1) A ball rolls along a flat level surface. The position of the ball (in meters) is given by the following: **x(t) = 5t + 5**. Note that t is measured in seconds.

a) What is the instantaneous and average velocity of the ball while it is on the level surface?

Ans. $v(t) = \frac{dx}{dt} = \frac{d}{dt}(5t+5) = 5\frac{m}{s}$

b) What is the acceleration of the ball?

Ans. $a(t) = \frac{dv}{dt} = \frac{d}{dt}(5) = 0\frac{m}{s^2}$

c) How much time does it take the ball to roll 10 m?

Ans. $\Delta t = \frac{\Delta x}{v} = \frac{10m}{\left(5\frac{m}{s}\right)} = 2s$

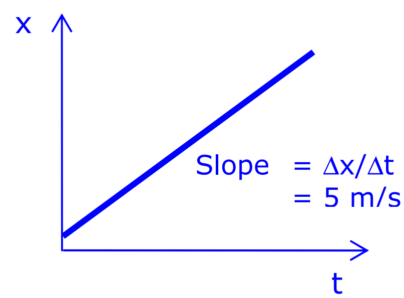
d) How far does the ball roll in 5 seconds?

Ans. $\Delta x = v\Delta t = \left(5\frac{m}{s}\right)(5s) = 25m$

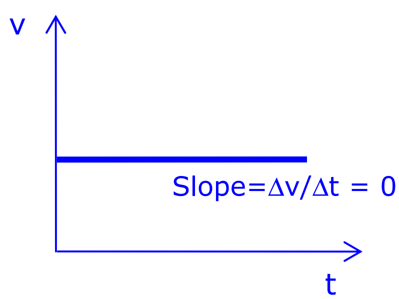
e) What is the acceleration of the ball?

Ans. a = 0 m/s^2

f) Sketch a position vs. time graph for the ball while it is rolling.



e) Sketch a velocity vs. time graph for the ball while it is rolling.



2) A bass swims along a straight line with a velocity given by: **v = v(t) = 3.5 m/s**.

a) What is the acceleration of the fish?

Ans. $a(t)=\frac{dv}{dt}=\frac{d}{dt}\left(3.5\frac{m}{s}\right)=0\frac{m}{s^2}$

b) What is the position of the fish as a function of t?

$$\int_{x_o}^{x} dx=\int_{t_o}^{t} v\,dt=\int_{t_o}^{t}\left(3.5\frac{m}{s}\right)dt$$

Ans. $x-x_o=\left(3.5\frac{m}{s}\right)(t-t_o)$

{when x_o=0 m & t_o=0 s} $x(t)=\left(3.5\frac{m}{s}\right)t$

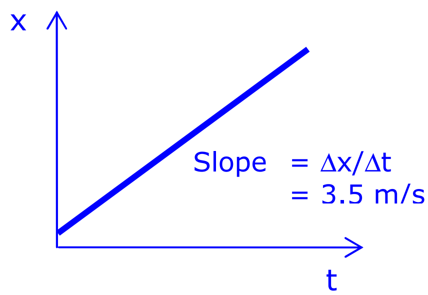
c) How much time does it take the bass to swim 5 m?

Ans. $\Delta t=\frac{\Delta x}{v}=\frac{5\ m}{\left(3.5\frac{m}{s}\right)}=1.43s$

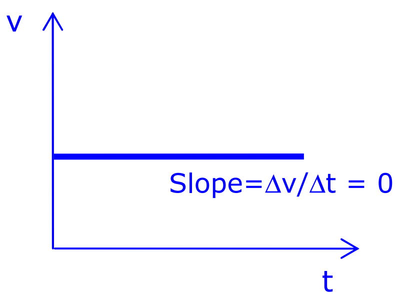
d) How far does the bass swim in 3 seconds?

Ans. $\Delta x=v\Delta t=\left(3.5\frac{m}{s}\right)(3\ s)=10.5\ m$

e) Sketch the position vs. time graph for the bass during this motion.



f) Sketch the velocity vs. time graph for the bass during this motion.



3) A coin, initially at rest, is dropped from an elevated position and freely falls to the ground with an acceleration of 9.8 m/s^2.

a) How fast is the coin traveling after 5 seconds?

Ans. $v = v_o + a\Delta t = 0 + \left(-9.8\frac{m}{s^2}\right)(5s) = -49\frac{m}{s}$

b) How far does the coin fall after 5 seconds?

Ans. $\Delta y = \bar{v}\Delta t = \frac{1}{2}\left(-49\frac{m}{s}\right)(5s) = -122.5m$ or $\Delta y = v_o\Delta t + \frac{1}{2}a\Delta t^2 = \frac{1}{2}\left(9.8\frac{m}{s^2}\right)(5s)^2 = 122.5m$

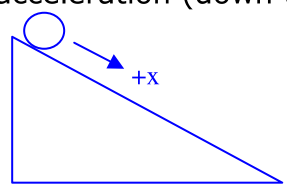
c) How much time does it take the coin to fall 10 m?

