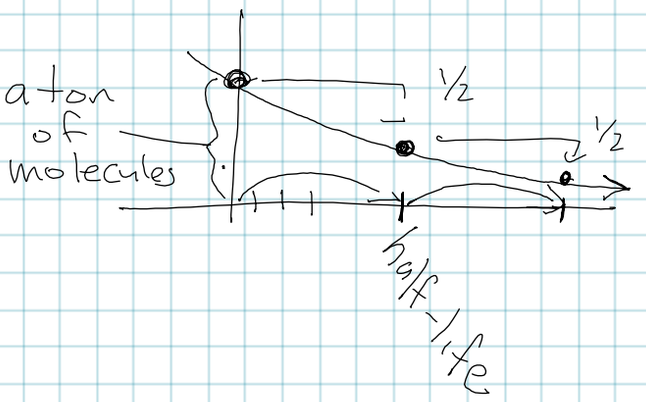
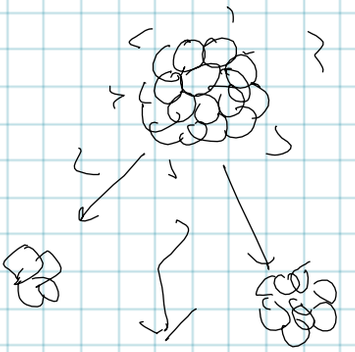
4.8 Common Applications

- Population Growth
- Radioactive Decay





the growth factor b is less than $\frac{1}{6}$!

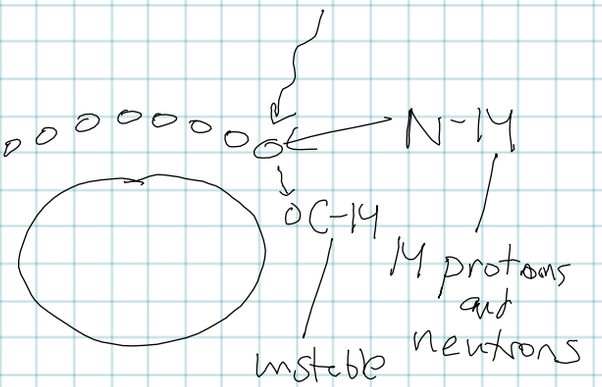
Ex Carbon-14

in the atmosphere

plants take it in

animals eat plants

every living thing has C-14.



As long as these things live, keep replenishing C-14.

Eventually things die

Ex A piece of cloth freshly made should have 8000 atoms of C-14 in it.

We find a piece of cloth in an ancient tomb. It has 6000 atoms of C-14 in it...

How old is the dig site?

For C-14, half-life is 5730 years.

$A(t) = a \cdot b^t$

goal: solve for this (t)

$6000 = 8000 \cdot b^t$

find b first.

$A(t) = a \cdot b^t$

$A(t) = 8000 \cdot b^t$

$4000 = 8000 \cdot b^{5730}$

$\frac{1}{2} = b^{5730}$

$\sqrt[5730]{\frac{1}{2}} = b$

$0.9998790 39 \approx b$

$6000 = 8000(0.9998790 39)^t$

solve for t!

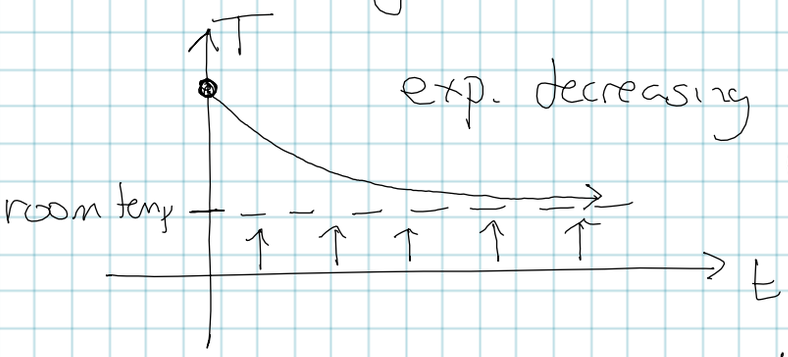
calculator

$t \approx 2738$

Roughly, tomb is 2738 yrs old.

Ex Newton's Law of Cooling

A hot object in a cool room



exp. decreasing function! (shifted up or down)

$$T(t) = a \cdot b^t + C$$

C ↑ room temp!

Newton's Law of Cooling $\longrightarrow$ $T(t) = a \cdot b^t + T_A$

T_A ↑ Ambient temp

Initial temp: T_0

Initial temp: $T(0) = a \cdot b^0 + T_A$

isolate a

$$T_0 = a + T_A$$

$$T_0 - T_A = a$$

Newton's Law of Cooling: $T(t) = (T_0 - T_A) \cdot b^t + T_A$

Ex A pie comes out at 375°F

The room is 68°F.

After 30 min, the pie is 120°F

- Find a model for T(t)
- How hot is it after only 5 min?
- How long until it's 100°F?

- $T(t) = (T_0 - T_A) \cdot b^t + T_A$

$T(t) = (375 - 68) \cdot b^t + 68$

$T(t) = 307 \cdot b^t + 68$

↑ need to figure out b.

$120 = T(30) = 307 \cdot b^{30} + 68$

$52 = 307 \cdot b^{30}$

$\frac{52}{307} = b^{30}$

$\sqrt[30]{\frac{52}{307}} = b$

$b \approx .94253$

So $T(t) = 307(.94253)^t + 68$

- $T(5) = 307(.94253)^5 + 68$

  $\approx 296\ ^\circ F$

- $100 = 307(.94253)^t + 68$

  $32 = 307(.94253)^t$

  ↓ calculator to solve

  $\longrightarrow \quad t \approx 38.2$ After 38.2 minutes, cooled to 100 °F.