To begin this exercise, use a computer to start-up Internet Explorer (or comparable web browser) and go to www.wiley.com/college/halliday. Select "Student Companion Site" for the 7th edition of Fundamentals of Physics then select on "Concept Simulations" from the left-side menu. Click on the "Concept Simulations" link then select "Vector Addition Using Components" from the left-side menu. *This simulation will be used to verify your graphical and calculation-based results.*

1) Draw a horizontal velocity vector where the magnitude is 10 m/s (use 1 cm = 2 m/s as a scale). Next, draw a second horizontal velocity vector (pointing in the same direction) with a magnitude of 10 m/s (with the tail at the head of the first vector).

a) Using a ruler and protractor, measure the magnitude of the resultant velocity.

b) Using mathematics, calculate the magnitude and direction of the resultant of velocity vector.

Ans. $\vec{R}=\vec{A}+\vec{B}=10\frac{m}{s}+10\frac{m}{s}=20\frac{m}{s}$ @ $0^o$ to the x-axis

c) Use the Vector Addition simulation to verify your results in (a) and (b). Do your results agree with the vector simulation? If not, explain.

d) Express the resultant vector in vector notation, using unit vectors **i** (x-direction) and **j** (y-direction).

Ans. $\vec{R}=\vec{A}+\vec{B}=\left(20\frac{m}{s}\right)\hat{i}+\left(0\frac{m}{s}\right)\hat{j}$

e) Now, let's rotate the x-axis (the direction of the **i** unit vector) 30$^o$ counter-clockwise. Calculate the x and y components for the resultant vector.

Ans.
$$R_x=R\cos30^o=\left(20\frac{m}{s}\right)\cos30^o=17.3\frac{m}{s}$$
$$R_y=R\sin30^o=\left(20\frac{m}{s}\right)\sin30^o=10\frac{m}{s}$$

f) Express the resultant vector from (e) in vector notation, using unit vectors **i** (x-direction) and **j** (y-direction).

Ans. $\vec{R}=\left(17.3\frac{m}{s}\right)\hat{i}+\left(10.0\frac{m}{s}\right)\hat{j}$

2) Draw a horizontal velocity vector where the magnitude is 10 m/s. Next, draw a vertical velocity vector with a magnitude of 10 m/s (with the tail at the head of the first vector).

a) Using a ruler and protractor, determine the measure and direction of the resultant velocity.

b) Using mathematics, calculate the magnitude and direction of the resultant of velocity vector.

Ans. $|\vec{R}|=\sqrt{\left(10\frac{m}{s}\right)^2+\left(10\frac{m}{s}\right)^2}=14.14\frac{m}{s}$ and $\theta=\tan^{-1}\left(\frac{10\frac{m}{s}}{10\frac{m}{s}}\right)=45^o$

c) Use the Vector Addition simulation to verify your results in (a) and (b). Do your results agree with the vector simulation? If not, explain.

d) Express the resultant vector in vector notation, using unit vectors **i** (x-direction) and **j** (y-direction).

Ans. $\vec{R}=\left(10\frac{m}{s}\right)\hat{i}+\left(10\frac{m}{s}\right)\hat{j}$

e) Rotate the x-axis (the direction of the **i** unit vector) 30° counter-clockwise. Calculate the x and y components for the resultant vector.

Ans.
$$R_x=\left(14.14\frac{m}{s}\right)\cos(15^o)=13.66\frac{m}{s}$$
$$R_y=\left(14.14\frac{m}{s}\right)\sin(15^o)=3.66\frac{m}{s}$$

f) Express the resultant vector from (e) in vector notation, using unit vectors **i** (x-direction) and **j** (y-direction).

Ans. $\vec{R}=\left(13.36\frac{m}{s}\right)\hat{i}+\left(-3.66\frac{m}{s}\right)\hat{j}$

3) Draw a velocity vector where the magnitude is 10 m/s and the direction is 30° above the horizontal. Next, draw a velocity vector with a magnitude of 10 m/s and a direction of 110° above the horizontal (with the tail at the head of the first vector).

a) Using a ruler and protractor, measure the magnitude and direction of the resultant velocity.

b) Using mathematics, calculate the magnitude and direction of the resultant of velocity vector.