Ans. $\Delta t = \sqrt{\frac{2\Delta y}{a}} = \sqrt{\frac{20m}{\left(9.8\frac{m}{s^2}\right)}} = 1.43s$

d) What is the velocity of the coin after it has fallen 10 m?

Ans. $v = v_o + a\Delta t = 0 + \left(-9.8\frac{m}{s^2}\right)(1.43s) = -14\frac{m}{s}$

4) A ball, initially at rest, rolls down a hillside with constant acceleration (down the hill) of 2.5 m/s^2.



a) How fast is the ball traveling after 5 seconds?

Ans. $v = v_o + a\Delta t = 0 + \left(2.5\frac{m}{s^2}\right)(5s) = 12.5\frac{m}{s}$

b) How far does the ball fall after 5 seconds?

Ans. $\Delta y = \bar{v}\Delta t = \left(\frac{12.5\frac{m}{s}}{2}\right)(5s) = 31.3m$ or $\Delta y = v_o\Delta t + \frac{1}{2}a\Delta t^2 = \frac{1}{2}\left(2.5\frac{m}{s^2}\right)(5s)^2 = 31.3m$

c) How much time does it take the ball to travel 10 m?

Ans. $\Delta t = \sqrt{\frac{2\Delta y}{a}} = \sqrt{\frac{20m}{\left(2.5\frac{m}{s^2}\right)}} = 2.83s$

d) What is the velocity of the ball after it has traveled 10 m?

Ans. $v = v_o + a\Delta t = 0 + \left(2.5\frac{m}{s^2}\right)(2.83s) = 7.07\frac{m}{s}$

e) What is the average velocity of the ball during this motion?

Ans. $\bar{v} = \left(\frac{v + v_o}{2}\right) = \frac{1}{2}\left(7.07\frac{m}{s}\right) = 3.54\frac{m}{s}$

5) A coin, is tossed upward from an elevated position (y_o= 5 m) and freely falls to the ground with a downward acceleration of 9.8 m/s^2. The coin has an initial upward velocity of 10 m/s.

a) How long does it take the coin to reach its highest point?

Ans. $\Delta t = \frac{\Delta v}{a} = \frac{(0 - 10\,\frac{m}{s})}{(-9.8\,\frac{m}{s^2})} = 1.02s$

b) What is the coin's velocity when it reaches its highest point?

Ans. v = 0 m/s

c) How long does it take the coin to reach the ground?

Ans. since $\Delta y = v_o\Delta t - \frac{1}{2} g\Delta t^2 \rightarrow -5\text{ m} = (10\text{ m/s})\Delta t - \frac{1}{2}(9.8\text{ m/s}^2)\Delta t^2$

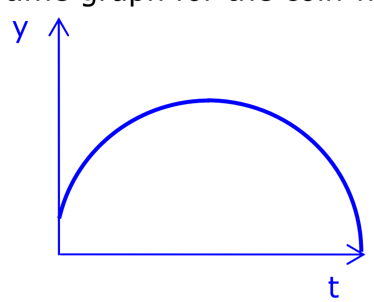
Use the quadratic equation to solve for t:

$$\Delta t = \frac{10\,\frac{m}{s} \pm \sqrt{(10\,\frac{m}{s})^2 - 4(4.9\,\frac{m}{s^2})(-5m)}}{9.8\,\frac{m}{s^2}} = 2.47s \text{ or } 0.42\text{ s} \rightarrow \Delta t = 2.47\text{ s}$$

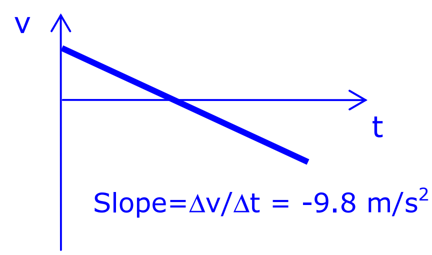
d) What is the velocity of the coin when it reaches the ground?

Ans. $v = v_o - g\Delta t = 10\,\frac{m}{s} + (-9.8\,\frac{m}{s^2})(2.47s) = -14.2\,\frac{m}{s}$

e) Sketch a position vs. time graph for the coin while it is in the air.



f) Sketch a velocity vs. time graph for the coin while it is in the air.



"Real World" Application

Road & Track magazine test drove a 1995 Corvette Convertible and published the following performance results:

Final Speed (USCS)	40 mph	60 mph	80 mph	100 mph
Speed (SI)	**17.9 m/s**	**26.8 m/s**	**35.8 m/s**	**44.7 m/s**
Time (from rest)	3.0 s	5.5 s	9.2 s	14.5 s

1) Convert the above final speed values to SI units and record in the table above.

