

Interest

We invest \$ P into an account nominally earning r ~~%~~ per year.
(as a %)

End of year 1

$$A = \overset{\substack{\text{we started} \\ \text{with}}}{P} + \overset{\substack{\text{interest} \\ \text{earned}}}{r \cdot P} \\ = P(1+r)$$

End of year 2

$$A = \overset{\substack{\text{had this} \\ \text{more interest}}}{P(1+r)} + \overset{\substack{\text{more interest}}}{P(1+r) \cdot r} \\ = P(1+r)(1+r) \\ = P(1+r)^2$$

End of year t :

$$A = P(1+r)^t$$

initial investment nominal interest rate measured in years.

Ex You invest \$5000 at 3% nominal interest, what is the balance after 4 years?

$$A(t) = 5000(1+0.03)^t$$

$$A(4) = 5000(1.03)^4 \approx \$5627.54.$$

Most banks compound interest monthly.

Advertise an interest rate (like 12%).

↓
nominal interest rate
"in name only"
applies for a whole year.

For monthly compounding: $\frac{12\%}{12}$ ← divide the nominal rate by n ($n=12$)
 $= 1\%$

↗
So each month you get 1% interest rate.

So... we invest \$P. After t years.

$P \left(1 + \frac{r}{n}\right)^{n \cdot t}$ ← Formula for compound interest.

Ex $5000 \left(1 + \frac{.12}{12}\right)^{12 \cdot t}$
 $= 5000 (1.01)^{12t}$
 $= 5000 (1.126825\dots)^t$

t years
→ 12t months
→ $\left(1 + \frac{.12}{12}\right)$ applied 12t times.
over the course of t years.

↓
effective annual interest rate is about 12.7%

Ex Invest \$7000 at a 3% nominal rate

(a) compound yearly
for 4 years.

$$7000(1.03)^4 \\ \approx \$7878.56$$

(b) compound monthly

$$7000\left(1 + \frac{0.03}{12}\right)^{12 \cdot 4} \\ \approx \$7891.30$$

(c) weekly

$$7000\left(1 + \frac{0.03}{52}\right)^{52 \cdot t} \\ \approx 7892.20$$

(d) daily

$$7000\left(1 + \frac{0.03}{365}\right)^{365 \cdot t} \\ \approx 7892.44$$

Effective Rate: $\longrightarrow$

Scenario: nominal rate r

Compounding

Frequency n

$$\left(1 + \frac{r}{n}\right)^n - 1$$

1-year growth
factor

effective growth rate.

Actual rate after one year.
Actual growth factor for
one year $\longrightarrow$

Growth factor for one
compounding cycle, and
raise to the n^{th} power.