

4.4 Logarithms

Ex Solve $2^x = 8$

$x = 3$ just by knowing
arithmetic

Ex Solve $2^x = 10$

not using graphing calculator

between
3 and 4

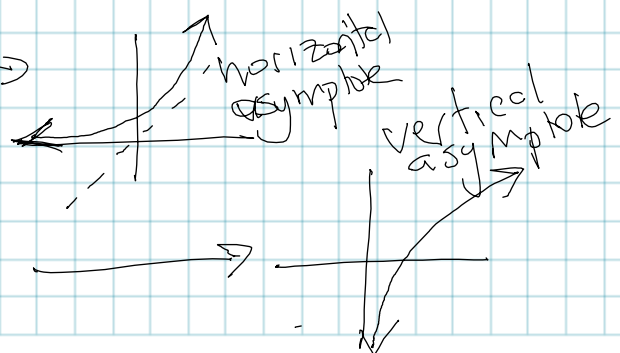
$$\begin{array}{l} 2^3 = 8 \\ 2^x = 10 \\ 2^4 = 16 \end{array}$$

Logarithms will help us solve
for x exactly.

We're trying to solve $f(x) = 10$

$$\Rightarrow x = f^{-1}(10)$$

With $f(x) = 2^x$ →



Define $f^{-1}(x) = \log_2(x)$ →

So Solve $2^x = 10$

$$f(x) = 10$$

$$x = f^{-1}(10)$$

$$x = \log_2(10)$$

calculator solved ✓
done ✓

$$x \approx 3.32 \dots$$

Ex

Solve $4^x = 7$

x is between 1 & 2.

$$x = \log_4(7)$$

"log base 4"

Fact

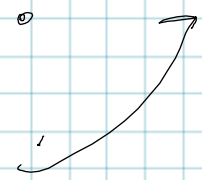
If you see $\log_b(x) = ?$

It is asking a question

"What must I raise b to,
to get x ?"

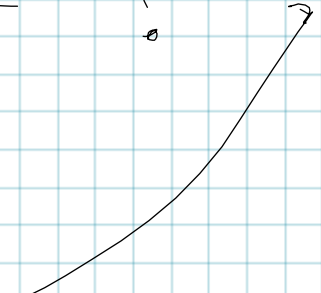
$$b^{(?) } = x$$

Ex $\log_2(16) = ?$ $\log_2(16) = 4$

$2^? = 16$ 4 

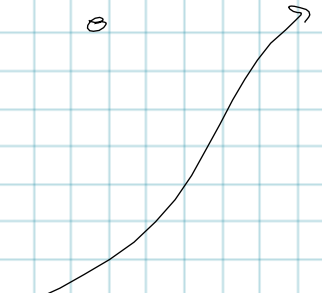
Ex $\log_3(81) = ?$ $\log_3(81) = 4$

$3^? = 81$

$3^4 = 81$ 

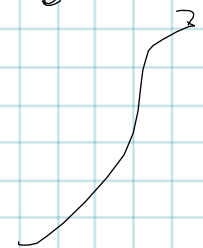
Ex $\log_7(1) = ?$ $\log_7(1) = 0$

$7^? = 1$

$7^0 = 1$ 

Ex $\log_2\left(\frac{1}{2}\right) = ?$ $\log_2\left(\frac{1}{2}\right) = -1$

$2^? = \frac{1}{2}$

$2^{-1} = \frac{1}{2}$ 

Ex $\log_5(\sqrt{5}) = ?$ $\log_5(\sqrt{5}) = \frac{1}{2}$

$5^? = \sqrt{5}$

$5^{1/2} = \sqrt{5}$

Ex $\log(0.001) = ?$ $\log(0.001) = -3$

$\log = \log_{10}$

$10^? = 0.001$

$10^{-3} = 0.001$

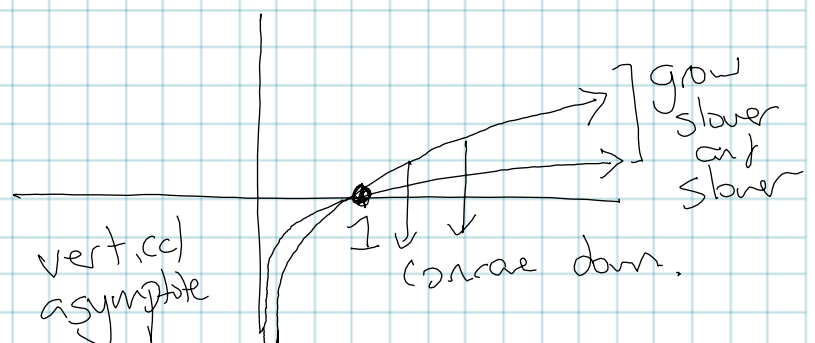
There are lots of log functions...

