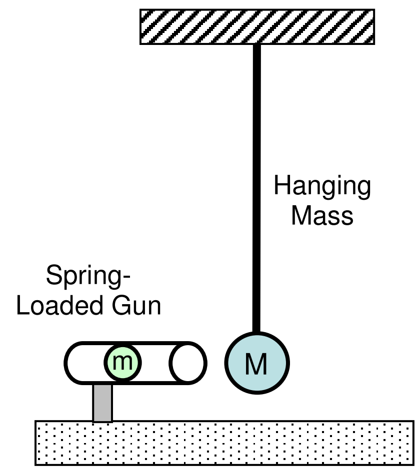
Experiment: The Ballistic Pendulum

Objectives:
- Apply the Law of Conservation of Momentum to an inelastic collision
- Apply the Law of Conservation of Mechanical Energy to a simple pendulum
- Determine then verify the muzzle velocity of a launched projectile

Equipment:
- Ballistic Pendulum apparatus
- Meter stick
- Digital gram scale
- Carbon paper

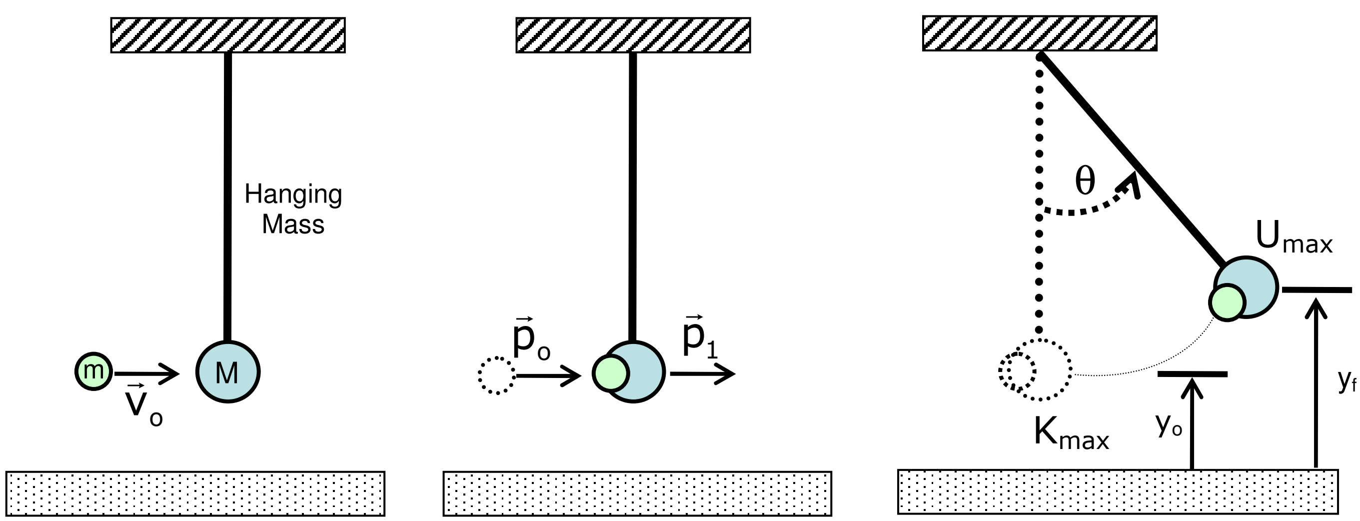
Figure 1: The Ballistic Pendulum



Introduction:
The ballistic pendulum is a device commonly used to measure the muzzle velocity of firearms. When the gun is fired, the propelled projectile impacts the hanging mass (a wood block or trap) of a simple pendulum. The momentum of the projectile just prior to impact, according to the Law of Conservation of Linear Momentum, is equal to the post collision momentum of the ball and pendulum combined (inelastic collision). Moreover, the collision results in the rotation of the pendulum. The height of the pendulum swing is related to the initial velocity of the pendulum at the lowest position of the swing, according to the Law of Conservation of Mechanical Energy. The main phases of the ballistic pendulum operation are shown below in Figure 2.

In this experiment, you will utilize the ballistic pendulum to determine the muzzle velocity of a launched projectile all the while applying the concepts associated with the conservation laws for linear momentum and mechanical energy.

Figure 2: Firing the Ballistic Pendulum



1. Firing the gun, the projectile (m) impacts the pendulum trap (M)

2. The projectile collision with the pendulum is inelastic and momentum is conserved

3. The projectile/pendulum rises as KE is converted to PE and mechanical energy is conserved

Preliminary Questions:

1. Consider a projectile (0.1 kg) fired into a ballistic pendulum (0.5 kg, r_{cm} = 0.3 m), the resulting collision is inelastic (see Figure 2) and the pendulum swings to a maximum angular deflection of 20^o. Assume r_{cm} is the same with or without the projectile attached. What is the maximum PE for the pendulum?

2. Determine the maximum KE for the pendulum and use this value to calculate the initial velocity of the projectile-pendulum just after the collision.

