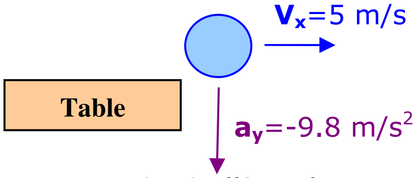
1) A ball rolls off a flat level table with a horizontal velocity (initial) of 5 m/s. The height of the table is 1.0 m. Neglect air resistance.

a) Sketch a vector diagram of the ball just as it leaves the table.



b) Express the ball's velocity vector, **v**(t), in component form, as it falls.

Ans. $\vec{v}(t)=v_o\hat{i} - g\Delta t\,\hat{j} \;\Rightarrow \vec{v}_o=\left(5\frac{m}{s}\right)\hat{i}$

c) How much time does it take for the ball to fall to the ground?

Ans. $\Delta t=\sqrt{\frac{2\Delta y}{a_y}}=\sqrt{\frac{2(-1.0m)}{\left(-9.8\frac{m}{s^2}\right)}}=0.45s$ *{note: v_{oy} = 0 m/s}*

d) Express the velocity vector of the ball just before it strikes the ground?

Ans. $\vec{v}=\left(5\frac{m}{s}\right)\hat{i} - \left(4.41\frac{m}{s}\right)\hat{j}$

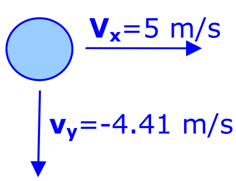
e) What is the horizontal velocity of the ball just before it hits the ground? Why?

Ans. $\vec{v}_x= 5\frac{m}{s}\;\hat{i}$

f) What is the magnitude of the ball's velocity just before it hits the ground?

Ans. $|\vec{v}|=\sqrt{\left(5\frac{m}{s}\right)^2+\left(-4.41\frac{m}{s}\right)^2}=6.67\frac{m}{s}$

g) Sketch a vector diagram of the ball just before it strikes the ground.



h) How far from the table does the ball strike the ground?

Ans. $\Delta x=v_x\Delta t=(5\tfrac{m}{s})(0.45s)=2.25m$

i) If the same ball were travelling with a initial horizontal speed of 10 m/s when it leaves the table, how long does it take for the ball to strike the ground?

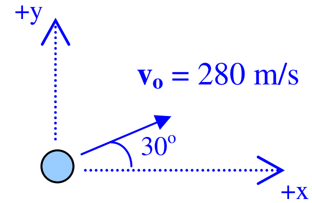
Ans. $\Delta t=\sqrt{\frac{2\Delta y}{a_y}}=\sqrt{\frac{2(-1.0m)}{\left(-9.8\frac{m}{s^2}\right)}}=0.45s$ *{same as above!}*

j) How far from the table would the ball in (f) strike the ground?

Ans. $\Delta x=v_x\Delta t=(10\tfrac{m}{s})(0.45s)=4.50m$

2) A bullet is shot from a gun with an initial velocity of 280 m/s at angle of 30° above the horizontal. The gun is positioned at ground level when the bullet is fired. Neglect the effects of air resistance.

a) Sketch a vector diagram of the bullet just as it leaves the gun.



b) Express the bullet's initial velocity vector in component form.

Ans. $\vec{v}_o = \left(280\frac{m}{s}\right)\cos 30^\circ \hat{i} + \left(280\frac{m}{s}\right)\sin 30^\circ \hat{j} = \left(242.5\frac{m}{s}\right)\hat{i} + \left(140\frac{m}{s}\right)\hat{j}$

c) How long does the bullet take to reach the top of its trajectory?

Ans. using the y-component of motion: $v_y - v_{oy} = -g\Delta t$

Therefore: $\Delta t = (v_y - v_{oy})/(-g) = (0\text{ m/s} - 140\text{ m/s})/(-9.8\text{ m/s}^2) = 14.3\text{ s}$

d) How high above the ground does the bullet reach at the top of its trajectory?