Ans. See above table.

2) What is the average acceleration of the Corvette from 0 to 40 mph?

Ans. $$\bar{a} = \frac{v - v_o}{\Delta t} = \frac{17.9\frac{m}{s} - 0\frac{m}{s}}{3.0s} = 6.0\frac{m}{s^2}$$

3) Treating the acceleration as constant, what is the equation for the position, x(t), of the Corvette as it accelerates from 0 to 40 mph? Assume $x_o = x(0) = 0$ m.

Ans.

$$\int_{v_o=0}^{v} dv = \int_{t_o}^{t} a\,dt = \int_{t_o}^{t} \left(6.0\frac{m}{s^2}\right) dt$$

$$v(t) = \left(6.0\frac{m}{s^2}\right) t$$

$$\int_{x_o}^{x} dx = \int_{t_o}^{t} v\,dt = \int_{t_o}^{t} \left(6.0\frac{m}{s^2}\right) t\,dt$$

$$x(t) = \frac{1}{2}\left(6.0\frac{m}{s^2}\right) t^2 \quad \left\{\text{when } v_o = 0\ \frac{m}{s},\ x_o = 0\text{ m} \ \&\ t_o = 0\text{ s}\right\}$$

4) What is the displacement of the Corvette as it accelerates from 0 to 40 mph?

Ans. $\Delta x = \frac{1}{2} a \cdot \Delta t^2 = (\frac{1}{2})(6.0\text{ m/s}^2)(3.0\text{ s})^2 = 27\text{ m}$

5) What is the average acceleration from 40 to 60 mph?

Ans. $$\bar{a} = \frac{v - v_o}{\Delta t} = \frac{26.8\frac{m}{s} - 17.9\frac{m}{s}}{2.5s} = 3.6\frac{m}{s^2}$$

6) Treating the acceleration as constant, what is the equation for the position, x(t), of the Corvette as it accelerates from 40 to 60 mph?

Ans.

$$\int_{v_o=17.9\frac{m}{s}}^{v=26.8\frac{m}{s}} dv = \int_{t_o=3s}^{t=5.5s} a\,dt = \int_{t_o=3s}^{t=5.5s} \left(3.6\frac{m}{s^2}\right) dt$$

$$v(t) = \left(3.6\frac{m}{s^2}\right) t - v_o = \left(3.6\frac{m}{s^2}\right) t - 17.9\frac{m}{s} \quad \{\text{for } 3.0\text{ s} \le t \le 5.5\text{ s}\}$$

$$\int_{x_o}^{x} dx = \int_{t_o}^{t} v\,dt = \int_{t_o}^{t} \left[\left(3.6\frac{m}{s^2}\right) t - 17.9\frac{m}{s}\right] dt$$

$$x(t) = \frac{1}{2}\left(3.6\frac{m}{s^2}\right) t^2 + \left(17.9\frac{m}{s}\right) t + 27m \quad \{\text{for } 3.0\text{ s} \le t \le 5.5\text{ s}\}$$

7) How far does the Corvette travel as it accelerates from 40 to 60 mph?

Ans. $\Delta x = v_o \cdot \Delta t + \frac{1}{2} \cdot a \cdot \Delta t^2 = (17.9 \text{ m/s})(2.5 \text{ s}) + (\frac{1}{2})(3.6 \text{ m/s}^2)(2.5 \text{ s})^2 = 56 \text{ m}$

8) What is the average acceleration from 0 to 100 mph?

Ans. $\bar{a} = \dfrac{v - v_o}{\Delta t} = \dfrac{44.7\frac{m}{s} - 0\frac{m}{s}}{14.5s} = 3.1\frac{m}{s^2}$

9) Using the average acceleration in (12), estimate how long it would take the Corvette to travel ¼ mile, starting from rest.

Ans. Convert the ¼ mile into SI units: $\Delta x = \left(\dfrac{0.25mi}{1}\right)\left(\dfrac{1.609km}{1mi}\right)\left(\dfrac{1000m}{1km}\right) = 402m$

Calculate the elapsed time: $\Delta t = \sqrt{\dfrac{2\Delta x}{a}} = \sqrt{\dfrac{2(402m)}{3.1\frac{m}{s^2}}} = 16.1s$

10) Estimate the final speed of the Corvette at the end of the quarter mile trial?

Ans. $v = \bar{a}\Delta t = \left(3.1\frac{m}{s^2}\right)(16.1s) = 49.9\frac{m}{s}$

Converting the units to mi/hr: $v = \left(\dfrac{49.9m}{1s}\right)\left(\dfrac{1mi}{1609km}\right)\left(\dfrac{3600s}{1hr}\right) = 111.7\frac{mi}{hr}$