Phy 211: General Physics I

Chapter 9: Center of Mass & Linear Momentum
Lecture Notes

Rene Descartes (1596-1650)

- Prominent French mathematician & philosopher
- Active toward end of Galileo's career
- Studied the nature of collisions between objects
- First introduced the concept of momentum
  - he defined momentum ("vis-à-vis") as the product of <u>weight</u> times <u>speed</u>
- Demonstrated the Law of Conservation of Momentum

Linear Momentum

- Linear momentum ($\vec{p}$) represents inertia in motion (Newton momentum as the "quantity of motion")
  - Conceptually, the effort required to bring a moving object to rest depends not only on its mass (inertia) but also on how fast it is moving

Definition: $\vec{p} = m\vec{v}$

- Momentum is a vector quantity with the same direction as the object's velocity
- SI units are kg·m/s

Newton's 1st Law revisited:

The momentum of an object will remain constant unless it is acted upon by a net force (or impulse)

Center of Mass

Center of Mass ($\vec{r}_{cm}$) refers to the average location of mass for a defined mass.

- To determine the center of mass, take the sum of each mass multiplied by its position vector and divide by the total mass of the system or

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \ldots + m_n\vec{r}_n}{m_1 + m_2 + m_3 + \ldots + m_n} = \frac{\sum_{i=1}^{n} m_i\vec{r}_i}{m_{sys}}$$

- Note, if the objects in the system are in motion, the velocity of the system (center of mass) is:

$$\vec{v}_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + \ldots + m_n\vec{v}_n}{m_1 + m_2 + \ldots + m_n} = \frac{\sum_{i=1}^{n} m_i\vec{v}_i}{m_{sys}}$$

- When $\Delta\mathbf{p}_{\mathbf{system}} = 0$ (i.e. $\mathbf{F}_{\mathbf{ext}} = 0$) then $\mathbf{v}_{cm}$ = constant
  - The motion of all bodies even if they are changing individually will always have values such that $\mathbf{v}_{cm}$ = constant

Impulse-Momentum Theorem

Newton's 2nd Law, can be rewritten as

$$\vec{F}_{net} = m\vec{a} = m\frac{d\vec{v}}{dt} = \frac{d(m\vec{v})}{dt} = \frac{d\vec{p}}{dt}$$

Rearranging terms:

$$d\vec{p} = \vec{F}_{net}dt \Rightarrow \int_{\vec{p}_o}^{\vec{p}} d\vec{p} = \int_{t_o}^{t} \vec{F}_{net}dt$$

$$\Delta\vec{p} = \int_{t_o}^{t} \vec{F}_{net}dt$$ this is the Net Impulse!!

Definition of Impulse associated with an applied force:

$$\vec{J} = \Delta\vec{p} = \int_{t_o}^{t} \vec{F}dt$$

- The SI units for impulse are N·s
- For a constant force (or average force), impulse simplifies to:

$$\vec{J} = \Delta\vec{p} = \vec{F}_{avg}\Delta t$$

- Therefore, Impulse represents simultaneously:
  1. The product of the force times the time: $\vec{F}_{avg}\Delta t$
  2. The change in linear momentum of the object: $\Delta\vec{p} = m\vec{v}_f - m\vec{v}_i$

Notes on Impulse

- Impulses always occur as action-reaction pairs
- The force·time relationship is observed in many "real world" examples:
  - Automobile safety:
    - Dashboards
    - Airbags
    - Crumple zones
  - Product packaging
    - Styrofoam spacers
  - Sports
    - Tennis: racket string tension
    - Baseball: "juiced" baseballs & baseball bats (corked & aluminum vs. wood)
    - Golf: the "spring-like" effect of golf club heads
    - Boxing gloves: (lower impulsive forces in the hands)

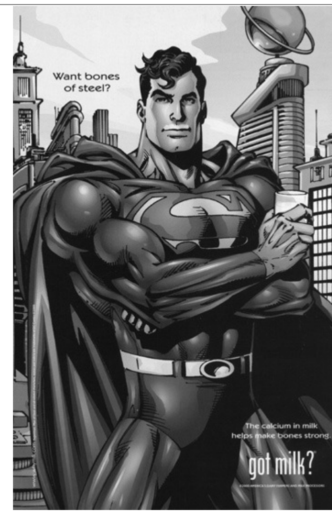
A Superman Problem

It is well known that bullets and missiles bounce off Superman's chest. Suppose a bad guy sprays Superman's chest with 0.003 kg bullets traveling at a speed of 300 m/s (fired from a machine gun at a rate of 100 rounds/min). Each bullet bounces straight back with no loss in speed.