Ans. $\Delta y = y - y_o = v_{oy}\Delta t - \frac{1}{2}\cdot g\Delta t^2 = (140\text{ m/s})(14.3\text{ s}) - (\frac{1}{2})(9.8\text{ m/s}^2)(14.3\text{s})^2 = 1000\text{ m}$

e) Sketch a vector diagram of the bullet at the top of its trajectory.

f) Express the bullet's velocity vector of the bullet, in component form, at the top of its trajectory?

Ans. $\vec{v} = \left(242.5\frac{m}{s}\right)\hat{i}$

g) What is the total "hang time" for the bullet?

Ans. Hang time = $\Delta t_{hang} = \Delta t_{up} + \Delta t_{down} = 2\Delta t_{up} = 28.6$ s

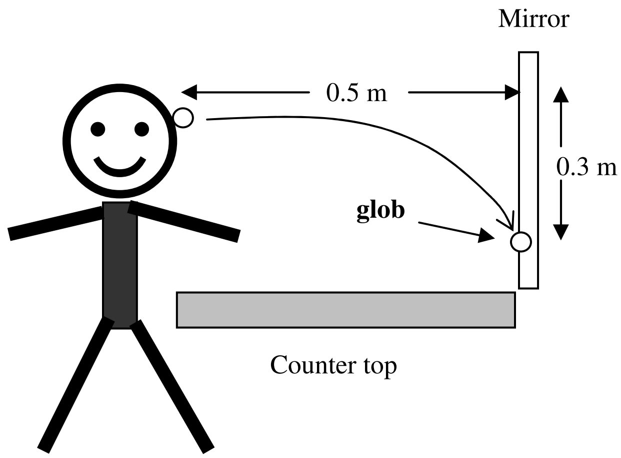
h) Express the displacement vector for the bullet, **r**(t), in component form.

Ans. $\vec{R}(t) = \left(242.5\frac{m}{s}\right)t\ \hat{i} + \left[\left(140\frac{m}{s}\right)t - \left(4.9\frac{m}{s^2}\right)t^2\right]\hat{j}$

i) How far from the gun does the bullet strike the ground?

Ans. $x - x_o = v_{ox}\Delta t = (242.5\text{ m/s})(28.6\text{ s}) = 6.94 \times 10^3$ m

3) In the bathroom, in a galaxy far, far away, Joey notices a really large zit residing on his forehead. Joey pops the zit and it shoots across the countertop and sticks to the mirror. When the zit was popped, Joey was standing 0.5 m from the mirror and the pus glob landed 0.3 m below the "launch" position. Assume there is no air resistance acting on the glob.



a) Express the velocity and displacement vectors, in component form, while the glob is in the air.

Ans. $\vec{v}=v_o\hat{i} + a_y t\,\hat{j} = v_o\hat{i} - gt\,\hat{j}$

$$\vec{R}(t)=v_o t\,\hat{i} + \left[\frac{1}{2}a_y t^2\right]\hat{j}= v_o t\,\hat{i} - \left(4.9\frac{m}{s^2}\right)t^2\,\hat{j}$$

b) How long does the glob fly through the air?

Ans. Using the vertical direction to find the time:

$y-y_o = v_{oy}\Delta t - \frac{1}{2}\cdot g\Delta t^2 = 0.3\text{ m},\ v_{oy} = 0\text{ m/s}$

therefore, $\Delta t=\sqrt{\frac{2\Delta y}{a}}=\sqrt{\frac{2(-0.3m)}{\left(-9.8\frac{m}{s^2}\right)}}=0.25s$

c) How fast ($\vec{v}_o$) was the pus glob traveling as it just leaves the pore?

Ans. In the horizontal direction: $\Delta x = x-x_o = v_{ox}\Delta t$, where $v_x = v_{ox}$ = constant

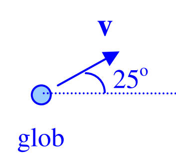
Solving for the velocity, $v_{ox}=\frac{\Delta x}{\Delta t}=\frac{0.5m}{0.25m}=2.0\frac{m}{s}$

d) If the same globule had shot from Joey's forehead at an initial angle of 25^o (above the horizontal), where would it land on the mirror (Yuck!!)?

