# Counting Digits

Which is bigger?

$2014^{2013}$ or $2010^{2017}$ ?

Come back later

| | $x$ | $\log(x)$ | |
|---|---|---|---|
| one digit | 1 | 0 | in $[0,1)$ |
| | 5 | 0.69... | |
| | 9 | 0.95... | |
| two digit | 10 | 1 | in $[1,2)$ |
| | 50 | 1.69... | |
| | 99 | 1.995... | |
| three digit | 100 | 2 | in $[2,3)$ |
| | 500 | 2.69... | |
| | 999 | 2.9995... | |

$$\boxed{\lfloor \log(n) \rfloor + 1 = \text{number of digits } n \text{ has}}$$

Ex $6,500,323$

how many digits? 7

Formula: $\lfloor \log(6,500,323) \rfloor + 1$

$= \lfloor 6.81\ldots\ldots \rfloor + 1$

$= 6 + 1$

$= 7$

Ex Which is bigger? $2014^{2013}$ or $2010^{2017}$?

Count digits instead of using these #s.

$\lfloor \log(2014^{2013}) \rfloor + 1$

$\lfloor 2013 \cdot \log(2014) \rfloor + 1$

$\lfloor 6651.07 \rfloor + 1$

$6651 + 1$

$6652$

$2014^{2013}$ has 6652 digits

$\lfloor \log(2010)^{2017} \rfloor + 1$

$\lfloor 2017 \cdot \log(2010) \rfloor + 1$

$\lfloor 6662.5 \rfloor + 1$

$6662 + 1$

$6663$

So $2010^{2017}$ power has more digits

$\Longrightarrow$ it is bigger!

Similarly, with very small numbers x,
like $\underbrace{0.00037}$ ...

Apply same formula: $\lfloor \log(0.00037) \rfloor + 1$

$\lfloor -3.43\ldots \rfloor + 1$

$= -4 + 1$

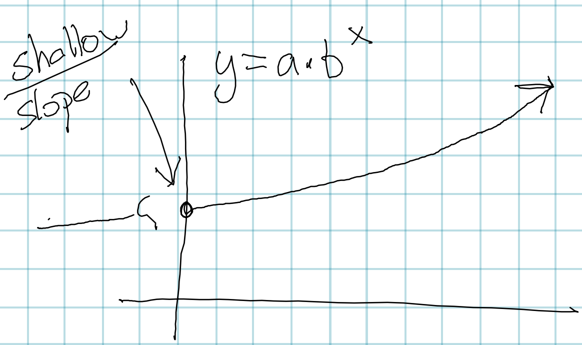
$-3 \rightarrow$ tells you how many leading 0's after decimal point.

Ports from 4.3–4.6

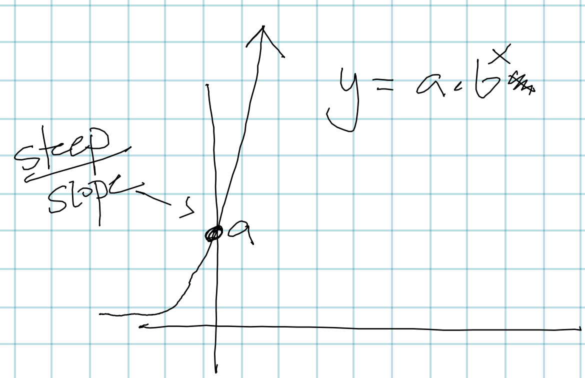
Euler's constant

There's a special number e, about 2.718... that we must study.