<u>Problems:</u>

a) What is the impulse exerted on Superman's chest by a single bullet?

b) What is the average force exerted by the stream of bullets on Superman's chest?



Collisions

A specific type of interaction between 2 objects. The basic assumptions of a collision:

1. Interaction is short lived compared to the time of observation
2. A relatively large force acts on each colliding object
3. The motion of one or both objects changes abruptly following collision
4. There is a clean separation between the state of the objects <u>before</u> collision vs. <u>after</u> collision

3 classifications for collisions:

– <u>Perfectly elastic:</u> colliding objects bounce off each other and no energy is lost due to heat formation or deformation (K_{system} is conserved)

– <u>Perfectly inelastic:</u> colliding objects stick together (K_{system} is not conserved)

– <u>Somewhat inelastic</u> (basically all other type of collisions): KE is not conserved

Conservation of Linear Momentum

The total linear momentum of a system will remain constant when no ***external*** *net force acts upon the system, or*

$$(\vec{p}_1 + \vec{p}_2 + \ldots)_{\text{before collision}} = (\vec{p}_1 + \vec{p}_2 + \ldots)_{\text{after collision}}$$

- ***Note:*** Individual momentum vectors may change due to collisions, etc. but the linear momentum for the system remains constant
- Useful for solving collision problems:
  - Where all information is not known/given
  - To simplify the problem
- Conservation of Momentum is even more fundamental than Newton's Laws!!

Conservation of Momentum (Examples)

- The ballistic pendulum
- 2 body collisions (we can't solve 3-body systems…☹)
  - Perfectly inelastic ($E_{\text{pre-collision}} \neq E_{\text{post-collision}}$)
  - Perfectly elastic ($E_{\text{pre-collision}} = E_{\text{post-collision}}$)
- Collisions in 2-D or 3-D:
  - Linear momentum is conserved by components:

$$(\vec{p}_1 + \vec{p}_2 + \ldots)_{\text{before collision}} = (\vec{p}_1 + \vec{p}_2 + \ldots)_{\text{after collision}}$$

By Components:

$$\left[(p_{1x} + p_{2x} + \ldots)\,\hat{i}\right]_{\text{before collision}} = \left[(p_{1x} + p_{2x} + \ldots)\hat{i}\right]_{\text{after collision}}$$

$$\left[(p_{1y} + p_{2y} + \ldots)\,\hat{j}\right]_{\text{before collision}} = \left[(p_{1y} + p_{2y} + \ldots)\hat{j}\right]_{\text{after collision}}$$

Notes on Collisions & Force

- During collisions, the forces generated:
  - Are short in duration
  - Are called <u>impulsive forces</u> (or impact forces or collision forces)
  - Often vary in intensity/magnitude during the event
  - Can be described by an average collision force:

$$\vec{F}_{Net} = \vec{F}_{avg} = \frac{\Delta\vec{p}}{\Delta t} \left\{ i.e. \ \frac{\text{impulse}}{\text{time}} \right\}$$

<u>Example:</u> a golf club collides with a 0.1 kg golf ball (initially at rest), Δt 0.01s. The velocity of the ball following the impact is 25 m/s.

The impulse exerted on the ball is:

$$\Delta\vec{p} = m\Delta\vec{v} = (0.1 \text{ kg})(25 \ \tfrac{m}{s} - 0 \ \tfrac{m}{s})\ \hat{i} = 2.5 \text{ N}\cdot\text{s}\ \hat{i}$$

The average impulsive force exerted on the <u>ball</u> is:

$$\vec{F}_{avg} = \frac{\Delta\vec{p}}{\Delta t} = \frac{2.5 \text{ N}\cdot\text{s}}{0.01 \text{ s}}\ \hat{i} = 250 \text{ N}\ \hat{i}$$

The average impulsive force exerted on the <u>club</u> is:

$$\vec{F}_{avg} = \frac{\Delta\vec{p}}{\Delta t} = \frac{-2.5 \text{ N}\cdot\text{s}}{0.01 \text{ s}}\ \hat{i} = -250 \text{ N}\ \hat{i}$$