Phy 211: General Physics I

Chapter 7: Kinetic Energy & Work
Lecture Notes

What is Energy?

- Energy is a scalar quantity associated with the state of an object (or system of objects).
  - Energy is a calculated value that appears in nature and whose total quantity in a system always remains constant and accounted for.
  - Energy is said to be "Conserved"
- Loosely speaking, energy represents the "fuel" necessary for changes to occur in the universe and is often referred to as the capacity to perform work.
- We can think of energy as the "currency" associated with the "transactions" (forces!) that occur in nature.
  - In mechanical systems, energy is "spent" as force transactions are conducted.
  - Alternatively, the exertion of force requires an "expenditure" of energy.
- The SI units for energy are called Joules (J)
  - In honor of James Prescott Joule

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James Prescott Joule (1818-1889)

- English inventor & scientist
- Interested in the efficiency of electric motors
- Described the heat dissipated across a resistor in electrical circuits (now known as Joules' Law)
- Demonstrated that heat is produced by the motion of atoms and/or molecules
- Credited with establishing the mechanical energy equivalent of heat
- Participated in establishing the "Law of Energy Conservation"

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Review: The Scalar (Dot) Product

- Two vectors $(\vec{A} \text{ and } \vec{B})$ can be multiplied to product a scalar resultant, called the scalar (or Dot) product.
- When using the magnitudes of the vectors: $\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}|\cos\phi$ where ϕ is the angle between vectors **A** and **B**
- When using vector components: $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y$
- Useful properties of scalar products:
  1. Commutative property: $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$
  2. Squaring vectors: $\vec{A} \cdot \vec{A} = A^2$
  3. Unit vectors: $\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$

Important Application: We will use the scalar product of the vectors of force and displacement to calculate work performed by force

Kinetic Energy

The energy associated with an object's state of motion

– Kinetic energy is a scalar quantity that is never negative in value

Definition:

Scalar form: $K = \frac{1}{2}mv^2 \quad \left\{ SI\ units:\ kg\left(\frac{m}{s}\right)^2 \right\}$

Vector form: $K = \frac{1}{2}m\vec{v}\cdot\vec{v} = \frac{1}{2}mv_x^2 + \frac{1}{2}mv_y^2 + \frac{1}{2}mv_z^2$

$$K = K_x + K_y + K_z$$

Key Notes:

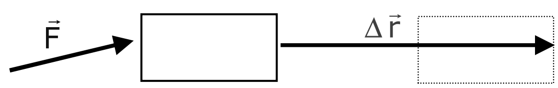
1. *The kinetic energy for the x, y, and z components are additive*
2. *Kinetic energy is relative to the motion of observer's reference frame, since speed and velocity are as well.*
3. *An object's kinetic energy depends more on its speed than its mass*
4. *Any change in an object's speed will affect a change in its kinetic energy*
5. *Unit comparison:* $1\ J = 1\ kg\left(\frac{m}{s}\right)^2$

Work

The energy transferred to/from an object by the exertion of a force

– Work is essentially a measure of useful physical output or

Work = Effort x Outcome

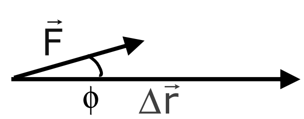


Definition:

standard form: $W = \vec{F}\cdot\Delta\vec{r} = F_x\Delta x + F_y\Delta y + F_z\Delta z$

or $W = |\vec{F}||\Delta\vec{r}|\cos\phi$

– SI Units for Work: N·m

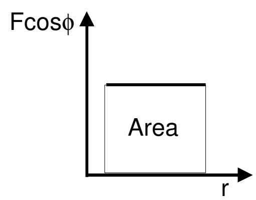


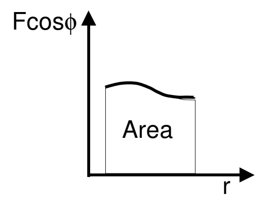
Notes:

1. Unit comparison: $1\ J = 1\ N\cdot m$

Work (cont.)

