Phy 211: General Physics I

Chapter 3: Vectors

Lecture Notes

Vectors & Scalars

Most physical quantities can categorized as one of 2 types (tensors notwithstanding):

1. **Scalar Quantities:**
   - described by a single number & a unit (s).

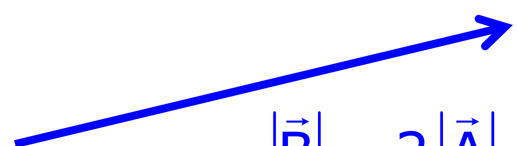
   Example: the length of the driveway is 3.5 m

2. **Vectors Quantities:**
   - described by a value (magnitude) & direction.

   Example: the wind is blowing 20 m/s due north

   - Vectors are represented by an arrow, where:
     1. the length of the arrow is proportional to the magnitude of the vector.

$$\left|\vec{A}\right| = 2\text{ m}$$

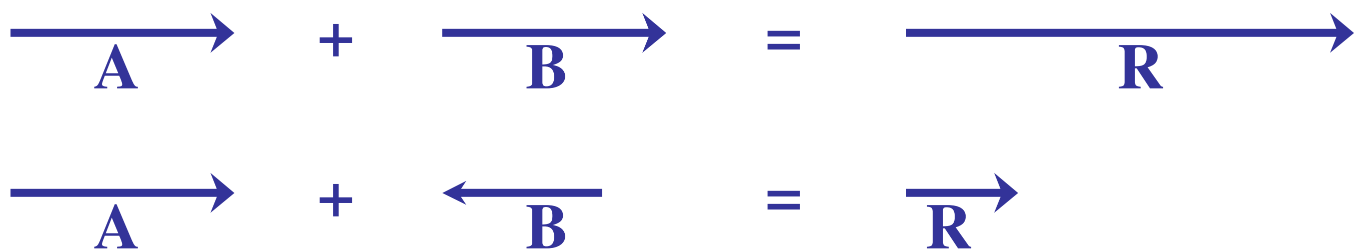


$$\left|\vec{B}\right| = 2\left|\vec{A}\right| = 4\text{m}$$

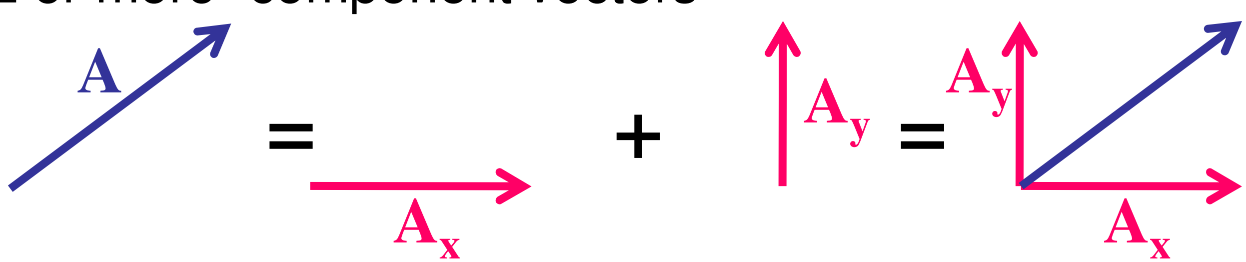
     2. The direction of the arrow represents the direction of the vector

Properties of Vectors

1. Only vectors of the same kind can be added together
2. 2 or more vectors can be added together to obtain a “resultant” vector
3. The “resultant” vector represents the combined effects of multiple vectors acting on the same object/system
   – Direction as well as magnitude must be taken into account when adding vectors
   – When vectors are co-linear they can be added like scalars