For b that are barely bigger than 1

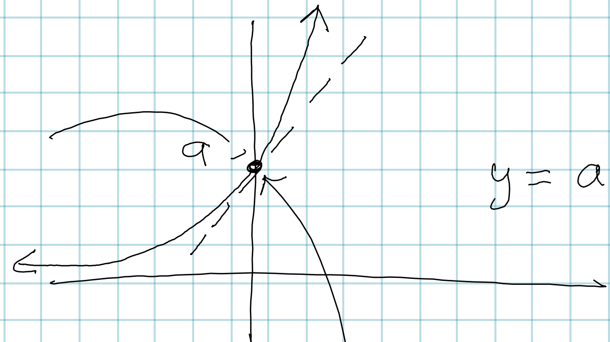



For b really big,



If growth factor $b$ equals $e \approx 2.718...$

initial value



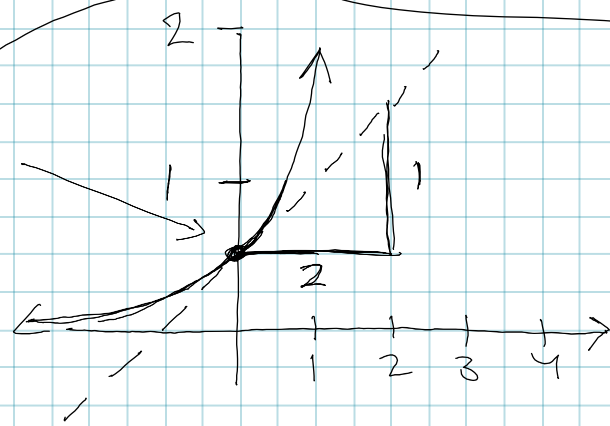

$y = a \cdot e^x$

$a$ is <u>also</u> the initial slope (when $b = e$)

<u>Ex</u> $f(x) = \frac{1}{2} \cdot e^x$ Sketch this...

$\frac{1}{2}$ is <u>also</u> initial slope

$a = \frac{1}{2}$




<u>Ex</u> A population of bacteria has 100 to start with. Initially, the growth rate is 100 bacteria per hour.

Write a formula for $P(t)$

$$P(t) = a \cdot b^t$$
$$= 100 \cdot b^t$$
$$= 100 \cdot e^t$$

initial value = initial slope

So $P(t) = 100 \cdot e^t$.

So what is the relative growth rate?

$\Longrightarrow$ what is $r$?

$\Longrightarrow$ $r = b - 1$

$\Longrightarrow$ $r = 2.718\ldots - 1$

$r = 1.718\ldots$

$r = 171.8\%$ / hour.

bacteria growing by 171.8% each hour.

Well, $\log_b(x)$.

↑

10, 2, 4, etc.

We have $\log_e(x)$

↓

log base $e \approx 2.718\ldots$

We write $\ln(x)$ instead.

logarithm natural

natural logarithm

$\ln(\ )$ is on all calculators.

Ex $\ln(e^2) = 2$

$\ln\left(\sqrt[4]{e}\right) = \frac{1}{4}$

$e^{\ln(x^2+2)} = x^2+2$

$\left(\ln(e^x) = x\right)$

$x$ must be positive

$\left(e^{\ln(x)} = x\right)$

Ex Solve $3 \cdot 1.8^t = 7$

Apply ln( ) $\longrightarrow$ $\ln(3 \cdot 1.8^t) = \ln(7)$

$$\ln(3) + \ln(1.8^t) = \ln(7)$$

$$\ln(3) + t \cdot \ln(1.8) = \ln(7)$$

$$t = \frac{\ln(7) - \ln(3)}{\ln(1.8)} \checkmark$$

## Continuous relative growth rate

When we look at exponential functions

$f(t) = a \cdot b^t$ $\qquad$ $b$ = growth factor

or

$f(t) = a(1+r)^t$ $\qquad$ $r$ = relative growth rate

or

$f(t) = a \cdot e^{k \cdot t}$ $\qquad$ $k$ = continuous relative growth rate

Ex $f(t) = 3(\underline{1.2})^t$

growth factor
$b = 1.2$

$f(t) = 3(1 + \underline{0.2})^t$

relative growth rate
is 20%

Now write f as

$f(t) = a \cdot e^{k \cdot t}$

? ?

$3(1.2)^t = a \cdot e^{kt}$ $a = 3$

$=$

$1.2^t = e^{kt}$ raise both sides to $1/t$

$1.2 = e^k$

$0.1823\ldots \approx \ln(1.2) = k$

So $f(t) = 3 \cdot e^{\ln(1.2) \cdot t}$

$= 3 \cdot e^{\underline{0.1823\ldots} t}$

Continuous relative growth rate

$k \approx 18.23\%$