{ Earth on }m_2}\right| = \left|\vec{F}_{G\ m_2\text{ on Earth}}\right|$$
$$\left|\vec{F}_{T\text{ rope on }m_1}\right| = \left|\vec{F}_{T\ m_1\text{ on rope}}\right|$$
$$\left|\vec{F}_{T\text{ rope on }m_2}\right| = \left|\vec{F}_{T\ m_2\text{ on rope}}\right|$$
$$\left|\vec{F}_{N\text{ by support on }m_2}\right| = \left|\vec{F}_{N\text{ by }m_2\text{ on support}}\right|$$

6. How would the pulley system above behave if the surface supporting M_2 were at an angle (θ=20°), pointing upward to the left? Assume there is no friction between M_2 and the incline.

a. Draw a diagram of the pulley system described in this problem



b. Draw a free body diagram for each mass.



c. Apply Newton's 2nd law to each mass.

Ans. *{mass 1}* $\vec{F}_{Net1} = \vec{F}_T - m_1\vec{g} = m_1\vec{a}_1$

or $F_{Net1} = F_{Net1\,y} = F_T - m_1 g = m_1 a_{1y}$ (note +y direction is upward)

{mass 2} $\vec{F}_{Net2} = m_2\vec{g} + \vec{F}_T + \vec{F}_N = m_2\vec{a}_2$

or $F_{Net2\,x} = m_2 g\sin\theta - F_T = m_2 a_{2x}$ (note +x direction down incline)

and $F_{Net2\,y} = F_N - m_2 g\cos\theta = 0$ N (note +y direction is normal to incline)

d. Determine an equation for the acceleration of the 2 mass system in terms of only the masses and g.

Ans. Solving for F_T in the equation for $F_{Net1\,y}$: $F_T = m_1 a_{1y} + m_1 g$

Substituting F_T into the equation for $F_{Net2\,x}$: $m_2 g\sin\theta - m_1 a_{1y} - m_1 g = m_2 a_{2x}$

Since the masses are attached their accelerations are equal: $a_{1y} = a_{2x} = a_{system}$

Solving for a_{system}: $m_2 g\sin\theta - m_1 a_{system} - m_1 g = m_2 a_{system}$

$a_{system} = (m_2 g\sin\theta - m_1 g)/(m_1 + m_2) = -1.03\ m/s^2$ *(i.e. up the incline!)*

e. What are the tension forces acting on each mass? Express the tension vectors in component form.

Ans. The magnitude of the tension force, $F_T = m_1 a_{1y} + m_1 g = 87.7$ N

or $F_T = \left(\dfrac{m_1}{m_1 + m_2}\right)(m_2 g\sin\theta - m_1 g) + m_1 g = 87.7\text{N}$

The tension force vectors:

$$\vec{F}_{T\text{ on }m_2} = \left[\left(\frac{m_1}{m_1 + m_2}\right)(m_1 g - m_2 g\sin\theta) - m_1 g\right]\ \hat{i}_2 = -87.7\text{ N }\hat{i}_2$$

And $$\vec{F}_{T\text{ on }m_1} = \left[\left(\frac{m_1}{m_1 + m_2}\right)(m_2 g\sin\theta - m_1 g) + m_1 g\right]\hat{j}_1 = 87.7\text{ N }\hat{j}_1$$

f. Express the normal force vector acting on the mass, in component form.

Ans. The magnitude of the normal force, $F_N = m_2 g\cos\theta = 184.2$ N

The normal force vector, $\vec{F}_N = (m_2 g\cos\theta)\ \hat{j}_2 = 184.2\text{ N }\hat{j}_2$

g. What is the angle of incline that will keep the system from moving?

Ans. When system is at rest, $a_{system} = 0\text{ m/s}^2$: $a_{system} = (m_2 g\sin\theta - m_1 g)/(m_1 + m_2) = 0\text{ m/s}^2$

Solving for $\sin\theta$: $m_2 g\sin\theta = m_1 g \rightarrow \sin\theta = m_1/m_2$

Solving for the angle θ: $\theta = \sin^{-1}(m_1/m_2) = \sin^{-1}(10/20) = 30^{o}$