$f(x) = \log_2(x)$

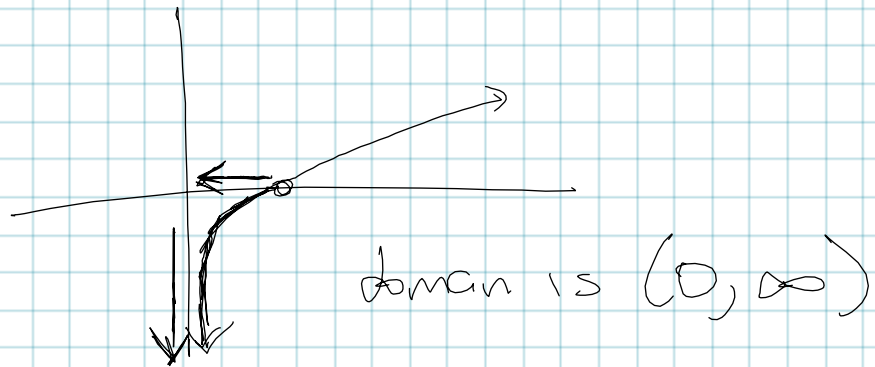
$g(x) = \log(x)$

$h(x) = \log_{1/2}(x)$

All have in common
 $\log_b(1) = 0$



As $x \rightarrow \infty$, $\log(x) \rightarrow \infty$



As $x \rightarrow 0$, $\log(x) \rightarrow -\infty$

Exponent Rules

$$a^m \cdot a^n = a^{m+n}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$(a^m)^n = a^{m \cdot n}$$

$$a^{-1} = \frac{1}{a}$$

Logarithm Rules

$$\bullet \log_b(b^x) = x$$

$$\bullet b^{\log_b(x)} = x$$

$$\bullet \log_b(s \cdot t) = \log_b(s) + \log_b(t)$$

$$\bullet \log_b\left(\frac{s}{t}\right) = \log_b(s) - \log_b(t)$$

$$\bullet \log_b(s^t) = t \cdot \log_b(s)$$

$$\bullet \log_b\left(\frac{1}{s}\right) = -\log_b(s)$$

• Change of base formula

$$\log_b(s) = \frac{\log_c(s)}{\log_c(b)}$$

Ex

Simplify

$$\log(x^5 y)$$

Went things
inside $\log()$
to be as simple
possible

$$\begin{aligned} &= \log((x^5) \cdot y) = \log(x^5) + \log(y) \\ &= 5 \cdot \log(x) + \log(y) \end{aligned}$$

Ex Simplify $\log_2 \left(\sqrt[5]{2^{9x+180}} \right)$

$$= \log_2 \left(\left(2^{9x+180} \right)^{1/5} \right) \quad \left[\log(s^t) \right]$$

$$= \frac{1}{5} \cdot \log_2 \left(2^{9x+180} \right)$$

$$= \frac{1}{5} (9x+180)$$

Ex Simplify $\log_3 \left(\frac{x^3}{x-3} \right)$ $\left[\log\left(\frac{s}{t}\right) \right]$

$$= \log_3(x^3) - \log_3(x-3)$$

$$= 3 \cdot \log_3(x) - \log_3(x-3)$$

Ex Solve for x $84 \cdot 5^x = 321$ note x is
up in exponent
↓
log!

Apply $\log$: $\log \left(\frac{84}{s} \cdot \frac{5^x}{t} \right) = \log(321)$

$$\log(84) + \log(5^x) = \log(321)$$

$$\log(84) + x \cdot \log(5) = \log(321)$$

$$x \cdot \log(5) = \log(321) - \log(84)$$

$$x = \frac{\log(321) - \log(84)}{\log(5)} \approx 0.833 \dots$$

Ex Solve for t : $3 \cdot 8^t = 2 \cdot 10^t$ use log!

$$\log(3 \cdot 8^t) = \log(2 \cdot 10^t)$$

$$\log(3) + \log(8^t) = \log(2) + \log(10^t)$$

$$\log(3) + \boxed{t} \cdot \log(8) = \log(2) + \boxed{t} \cdot \log(10) \quad \xrightarrow{1}$$

$$\boxed{t} \cdot \log(8) - \boxed{t} = \log(2) - \log(3)$$

$$t(\log(8) - 1) = \log(2) - \log(3)$$

$$t = \frac{\log(2) - \log(3)}{\log(8) - 1} \approx 1.817...$$

Ex Iodine-131 is radioactive
 has a ~~half-life~~ relative decay rate of
 8.28% per day. $(1 - 0.0828)$

If we had 10mg, how long until we have
 1mg?

$$A(t) = 10 \cdot (0.9172)^t$$

$$1 = 10 (0.9172)^t \quad !!$$

$$\log(1) = \log(10 \cdot (0.9172)^t)$$

$$0 = \log(10) + \log(0.9172^t)$$

$$0 = 1 + t \cdot \log(0.9172) \Rightarrow t = \frac{-1}{\log(0.9172)} \approx 26.7$$