Experiment: Ideal Gas Laws
(Archimedes Revisited)

Archimedes' Principle can be applied to an object immersed into a gas, such as a helium-filled balloon in the atmosphere. During this lab, we will use a video analysis method to determine the acceleration of an ascending balloon. From this information, we will calculate the mass of the gas inside the balloon. By measuring the pressure of the gas inside the balloon, the number of moles will be determined (ideal gas law). We will use the mass of the gas and the number of moles to estimate the effective molar mass of the gas mixture in the balloon.

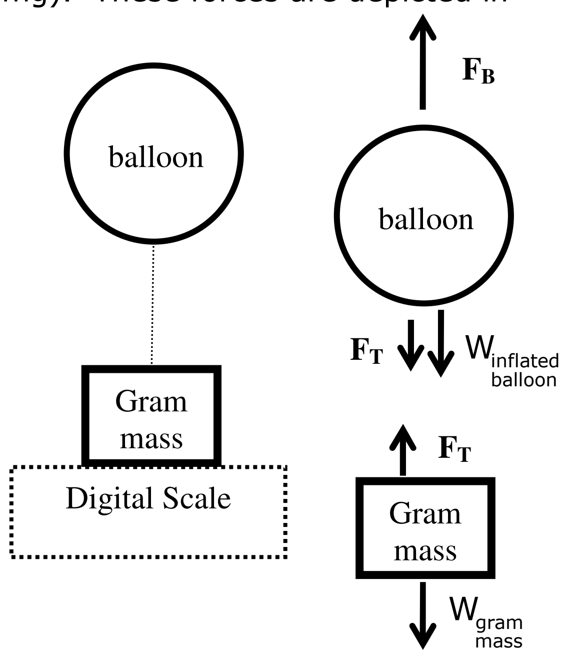
Objectives:

a. to apply the Ideal gas Law and Archimedes' Principle to an inflated balloon and determine the number of moles of gas in a balloon
b. to determine the density of the air
c. to calculate the effective molar mass for a mixture of gases (air and helium) and estimate the purity of the helium

Theory:
In a helium-filled balloon (see the figure), the number of moles of the enclosed gas can be determined from the pressure, volume and temperature of the gas:

(1) $$PV=nRT$$

Since the density of a helium filled balloon is less than the surrounding air, it is subject to an upward buoyant force (F_B) that is greater than its weight (mg). These forces are depicted in the above figure.



Attaching a small gram mass to the balloon and placing the mass on a digital gram scale, the scale reading will measure the difference between combined mass of the and the upward tension force due to the suspended balloon. Applying Newton's 2nd Law to the gram mass and the balloon respectively, normal force exerted on the gram mass (**F_N**), i.e. the scale reading, is obtained:

(2) $$F_{net} = F_B - W_{\substack{inflated\\balloon}} - F_T = 0 \qquad \text{(on balloon)}$$

Or $$F_T = F_B - W_{\substack{inflated\\balloon}}$$

(3) $$F_{net} = F_N - W_{\substack{gram\\mass}} + F_T = 0 \qquad \text{(on gram mass)}$$

Or $$F_N = W_{\substack{gram\\mass}} - F_T = W_{\substack{gram\\mass}} - F_B + W_{\substack{inflated\\balloon}}$$

Therefore: $$m_{\substack{inflated\\balloon}} = m_{\substack{deflated\\balloon}} + m_{gas} = \frac{F_N - W_{\substack{gram\\mass}} + F_B}{g} \rightarrow m_{gas} = \frac{F_N - W_{\substack{gram\\mass}} + F_B}{g} - m_{\substack{deflated\\balloon}}$$

The buoyant force, $F_B = \rho_{air} V g$, is described in accordance to Archimedes' Principle.

Notes:

V is the volume of the balloon

ρ_{air} is the air density outside the balloon

g is 9.8 m/s^2

Preliminary Questions

1. Find the density of the air on a sunny day (T = 27°C and P = 101,325 Pa). Note: The molar mass for air is 28.89 g/mol (or 28.89 kg/kmol). *Hint: Use Ideal Gas Law…*

2. A balloon filled with Helium has a volume V=10^{-2} m^3. Find the buoyant force acting on the balloon on the sunny day described by Question 1.

3. Assume that the total mass of the inflated balloon is 8.0 g. Draw a free-body diagram and apply Newton's Second law to find the tension force measured by the force sensor.

