Rotational Variables

1) Write out the expression for the average angular (\(\omega_{avg}\)), in terms of the angular displacement (\(\theta\)) and elapsed time (\(\Delta t\)).

2) Write out the expression for the average angular acceleration (\(\alpha_{avg}\)), in terms of the angular velocity (\(\omega\)) and elapsed time (\(\Delta t\)).

3) Write an expression for the average angular velocity (\(\omega_{avg}\)) in terms of initial and final angular velocities (\(\omega_0\) and \(\omega\), respectively), where the angular acceleration is constant.

4) Combine your answers to the above exercises and derive the rotational kinematic equations in terms of the angular variables (\(\theta, \omega, \alpha\)). Assume the angular acceleration is constant.

Rotational Motion

5) An ant is standing on a moving CD, 2 cm from the center. The CD is moving at 80 rpm.
   a. When the CD makes a 45.0° revolution, what is the length of the ant’s path?
   b. What is the ant’s actual linear displacement (\(\Delta \vec{r}\))?
   c. What is the rotational speed of the CD expressed in radians per sec?
   d. The ant moves to a new position 4.0 cm from the center.
      i. What is the angular velocity of the ant compared to its speed at \(r = 2.0\) cm?
      ii. What is the tangential speed of the ant compared to its speed at \(r = 2.0\) cm?
   e. The ant then moves to the center of the CD. What is the angular velocity and tangential speed of the ant as the CD rotates?
Moment of Inertia

6) Calculate the moment of inertia (in SI units) for the following objects:

a. A 2 kg mass attached to a thin 0.15 m long string (a simple pendulum).

b. A 0.5 kg hollow sphere with a 0.10 m diameter.

c. Two 10 kg solid spheres (r=0.05 m) attached by a mass-less 0.5 m bar (simple dumbbell), rotated about the middle of the bar.

d. Two 10 kg solid spheres (r=0.05 m) attached by a mass-less 0.5 m bar (simple dumbbell), rotated about the center of one of the spheres.

e. A meter stick (0.15 kg) rotated about its center of mass.

f. A meter stick (0.15 kg) rotated about one end.

Torque:

7) Two people are sitting on a 20 kg teeter-totter (L = 7.0 m), Person A (60 kg) and Person B (30 kg).

a. Person A sits 1.5 m from the fulcrum. What is the torque exerted by Person A on the teeter-totter (assume the mass of the teeter-totter itself can be neglected)?

b. For the teeter-totter to be balanced, and in mechanical equilibrium, with Person B sitting on the opposing end, what torque must Person B apply to the teeter-totter? Draw a free body diagram of the 2-person teeter-totter system.

c. How far from the fulcrum must Person B sit so that the teeter-totter balances?

d. Re-calculate all of the torques using a fulcrum located at one end of the balanced teeter-totter.
Kinetic Energy & Rotation:

8) Determine the rotational kinetic energy for the following objects, rotated about the axis described above.

a. A mass attached to a string and rotating at 25 rad/sec.

b. A hollow sphere rotating about its center at 15 rad/sec.

c. Two 10 kg solid spheres (r=0.05 m) attached by a massless 0.5 m bar (simple dumbbell), rotated about the middle of the bar at 15 rad/sec.

d. Two 10 kg solid spheres (r=0.05 m) attached by a massless 0.5 m bar (simple dumbbell), rotated about the center of one of the spheres at 15 rad/sec.

e. A meter stick (0.15 kg) rotated about its center of mass, at 5 rpm.

f. A meter stick (0.15 kg) rotated about one end, at 5 rpm.

9) Determine the rotational kinetic energy of a 0.15 kg golf ball, radius of 2.0 cm.

a. Treat the golf ball as a solid sphere that is rotating at 1000 rpm and simultaneously traveling at 10 m/s, what is the rotational kinetic energy of the ball?

b. What is the total kinetic energy of the ball?
10) Consider 2 attached masses (Bugs and Wiley E.) suspended by a simple (solid cylindrical) pulley (2 kg). Bugs has a mass of 4 kg and Wiley E. has a mass of 8 kg. The radius of the pulley is 0.1 m. Starting from rest, Wiley E. drops 1 m and Bugs ascends the same amount.

a. Draw a force vector diagram for each object (including the pulley). *Note: the tension forces on each end of the rope are not equal.*

b. Apply Newton’s 2nd Law to Wiley E. and Bugs. Use it to solve for the tension forces, $T_1$ and $T_2$, in terms of $g$, $m_{Wiley}$ and $m_{Bugs}$.

c. Apply Newton’s 2nd Law (torque) to the pulley.

d. Use the expressions for the tension forces in (b) and the torque equation obtained in (c) to solve for the acceleration of the system (a).

e. What is the magnitude of the tension force at each end of the rope, $T_1$ and $T_2$, respectively?

f. What is KE for Wiley E. and Bugs, respectively, at the end of the 1 m displacement?

g. What is the rotational KE of the pulley at the end of the 1 m displacement?

h. Calculate the total KE and mechanical energy of the system at the end of the 1 m displacement. Compare this value with the total KE for the same system where the pulley is mass-less.