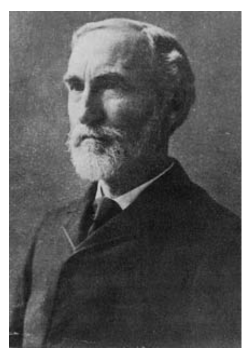
Phy 211: General Physics I

Chapter 3: Vectors

Lecture Notes

J. Willard Gibbs (1839-1903)



- Considered one of the greatest scientists of the 19th century
- Major contributions in the fields of:
  - Thermodynamics & Statistical mechanics
    - Formulated a concept of thermodynamic equilibrium of a system in terms of energy and entropy
  - Chemistry
    - Chemical equilibrium, and equilibria between phases *(I'm sure you've heard of the Gibb's Free Energy…)*
  - Mathematics
    - Developed the foundation of vector mathematics

Vectors & Scalars

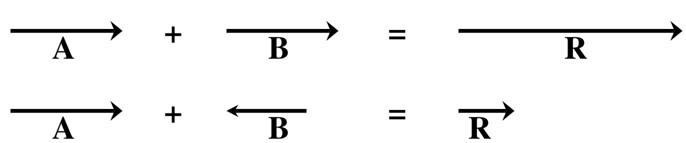
- Most physical quantities can categorized as one of 2 types (tensors notwithstanding):
- **Scalars:**
  - described by a single number & a unit (s).

  Example: the length of the driveway is 3.5 m
- **Vectors:**
  - described by a value (magnitude) & direction.

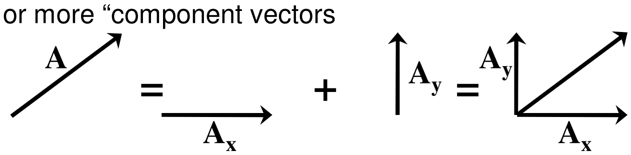
  Example: the wind is blowing 20 m/s due north
  - Vectors are represented by an arrow:
    - the length of the arrow is proportional to the magnitude of the vector.
    - The direction of the arrow represents the direction of the vector

Properties of Vectors

- Only vectors of the same kind can be added together
- 2 or more vectors can be added together to obtain a "resultant" vector
- The "resultant" vector represents the combined effects of multiple vectors acting on the same object/system
  - Direction as well as magnitude must be taken into account when adding vectors
  - When vectors are co-linear they can be added like scalars



- Any single vector can be treated as a "resultant" vector and represented as 2 or more "component vectors

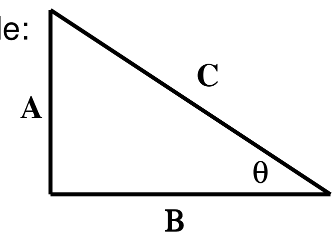


- To add vectors of this type requires sophisticated mathematics or use of graphical techniques

Trigonometry Review

(remember: SOHCAHTOA)

- The relationships between the sides and angles of right triangles are well defined
- Consider the following right triangle:



Three primary "trig" relations:

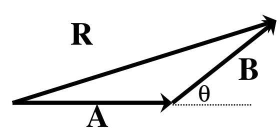
- Sine: $\sin \theta$ = opposite/hypotenuse = A/C
- Cosine: $\cos \theta$ = adjacent/hypotenuse = B/C
- Tangent: $\tan \theta$ = opposite/adjacent = A/B

Vector Addition

A. Graphic Method

- To add 2 vectors, place them tail-to-head, without changing their direction; the sum (resultant) is the vector obtained by connecting the tail of the first vector with the head of the second vector
- **R**=**A**+**B** means "the vector **R** is the sum of vectors **A** and **B**"
- $R \neq A+B$: the magnitude of the vector **R** is NOT equal to the sum of the magnitudes of vectors **A** and **B**

$$R^2 = A^2 + B^2 + 2AB\cos\theta$$



- Note:
  - For co-linear vectors pointing in the same direction, **R** = **A** + **B**
  - For co-linear vectors pointing in opposite directions, **R** = **A** - **B**

Vector Addition (cont.)

B. Component Method

- Express each vector as the sum of 2 perpendicular vectors. The direction of each component vector should be the same for both vectors. It is common to use the horizontal and vertical directions (These vectors are the horizontal and vertical *components* of the vector)

Example:

vector **A** ⟶ $\mathbf{A_x}$ (horizontal) and $\mathbf{A_y}$ (vertical) or $\mathbf{A} = \mathbf{A_x} + \mathbf{A_y} = A_x\mathbf{i} + A_y\mathbf{j}$

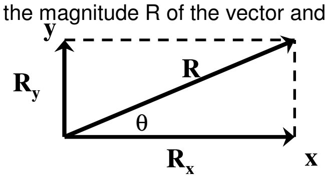
vector **B** ⟶ $\mathbf{B_x}$ (horizontal) and $\mathbf{B_y}$ (vertical) or $\mathbf{B} = \mathbf{B_x} + \mathbf{B_y} = B_x\mathbf{i} + B_y\mathbf{j}$

- *Note: The unit vectors **i** and **j** indicate the directions of the vector components*
- The common *component* vectors for A & B can now be added together like scalars to obtain the component vectors for the resultant vector:
  $R_x = A_x + B_x$ **and** $R_y = A_y + B_y$

  And thus: $\mathbf{R} = \mathbf{R_x} + \mathbf{R_y} = R_x\mathbf{i} + R_y\mathbf{j}$
- The magnitude of the resultant is then obtained from the component vectors by using the Pythagorean Theorem: $\mathbf{R}^2 = \mathbf{R_x}^2 + \mathbf{R_y}^2$
- To calculate the components, we need to know the magnitude R of the vector and the angle a it makes with the horizontal direction:

cos θ=R_x/R, since R_x=Rcos θ

sin θ=R_y/R, since R_y=Rsin θ



The Scalar (Dot) Product

- Two vectors ($\vec{A}$ and $\vec{B}$) can be multiplied to product a scalar resultant, called the scalar (or Dot) product.
- When using the magnitudes of the vectors: $\vec{A} \bullet \vec{B} = |\vec{A}||\vec{B}|\cos\phi$

  where ϕ is the angle between vectors **A** and **B**
- When using vector components: $\vec{A} \bullet \vec{B} = A_x B_x + A_y B_y$
- Useful properties of scalar products:

$$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$$
$$\vec{A} \cdot \vec{A} = A^2$$
$$\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$$

Example: The scalar product of the vectors of force and displacement is used to calculate work performed by the force

The Vector (Cross) Product

- Two vectors($\vec{A}$ and $\vec{B}$) can be multiplied to produce a vector resultant, called the vector (or cross) product.
- When using the magnitudes of the vectors:

$$\vec{A}\times\vec{B}=|\vec{A}||\vec{B}|\sin\theta$$

  where q is the angle between vectors **A** and **B**
- The direction of the vector product is perpendicular to the plane of the vectors **A** & **B**
- When using vector components:

$$\vec{A}\times\vec{B}=\left(A_yB_z-A_zB_y\right)\hat{i}+\left(A_zB_x-A_xB_z\right)\hat{j}+\left(A_xB_y-A_yB_x\right)\hat{k}$$

Notes

- *The presence of the vector product implies that 3 spatial dimensions are specified*
- *The vector product is perpendicular to both* **A** *and* **B**