4. Any single vector can be treated as a “resultant” vector and represented as 2 or more “component vectors

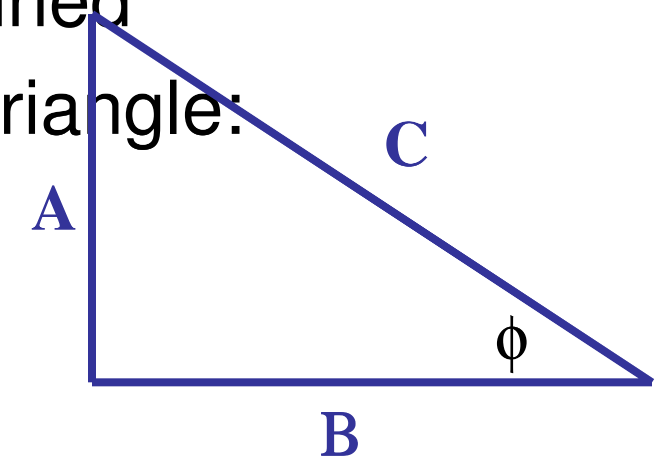


5. To add vectors of this type requires sophisticated mathematics or use of graphical techniques

Trigonometry Review

(remember: SOHCAHTOA)

1. The relationships between the sides and angles of right triangles are well defined
2. Consider the following right triangle:



Three primary "trig" relations (relative to ϕ):

- Sine of ϕ: $\sin \phi$ = opposite/hypotenuse = A/C
- Cosine of ϕ: $\cos \phi$ = adjacent/hypotenuse = B/C

Adding Vectors ($\perp$ or otherwise)

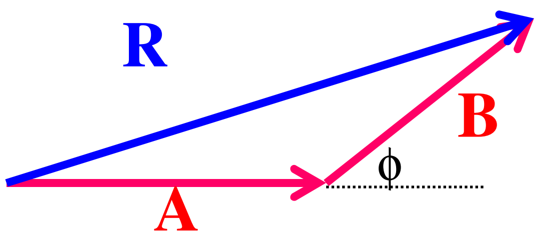
A. Graphic Method

To add 2 vectors, place them tail-to-head, without changing their direction; the sum (resultant) is the vector obtained by connecting the tail of the first vector with the head of the second vector

a. $\vec{R} = \vec{A} + \vec{B}$ means "the vector **R** is the sum of vectors **A** and **B**"

b. *Note:* $|\vec{R}| \neq |\vec{A}| + |\vec{B}|$, the magnitude of the vector **R** is NOT necessarily equal to the sum of the magnitudes of vectors **A** and **B**. In general:

$$|\vec{R}|^2 = |\vec{A}|^2 + |\vec{B}|^2 + 2|\vec{A}| \cdot |\vec{B}| \cos \phi$$



Other Notes:

1. For co-linear vectors pointing in the same direction, $|\vec{R}| = |\vec{A}| + |\vec{B}|$

2. For co-linear vectors pointing in opposite directions, $|\vec{R}| = \left||\vec{A}| - |\vec{B}|\right|$

Vector Addition (cont.)

B. Component Method

1. Express each vector as the sum of 2 "component" vectors. The direction of each component vector should be the same for both vectors. It is common to use the horizontal and vertical directions (These vectors are the horizontal and vertical *components* of the vector)

Example:

vector **A** $\longrightarrow$ $\mathbf{A_x}$ (horizontal) and $\mathbf{A_y}$ (vertical) or $\vec{A} = \vec{A}_x + \vec{A}_y = A_x\hat{i} + A_y\hat{j}$

vector **B** $\longrightarrow$ $\mathbf{B_x}$ (horizontal) and $\mathbf{B_y}$ (vertical) or $\vec{B} = \vec{B}_x + \vec{B}_y = B_x\hat{i} + B_y\hat{j}$

*Note: The unit vectors **i** and **j** indicate the directions of the vector components*

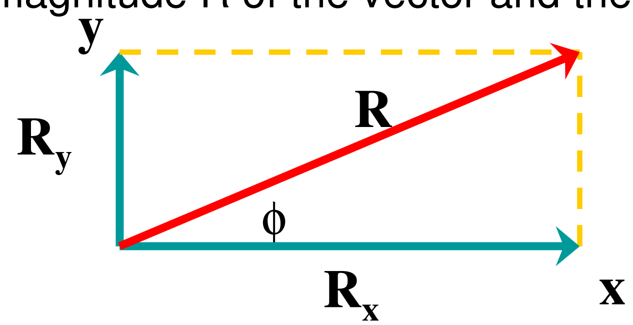
2. The magnitudes for corresponding *component* vectors for **A** & B can now be added together like scalars to obtain the component vectors for the resultant vector:

$R_x = A_x + B_x$ **and** $R_y = A_y + B_y$

And thus: $\vec{R} = \vec{R}_x + \vec{R}_y = R_x\hat{i} + R_y\hat{j}$

The magnitude of the resultant is then obtained from the component vectors by using the Pythagorean Theorem: $|\vec{R}| = |\vec{R}_x| + |\vec{R}_y| = \sqrt{R_x^2 + R_y^2}$

3. To calculate the components, we need to know the magnitude R of the vector and the angle a it makes with the horizontal direction:



$\cos\phi = R_x/R$, **since $R_x = R\cos\phi$**

$\sin\phi = R_y/R$, **since $R_y = R\sin\phi$**

The Scalar (Dot) Product

- Two vectors ($\vec{A}$ and $\vec{B}$) can be multiplied to product a scalar resultant, called the scalar (or Dot) product.
- When using the magnitudes of the vectors: $\vec{A} \bullet \vec{B} = \left|\vec{A}\right|\left|\vec{B}\right| \cos\phi$

  where ϕ is the angle between vectors **A** and **B**

- When using vector components: $\vec{A} \bullet \vec{B} = A_x B_x + A_y B_y$

- Useful properties of scalar products:

$$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$$
$$\vec{A} \cdot \vec{A} = A^2$$
$$\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$$

Example: The scalar product of the vectors of force and displacement is used to calculate work performed by the force

The Vector (Cross) Product

1. Two vectors ($\vec{A}$ and $\vec{B}$) can be multiplied to produce a vector resultant, called the vector (or cross) product.
2. When using the magnitudes of the vectors:

$$\vec{A}\times\vec{B}=\left|\vec{A}\right|\left|\vec{B}\right|\sin\phi$$

   where ϕ is the angle between vectors **A** and **B**
3. The direction of the vector product is perpendicular to the plane of the vectors **A** & **B**
4. When using vector components:

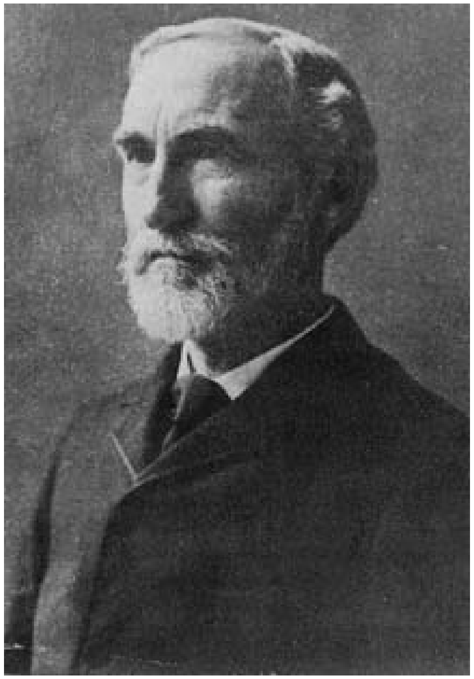
$$\vec{A}\times\vec{B}=\left(A_yB_z-A_zB_y\right)\hat{i}+\left(A_zB_x-A_xB_z\right)\hat{j}+\left(A_xB_y-A_yB_x\right)\hat{k}$$

Notes:

a. The presence of the vector product implies that 3 spatial dimensions are specified

b. The vector product is perpendicular to both ***A*** *and* ***B***

J. Willard Gibbs (1839-1903)



- Considered one of the greatest scientists of the 19th century
- Major contributions in the fields of:
  - Thermodynamics & Statistical mechanics
    - Formulated a concept of thermodynamic equilibrium of a system in terms of energy and entropy
  - Chemistry
    - Chemical equilibrium, and equilibria between phases *(I'm sure you've heard of the Gibb's Free Energy…)*
  - Mathematics
    - Developed the foundation of vector mathematics

Portland Community College

Physics Humor

1. What do you get when you cross an apple with a grape?

   Ans. $(\text{Apple})(\text{Grape})\sin\theta$

2. What do you get when you cross an apple with a alligator?

   Ans. Nothing, alligators are scalar…