- *The general definition of Work, for a varying (as well as constant) force*: $W = \int_0^W dW = \int_{r_o}^{r} \vec{F}\cdot d\vec{r}$
- Graphically, work is the “area” beneath the Force vs. Displacement graph





$$dW = \vec{F}\cdot d\vec{r} \Rightarrow W = \int_0^W dW = \int_{r_o}^{r} \vec{F}\cdot d\vec{r}$$

Work & Kinetic Energy

- The net work performed on an object is related to the net force: $W_{Net} = \vec{F}_{Net}\cdot\Delta\vec{r} = m(\vec{a}\cdot\Delta\vec{r})$
- When $\vec{F}_{Net} \neq 0$, a change in state of motion & kinetic energy is implied: $W_{Net} = \int \vec{F}_{Net}\cdot d\vec{r} = K - K_o = \Delta K$

Derivation:

$$W_{Net} = \int dW = \int \vec{F}_{Net}\cdot d\vec{r} = \int m(\vec{a}\cdot d\vec{r})$$

$$W_{Net} = \int m(\vec{a}\cdot d\vec{r}) = \int m\left(\frac{d\vec{v}}{dt}\cdot d\vec{r}\right) = \int m\left(\frac{d\vec{v}}{d\vec{r}}\cdot\frac{d\vec{r}}{dt}\cdot d\vec{r}\right)$$

$$W_{Net} = \int m\left(d\vec{v}\cdot\frac{d\vec{r}}{dt}\right) = \int m(\vec{v}\cdot d\vec{v}) = \frac{1}{2}m(\vec{v}\cdot\vec{v})\Big|_{v_o}^{v}$$

$$W_{Net} = \frac{1}{2}m(\vec{v}\cdot\vec{v}) - \frac{1}{2}m(\vec{v}_o\cdot\vec{v}_o)$$ **The Work-Energy Theorem!**

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Work Performed by Gravitational Force

For a falling body (no air drag):

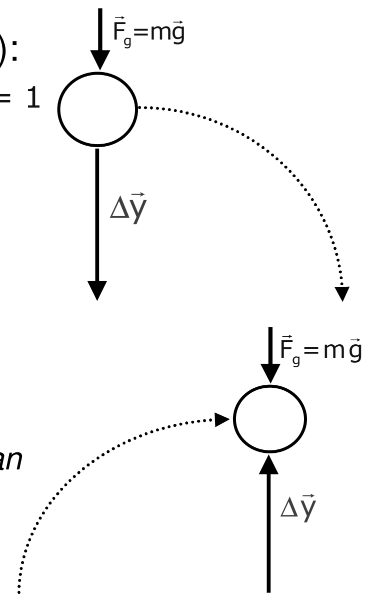
$W_y = \vec{F}_g \cdot \Delta\vec{y} = -mg\Delta y$ since $\cos\phi = 1$

$W_x = \vec{F}_g \cdot \Delta\vec{x} = 0$ since $\cos\phi = 0$

$W_g = W_x + W_y = -mg\Delta y$

- Gravitational force only performs work in the vertical direction:
  1. W_g is + when Δy is –
  2. W_g is - when Δy is +

What about gravitational force on an incline?



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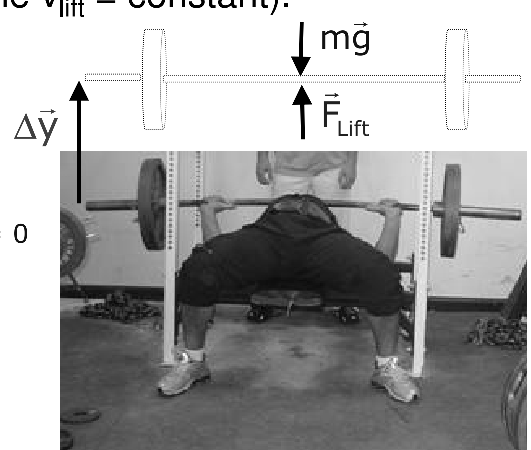
Work Performed during Lifting & Lowering

- Consider Joey "blasting" his pecs with a bench press workout (assume v_{lift} = constant).

Given: $m_{bar} = 100$ kg

$m_{bar}\vec{g} = -980\ \text{N}\ \hat{j}$

$\Delta y = 1\text{m}\ \hat{j}$



Applying Newton's 2nd Law:

$\vec{F}_{Net} = (F_{Lift} - m_{bar}g)\hat{j} = m_{bar}\vec{a} = 0$

$F_{Lift} = m_{bar}g = 980$ N

The Work performed:

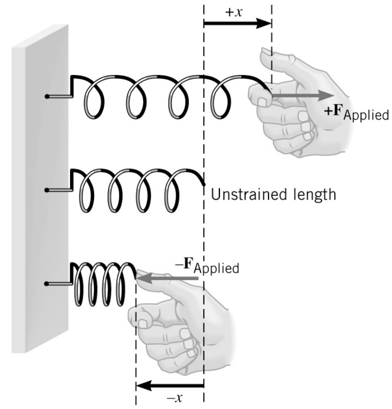
$W_g = m_{bar}\vec{g} \cdot \Delta\vec{y} = -980\ \text{N}\cdot\text{m}$

