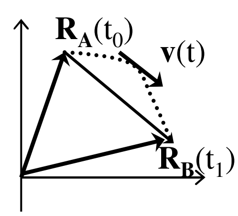
Phy 211: General Physics I

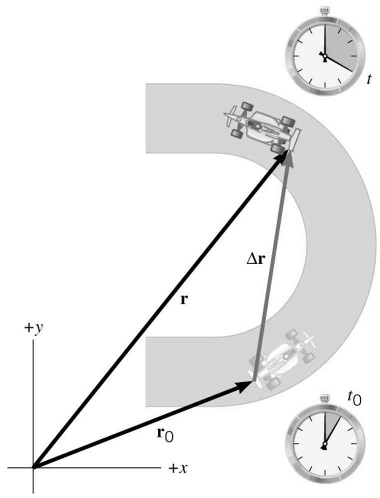
Chapter 4: Motion in 2 & 3 Dimensions
Lecture Notes

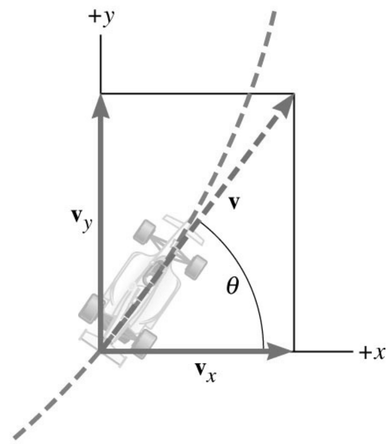
2-Dimensional Motion

- A motion that is not along a straight line is a two-dimensional motion.
- The position, displacement, velocity and acceleration vectors are necessarily not on the same directions; the rules for vector addition and subtraction apply
- The instantaneous velocity vector (**v**) is always tangent to the trajectory <u>but</u> the average velocity ($\mathbf{v}_{avg}$) is always in the direction of $\Delta\mathbf{R}$
- The instantaneous acceleration vector (**a**) is always tangent to the slope of the velocity at each instant in time <u>but</u> the average acceleration ($\mathbf{a}_{avg}$) is always in the direction of $\Delta\mathbf{v}$



Example of 2-D Motion





Displacement, Velocity & Accelerations

- When considering motion problems in 2-D, the definitions for the motion vectors described in Chapter 2 still apply.
- However, it is useful to break problems into two 1-D problems, usually
  - Horizontal (x)
  - Vertical (y)
- **Displacement:** $\Delta\vec{r}=\vec{r}-\vec{r}_o=(x-x_o)\hat{i}+(y-y_o)\hat{j}$
- **Velocity:** $\vec{v}=\frac{d\vec{r}}{dt}=\frac{d\vec{r}_x}{dt}\hat{i}+\frac{d\vec{r}_y}{dt}\hat{j}$
- **Acceleration:** $\vec{a}=\frac{d\vec{v}}{dt}=\frac{d\vec{v}_x}{dt}\hat{i}+\frac{d\vec{v}_y}{dt}\hat{j}$
- Use basic trigonometry relations to obtain vertical & horizontal components for all vectors

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Equations of Kinematics in 2-D

When $t_o = 0$ and **a** is constant:

Horizontal:

- $v_x - v_{ox} = a_x t$
- $x - x_o = \frac{1}{2}(v_{ox} + v_x)t$
- $x - x_o = v_{ox}t + \frac{1}{2} a_x t^2$
- $v_x^2 - v_{ox}^2 = 2a_x\Delta x$

Vertical:

- $v_y - v_{oy} = a_y t$
- $y - y_o = \frac{1}{2}(v_{oy} + v_y)t$
- $y - y_o = v_{oy}t + \frac{1}{2} a_y t^2$
- $v_y^2 - v_{oy}^2 = 2a_y\Delta y$

Remember: when $t_o \neq 0$, replace t with Δt in the above equations

Projectile Motion is the classic 2-D motion problem

- Vertical motion is treated the same as free-fall
  - $\mathbf{a_y} = -\mathbf{g} = -9.8$ m/s^2 {downward}
- Horizontal motion is independent of vertical motion but connected by time but no acceleration vector in horizontal direction
  - $\mathbf{a_x} = 0$ m/s^2
- As with Free Fall Motion, air resistance is neglected