Ans. Assume v_o is the same as above (v_o = 2.0 m/s), solve for the x- and y- components:

$v_{ox} = v_o\cos 25^o = (2.0\text{ m/s})\cdot\cos25^o = 1.8\text{ m/s}$

$v_{oy} = v_o\sin 25^o = (2.0\text{ m/s})\cdot\sin25^o = 0.85\text{ m/s}$



The flight time is: $\Delta t=\frac{\Delta x}{v_{ox}}=\frac{0.5m}{1.8\frac{m}{s}}=0.28s$

Therefore, the vertical displacement of the glob when it hits the mirror is:

$\Delta y = y-y_o = v_{oy}\Delta t - \frac{1}{2}\cdot g\Delta t^2 = (0.85\text{ m/s})(0.28\text{ s}) - (0.5)(9.8\text{ m/s}^2)(0.28\text{ s})^2 = -0.15\text{ m}$

4) Consider two identical golf balls struck by different loft golf clubs. Ball 1 leaves the ground at an initial angle of 60^o (above the horizontal) and ball 2 has an initial angle of 45^o. Both balls have an initial speed of 45 m/s. Neglect the effects of spin and air resistance on the golf balls.

a) What are the initial velocity vectors for each ball in component form?

b) Which ball has the greater hang time? Calculate it for both balls.

Ans. Since both balls will be subject to the same gravitational acceleration (a_y = -g), the ball with the greatest initial vertical velocity will have the greatest hang time:

Ball 1: $v_{oy} = v_o \sin 60^o = (45\text{ m/s})\cdot\sin 60^o = 39.0\text{ m/s}$,

$\Delta t_{hang} = \Delta v_y/a_y = (-v_{oy} - v_{oy})/a_y = (-78.0\text{ m/s})/(-9.8\text{ m/s}^2) = 7.95\text{ s}$

Ball 2: $v_{oy} = v_o \sin 45^o = (45\text{ m/s})\cdot\sin 45^o = 31.8\text{ m/s}$,

$\Delta t_{hang} = \Delta v_y/a_y = (-v_{oy} - v_{oy})/a_y = (-63.6\text{ m/s})/(-9.8\text{ m/s}^2) = 6.49\text{ s}$

→ Ball 1 will have greater hang time

c) Which ball travels farther (horizontally) while in the air? Calculate the "range" for both balls.

Ans. Ball 1: $\Delta x = v_{ox}\Delta t = (45\text{ m/s})\cdot\cos 60^o\cdot(7.95\text{ s}) = 178.9\text{ m}$

Ball 2: $\Delta x = v_{ox}\Delta t = (45\text{ m/s})\cdot\cos 45^o\cdot(6.49\text{ s}) = 206.6\text{ m}$

→ Ball 2 travels farther

Alternatively, this problem can be solved using the "range" equation: $\Delta x = \dfrac{2v_o^2\sin\theta\cos\theta}{g}$

Ball 1: $\Delta x = (2)(45\text{ m/s})^2\sin 60^o\cos 60^o/(9.8\text{ m/s}^2) = 178.9\text{ m}$

Ball 2: $\Delta x = (2)(45\text{ m/s})^2\sin 45^o\cos 45^o/(9.8\text{ m/s}^2) = 206.6\text{ m}$

d) Using the 60^o loft golf club, what initial speed and velocity (in component form) must the golf ball have to travel 70 yards in the air?