Part 1: Initial Measurements

1. The balloon is not a perfect sphere. Measure the larger and the smaller circumference then average the results. Enter the average circumference in the table below.
2. From this information, find the average radius of the balloon and the volume of the balloon. Enter your result in the table.

Table 1: Preliminary Measurements

Average Circumference	Effective Radius	Volume	Pressure	T_{room} (°C)	T_{room} (K)	# moles

3. Measure the pressure of the gas inside the balloon with a pressure sensor. Enter the pressure in the above table.
4. Measure the room temperature and record the value.
5. From this information and using the volume of the balloon calculated above, find the number of moles of gas in the balloon.
6. The molar mass of the air is approximately 28.89 g/mol. Using the ideal gas equation, calculate the density of the air. Enter your result here:

   Density of air = ρ_{air} = _________________

7. Calculate the buoyant force acting on the balloon, using the volume of the balloon calculated in step 1 and the density of air (refer to equation 3):

   Buoyant force = F_B = _________________

Part 2: Gram Scale

In order to find the mass of the helium in the balloon, you will have to measure the normal force exerted on the digital gram scale associated with gram mass and the suspended balloon.

8. Zero the digital scale and verify that it is connected.
9. Place a 50 gram mass on the scale then record the value in Table 2.
10. Attach the inflated balloon to the 50 gram mass and place it back on the scale. Attach the inflated balloon a 50 gram mass and place it on the scale then collect several seconds of the scale reading. Use the statistics function to calculate the average value and uncertainty of the force reading. Record the value in Table 2.
11. Using equation (2) above, determine the mass of the inflated balloon (which will include the mass of the gas inside) and the uncertainty. Record in Table 2.
12. Deflate the balloon and measure its deflated mass with a digital scale. Using the result obtained in step 12, find the mass of the gas inside the inflated balloon. Record in Table 2.
13. Using the number of moles of gas and the mass of the gas, estimate the molar mass (μ_{gas}) of the gas (w/ uncertainty). Record in Table 2.

Table 2: Digital Scale Mass Measurements

$m_{gram\ mass}$ **(in kg)**	$m_{gram\ mass} + m_{inflated\ balloon}$ **(in kg)**	$m_{deflated\ balloon}$ **(in kg)**			
$W_{gram\ mass}$ **(in N)**	F_N **(in N)**	F_B **(in N) {From prev page}**	m_{gas} **(in kg)**	**Molar mass (μ_{gas})**	**Mole Fraction (f_{He})**

One Last Thing:

For a mixture of 2 gases, say He and air, the effective molar mass would be given by:

(4) $$\mu_{gas} = \frac{m_{gas}}{n} = \frac{m_{He} + m_{air}}{n} = \frac{\mu_{He} n_{He} + \mu_{air} n_{air}}{n} = \mu_{He}\frac{n_{He}}{n} + \mu_{air}\frac{n_{air}}{n}$$

where:

n is the total molar quantity of the combined gases (i.e. $n = n_{He} + n_{air}$)
μ_{He} and μ_{air} are the molar masses of the 2 gases in the mixture, respectively
n_{He} and n_{air} are the molar quantities for each gas

The molar fraction of the first gas (f_1) is given by $f_{He} = \frac{n_{He}}{n}$, therefore the molar fraction for the second gas is $f_{air} = \frac{n_{air}}{n} = 1 - f_{He}$. Combining these 2 equations, leads to an expression for f_{He} in terms of the respective molar masses:

(5) $$f_{He} = \frac{\mu_{gas} - \mu_{air}}{\mu_{He} - \mu_{air}}$$

Estimate the molar fraction of the helium in this gas (i.e. the purity of the helium), assuming that the gas in the inflated the balloon contains only Helium and air.

Notes:

Molar mass for Helium = μ_{He} = 4.003 kg/kmol (or g/mol)
Molar mass for air = μ_{air} = 28.89 kg/kmol (or g/mol)

Molar mass of mixture = μ_{gas} = ____________ (from above)

Mole Fraction of He = f_{He} =___________________ *(The purity of the He gas)*

Analysis Questions:

1. Compare the density of the air calculated in step 6 above (using the Ideal Gas Law) with the accepted value in the textbook. Based on these values, is air an ideal gas?

2. According to the manufacturer, the purity of the Helium inside the balloon is 99%. Compare the labeled purity with the molar fraction calculated above. Find the % Error.