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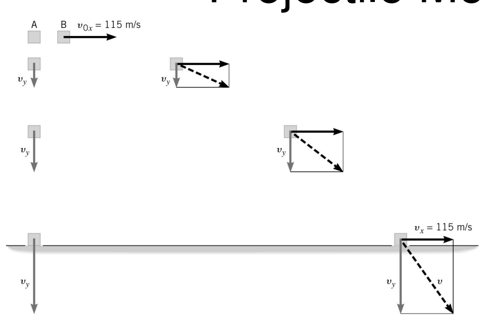
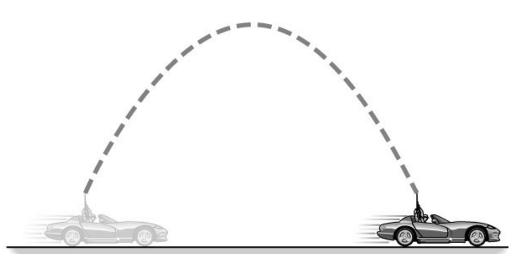
Projectile Motion Notes





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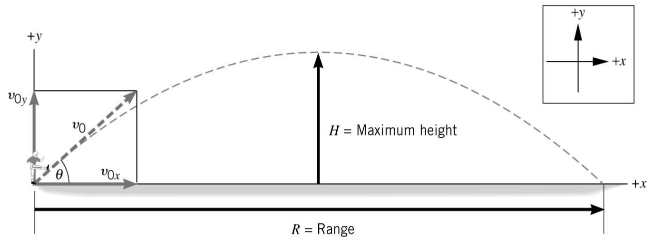
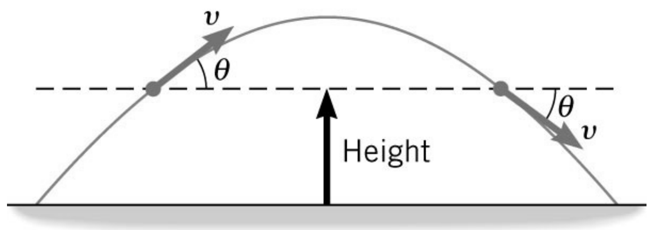
Projectile Motion Notes (cont.)





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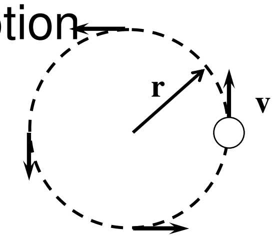
Uniform Circular Motion



- object travels in a circular path
- radius of motion (r) is constant
- speed (v) is constant
- velocity changes as object continually changes direction
- since **v** is changing (v is not but direction is), the object must be accelerated: $\vec{a}_c = \vec{a}_x + \vec{a}_y = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j}$
- the direction of the acceleration vector is always toward the center of the circular motion is referred to as the <u>centripetal acceleration</u>
- the magnitude of centripetal acceleration: $|\vec{a}_c| = a_c = \frac{v^2}{r}$

Derivation of Centrifugal Acceleration

Consider an object in uniform circular motion, where:

- r = constant {radius of travel}
- v = constant {speed of object}

The magnitude of the centripetal acceleration ($\vec{a}_c = \frac{d\vec{v}}{dt}$) can be obtained by applying the Pythagorean theorem:

$$|\vec{a}_c| = \sqrt{a_x^2 + a_y^2}$$

The components of the velocity vector continuous change as the objects travels around the circle and are related to the angle, θ:

$$|\vec{a}_c| = \frac{d}{dt}(-v\sin\theta)\hat{i} + \frac{d}{dt}(v\cos\theta)\hat{j} = \sqrt{\left[\frac{d}{dt}\left(-v\frac{y}{r}\right)\right]^2 + \left[\frac{d}{dt}\left(v\frac{x}{r}\right)\right]^2}$$

$$|\vec{a}_c| = \sqrt{\left[\left(-\frac{v}{r}\right)\frac{dy}{dt}\right]^2 + \left[\left(\frac{v}{r}\right)\frac{dx}{dt}\right]^2}$$

$$|\vec{a}_c| = \left(\frac{v}{r}\right)\sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}$$

$$|\vec{a}_c| = \left(\frac{v}{r}\right)\sqrt{v_y^2 + v_x^2} = \frac{v^2}{r}$$