3. Apply the Law of Conservation Momentum to calculate the initial momentum of the projectile.

4. Determine the velocity of the projectile.

Part 1 Ballistic Pendulum: (Procedure)

1. Measure the mass of the metal ball. Record in data Table 1.
2. Measure the mass of the pendulum. Record in the data Table 1.
3. Measure the length of the pendulum. Record in the data Table 1.
4. Cock the trigger of the spring loaded gun and load the metal ball.
5. Fire the projectile into the ballistic pendulum and record the deflection angle of the pendulum.
6. Repeat steps 4 and 5 for a total of 5 trials.
7. Using the angle measurements and the pendulum length (r), determine the vertical displacement (Δy) values for each trial. Record in Table 1.

Table 1: Ballistic Pendulum (Raw Data)			
Mass of projectile:		**Mass of pendulum:**	
Pendulum Length (r):			
Trial	**Angle (θ)**	**Δy**	

Part 1 Ballistic Pendulum: (Analysis)

1. Using the data Table 1, determine the maximum gravitational potential energy for the pendulum in each trial. Record values in Table 2.
2. Using the gravitational potential energy values Table 2, determine the maximum kinetic energy for each trial.
3. Using the kinetic energy values, calculate the velocity (v_1) of the pendulum/projectile system corresponding to the bottom of the swing, for each trial. Record values in Table 2.
4. For each trial, calculate the momentum (p_1) of the pendulum-projectile system. This is the system momentum following the projectile collision with the pendulum. Record values in Table 2.
5. Apply the Law of Conservation of Linear Momentum to projectile-pendulum collision determine the pre-collision momentum of the fired projectile, for each trial. Record values in Table 2.
6. Determine the initial projectile velocity (v_o) for each trial then calculate the average velocity. Record in Table 2.
7. Estimate the uncertainty ($\frac{\pm \text{Range}}{2}$) for the v_o values, using the min-max method.

Table 2: Ballistic Pendulum (Analysis)						
			After Collision (pendulum & projectile)		**Before Collision** (projectile)	
Trial	**U_{max}**	**K_{max}**	**v_1**	**p_1**	**p_o**	**v_o**
1						
2						
3						
4						
5						
				Average v_o:		
				Uncertainty ($\pm \delta v_o$):		

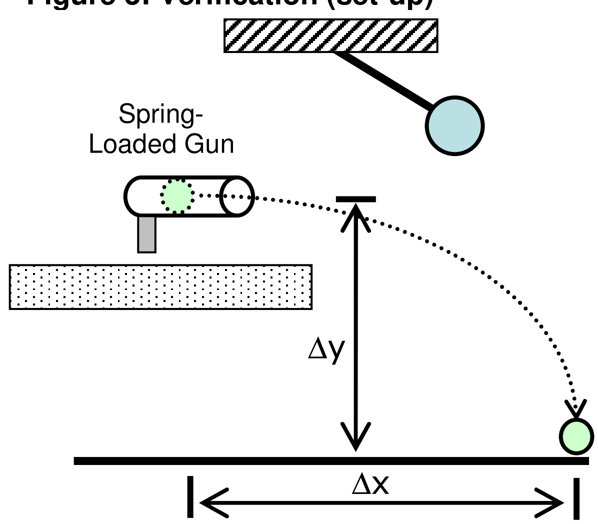
Part 2 Verification: (Procedure)

1. Depending on the Ballistic pendulum model you are using, either: (i) remove the gun assembly from the pendulum; or (ii) position the pendulum from the gun assembly line of fire.
2. Position the gun so that when fired the projectile will land safely on the floor.
3. Measure the vertical distance (Δy) of the projectile from the floor. Record value in Table 3.
4. Fire a practice shot. Locate the landing point on the floor then secure a piece of white paper to the floor (centered about the landing point). Cover the sheet with a piece of carbon paper.
5. Fire the projectile at the carbon paper landing point. Repeat for a total of 5 trials.
6. Measure the horizontal displacement (Δx) for each trial and record values in Table 3.
7. For each trial, calculate the initial velocity (v_o) of the projectile using the Δx and Δy values. Do you remember how these variables are related to the initial velocity?
8. Estimate the uncertainty ($\frac{\pm \text{Range}}{2}$) for the v_o values. Use the min-max method.

Table 3: Verification

Trial #	**Δy**	**Δx**	**v_o**
	Average v_o:		
	Uncertainty ($\pm \delta v_o$):		

Figure 3: Verification (set-up)



Questions:

1. Why is mechanical energy not conserved in the collision of the projectile with the pendulum?

2. What kind of energy does the spring have when it is compressed? Where did it get it from?

3. What extra information about the setup would you need to calculate the work done to compress the spring in the gun?

4. Apply the Law of Conservation of Linear Momentum to the inelastic collision (step 2 in Figure 2) to derive a relation between v_o for the projectile and v_1 for the projectile-pendulum system (including the corresponding masses).

5. In step 3 of Figure 2, the projectile/pendulum system swings to a maximum height. Apply the Law of Conservation of Mechanical Energy to derive a relationship between the final (maximum) angle of the pendulum rotation and the initial velocity (v_1) of the pendulum.

6. From your results in 4 and 5 (above), obtain a relationship between v_o for the projectile and the maximum angle of the pendulum deflection.

Sadistic Extension:

7. Derive an equation for the uncertainty of the projectile velocity (δv_o) associated with the equation obtained in question 6.

8. Using your measurement data for the ballistic pendulum, calculate the uncertainty (δv_o) for v_o. How does this value compare to uncertainty value determined in Table 2?