$W_{Lift} = F_{Lift} \cdot \Delta\vec{y} = 980\ \text{N}\cdot\text{m}$

$W_{Net} = \vec{F}_{Net} \cdot \Delta\vec{y} = W_{Lift} + W_g = 0$

Work Performed by Elastic Force

- The elastic (or "restoring") force associated with the compression (or stretching of a simple spring increases linearly with the amount of deformation (this is Hooke's Law)
- The following assumptions are made:
  - The spring is ideal & massless
  - The spring obeys Hooke's Law

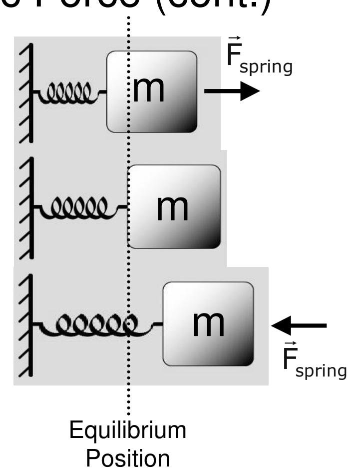


$\vec{F}_{spring} = -k\Delta\vec{r}_{stretch} = -k\vec{x}$ where $\Delta\vec{r}_{stretch} = \Delta\vec{x} = x$

where $\vec{x}_o = 0$ is the equilibrium position of the spring and k is the spring constant (in N/m)

Work Performed by Elastic Force (cont.)

Consider a spring (k=10 N/m) attached to a wall at one end and a mass (1 kg) at the other. The block is free to slide on a frictionless surface. The block/spring system is compressed 0.5 m then released.



The work performed by the spring force is:

$$W_{spring} = \int F_{spring} \cdot d\vec{x}$$

$$W_{spring} = -\int_{x_o}^{x} k\vec{x} \cdot d\vec{x} = -\frac{1}{2}k\left(\vec{x} \cdot \vec{x}\right)\Big|_{x_o}^{x} = -\frac{1}{2}k\left(x^2 - x_0^2\right)$$

$$W_{spring} = \frac{1}{2}kx_0^2 - \frac{1}{2}kx^2$$

Power

- A measure of work effectiveness
- The time rate of energy transfer (work) due to an exerted force:

Average Power: $P_{avg} = \dfrac{W}{\Delta t} = \dfrac{\vec{F}\cdot\Delta\vec{r}}{\Delta t}$

Instantaneous Power: $P = \dfrac{dW}{dt} = \dfrac{\vec{F}\cdot d\vec{r}}{dt}$

Average Net Power: $P_{Net} = \dfrac{\Delta W_{Net}}{\Delta t} = \dfrac{\vec{F}_{Net}\cdot d\vec{r}}{\Delta t} = \dfrac{\Delta K}{\Delta t}$

- *SI units:* The Watt (1 W = 1 J/s)
- **Note:** Power is also related to Force & Velocity:

$$P = \frac{dW}{dt} = \frac{\vec{F}\cdot d\vec{r}}{dt} = \vec{F}\cdot\vec{v}$$

Power (cont.)

- The same work output can be performed at various power rates.

Example 1: Consider 100 J of work output accomplished over 2 different time intervals:

100 J over 1 s: $P = \dfrac{100\ \text{J}}{1\ \text{s}} = 100\ \text{W}$ **100 J over 100 s:** $P = \dfrac{100\ \text{J}}{100\ \text{s}} = 1\ \text{W}$

Example 2a: An 900 kg automobile accelerates from 0 to 30 m/s in 5.8 s. What is the average net power?

$$P_{net} = \frac{K\text{-}K_o}{\Delta t} = \frac{\frac{1}{2}(900\ \text{kg})\left[(30\,\tfrac{m}{s})^2 - (0\,\tfrac{m}{s})^2\right]}{5.8\ \text{s}} = 6.98\times10^4\ \text{W or } 93.6\ \text{hp}$$

Example 2b: At the 30 m/s, how much force does the road exert on this vehicle? Use the same power as 2a.

$$P = \vec{F}\cdot\vec{v} = 6.98\times10^4\ \text{W} \Rightarrow |F| = \frac{P}{|v|} = \frac{6.98\times10^4\ \text{W}}{30\,\tfrac{m}{s}} = 2330\ \text{N}$$