Ans. $|\vec{R}| = \sqrt{\left(10\frac{m}{s}\right)^2 + \left(10\frac{m}{s}\right)^2 - 2\left(10\frac{m}{s}\right)\left(10\frac{m}{s}\right)\sin(100^\circ)} = 12.86\frac{m}{s}$ and

$$\theta = \sin^{-1}\left(\frac{\left(10\frac{m}{s}\right)\sin(100^\circ)}{15.3\frac{m}{s}}\right) + 30^\circ = 70^\circ$$

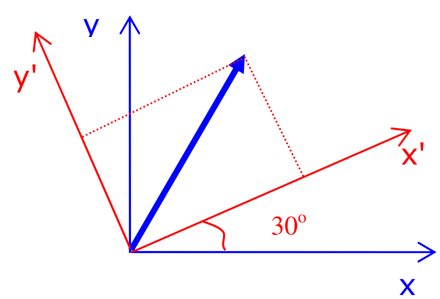
c) Use the Vector Addition simulation to verify your results in (a) and (b). Do your results agree with the vector simulation? If not, explain.

d) Express the resultant vector in vector notation, using unit vectors **i** (x-direction) and **j** (y-direction).

Ans. $\vec{R} = \left(\left(12.86\frac{m}{s}\right)\cos70^\circ\right)\hat{i} + \left(\left(12.86\frac{m}{s}\right)\cos70^\circ\right)\hat{j} = \left(5.24\frac{m}{s}\right)\hat{i} + \left(14.39\frac{m}{s}\right)\hat{j}$

e) Rotate the x-axis (the direction of the **i** unit vector) 30° counter-clockwise. Calculate the x and y components for the resultant vector.

Ans.
$$R_x = \left(15.3\frac{m}{s}\right)\cos(40^\circ) = 11.74\frac{m}{s}$$
$$R_y = \left(15.3\frac{m}{s}\right)\sin(40^\circ) = 9.85\frac{m}{s}$$




f) Express the resultant vector from (e) in vector notation, using unit vectors **i** (x-direction) and **j** (y-direction).

Ans. $\vec{R} = \left(11.74\frac{m}{s}\right)\hat{i} + \left(9.85\frac{m}{s}\right)\hat{j}$

4) Draw a velocity vector with a magnitude of 20 m/s directed 60° above the horizontal direction.

a) Using the ruler and graph scale, measure the magnitude of the x- and y- component vectors.

b) Using trigonometric relationships, calculate the x- and y-components of the vector.

Ans.
$$R_x = \left(20\frac{m}{s}\right)\cos\left(60^\circ\right) = 10\frac{m}{s}$$
$$R_y = \left(20\frac{m}{s}\right)\sin\left(60^\circ\right) = 17.3\frac{m}{s}$$

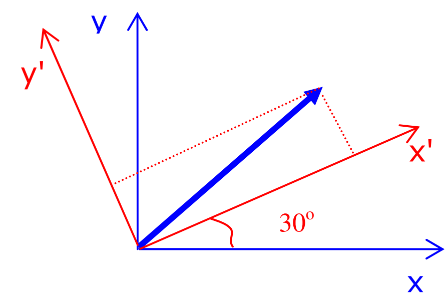
c) Express the vector in vector notation, using unit vectors **i** (x-direction) and **j** (y-direction).

Ans. $\vec{R} = \left(10\frac{m}{s}\right)\hat{i} + \left(17.3\frac{m}{s}\right)\hat{j}$

d) How do your graphical and mathematical results compare?

e) Use the Vector Addition simulation to verify your results in (a) and (b). How do your results agree with the vector simulation?

f) Rotate the x-axis (the direction of the **i** unit vector) 30° counter-clockwise. Calculate the x and y components for the resultant vector.




Ans.
$$R_x = \left(20\frac{m}{s}\right)\cos\left(30^\circ\right) = 17.3\frac{m}{s}$$
$$R_y = \left(20\frac{m}{s}\right)\sin\left(30^\circ\right) = 10\frac{m}{s}$$

g) Express the resultant vector from (e) in vector notation, using unit vectors **i** (x-direction) and **j** (y-direction).