Ans. The "range" equation is appropriate to solve this problem: $\Delta x = \dfrac{2v_o^2\sin\theta\cos\theta}{g}$

$$v_o = \sqrt{\frac{\Delta x g}{2\sin\theta\cos\theta}} = \sqrt{\frac{(64\text{m})\left(9.8\frac{\text{m}}{\text{s}^2}\right)}{2\sin 60^o\cos 60^o}} = 26.9\frac{\text{m}}{\text{s}}$$

{Note: Δx = 70 yards = 64 m}

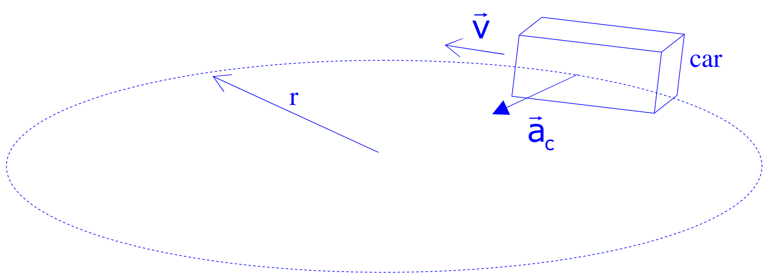
e) Explain why a ball hit with the 45^o golf club will likely roll further once the ball hits the ground compared to the 60^o golf club.

Ans. The ball hit with the 45^o club will have a greater horizontal velocity when it makes impact with the ground.

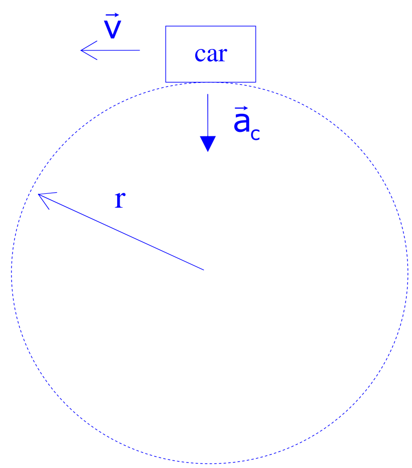
5. **Lateral Acceleration.** According to Road & Track magazine, the maximum lateral acceleration of the Corvette Convertible is 0.98 g. To measure lateral acceleration, the Corvette is driven around a flat 200 ft radius circular track (the "skidpad") at the highest speed possible until the tires lose grip with the road.

a. Draw a simple vector diagram for the car during this road test.

Side view



Looking down on car



b. What is the maximum lateral acceleration in SI units?

Ans. The lateral (or centripetal acceleration): $a_c = 0.98\text{ g} = 9.60\text{ m/s}^2$

c. What speed would correspond to the acceleration calculated in (b).

Ans. Since the centripetal acceleration is: $a_c = \frac{v^2}{r} = 9.60\ \frac{\text{m}}{\text{s}^2}$

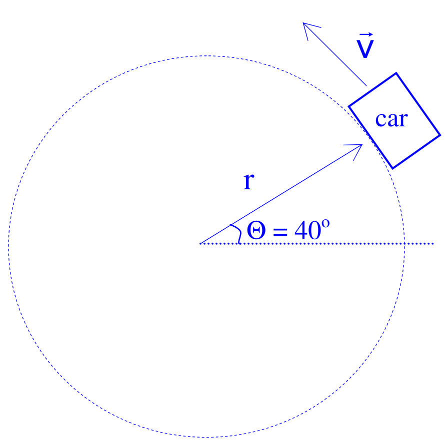
and the radius of the track is: $r = (200\text{ ft})(1\text{ m}/3.281\text{ ft}) = 61.0\text{ m}$

$$v = \sqrt{a_c r} = \sqrt{\left(9.60\ \frac{\text{m}}{\text{s}^2}\right)(61.0\text{ m})} = 24.2\ \frac{\text{m}}{\text{s}}$$

d. Express the velocity vector for the Corvette when it has traveled 40° around the track.

Ans. $\vec{v} = (-v_o\cos 40^\circ)\hat{i} + (-v_o\sin 40^\circ)\,\hat{j} = \left(-18.5\ \frac{\text{m}}{\text{s}}\right)\hat{i} + \left(15.6\ \frac{\text{m}}{\text{s}}\right)\hat{j}$

Looking down on car



Component View