Ans. $\vec{R} = \left(17.3\frac{m}{s}\right)\hat{i} + \left(10\frac{m}{s}\right)\hat{j}$

5) Consider two vectors, $\vec{F}$ and $\vec{s}$:

$\vec{F}$ = 15 N @ 25° above the x-axis

$\vec{s}$ = 5 m along the x-axis

a) Using graph paper, sketch the vectors $\vec{F}$ and $\vec{s}$

b) Calculate the scalar product, $\vec{F}\bullet\vec{s}$, using the vector magnitudes (don't forget the units).

Ans. $\vec{F}\bullet\vec{s} = F\cdot s\cdot\cos\left(25^\circ\right) = (15N)(5m)\cos\left(25^\circ\right) = 67.97\ N\cdot m$

c) Calculate the components for the vectors $\vec{F}$ and $\vec{s}$, using the horizontal and vertical directions for the unit vectors.

Ans.
$$F_x = (15N)\cos\left(25^\circ\right) = 13.59N \qquad s_x = 5m$$
$$F_y = (15N)\sin\left(25^\circ\right) = 6.34N \qquad s_y = 0m$$

d) Calculate the scalar product, $\vec{F}\bullet\vec{s}$, using the vector components (don't forget the units).

Ans. $\vec{F}\bullet\vec{s}=F_x\cdot s_x+F_y\cdot s_y=(13.59N)(5m)=67.97\ N\cdot m$

e) Calculate the components for the vectors $\vec{F}$ and $\vec{s}$, when the $\hat{i}$ and $\hat{j}$ directions are rotated 30° counterclockwise w/r to the horizontal and vertical directions. Determine the scalar product.

Ans.
$$F_x=(15N)\cos(-5^\circ)=14.94N \qquad s_x=(5m)\cos(-30^\circ)=4.33m$$
$$F_y=(15N)\sin(-5^\circ)=-1.31N \qquad s_y=(5m)\sin(-30^\circ)=-2.5m$$

f) How do the scalar products using the 3 methods?

Ans. The 3 methods produce the same result…

6) Consider two vectors, $\vec{F}$ and $\vec{s}$:

$$\vec{F}=(25\ N)\hat{i}+(15N)\hat{j} \quad \text{and} \quad \vec{s}=(3\ m)\hat{i}+(4m)\hat{j}$$

a) Using graph paper, sketch the vectors $\vec{F}$ and $\vec{s}$

b) Calculate the scalar product, $\vec{F}\bullet\vec{s}$, by components.

Ans. $\vec{F}\bullet\vec{s}=F_x\cdot s_x+F_y\cdot s_y=(25N)(3m)+(15N)(4m)=135\ N\cdot m$

c) Determine the magnitude and direction of each vector.

Ans.
$$\left|\vec{F}\right|=\sqrt{(25N)^2+(15N)^2}=29.15N \text{ and } \theta=\tan^{-1}\left(\frac{15N}{25N}\right)=30.96^\circ$$
$$\left|\vec{s}\right|=\sqrt{(3m)^2+(4m)^2}=5m \text{ and } \theta=\tan^{-1}\left(\frac{4m}{3m}\right)=53.13^\circ$$

d) Calculate the scalar product by using the magnitude of the vectors.

Ans. $\vec{F}\bullet\vec{s}=F\cdot s\cdot\cos(53.13^\circ-30.96^\circ)=(29.15N)(5m)\cos(22.17^\circ)=135\ N\cdot m$

e) Calculate the components for the vectors $\vec{F}$ and $\vec{s}$, when the $\hat{i}$ and $\hat{j}$ directions are rotated so that $\vec{s}$ and $\hat{i}$ are directed along the horizontal direction.

Ans.
$$F_x=(29.15N)\cos(22.17^\circ)=26.99N \qquad s_x=5m$$
$$F_y=(29.15N)\sin(22.17^\circ)=11.00N \qquad s_y=0m$$