Ex $f(t) = 4^{t+1} \cdot 2^{3t}$

Rewrite in $a \cdot b^t$ form.

make factors have same base

$f(t) = (2^2)^{t+1} \cdot 2^{3t}$

all your base belongs to us

$= 2^{2t+2} \cdot 2^{3t}$

$= 2^{5t+2}$

$= 2^{5t} \cdot 2^2$

$= 4 \cdot 32^t$

a b ✓

$2^{3t} = (2^3)^t$

$= 8^t$

$f(t) = 4^{t+1} \cdot 8^t$

dead end

Alternative

$f(t) = 4^{t+1} \cdot 2^{3t}$

Find a....

Same as $f(0)$

$f(0) = 4^{0+1} \cdot 2^{3 \cdot 0}$

$= 4^1 \cdot 2^0$

$= 4$

$t=0$ $\quad$ $t=1$

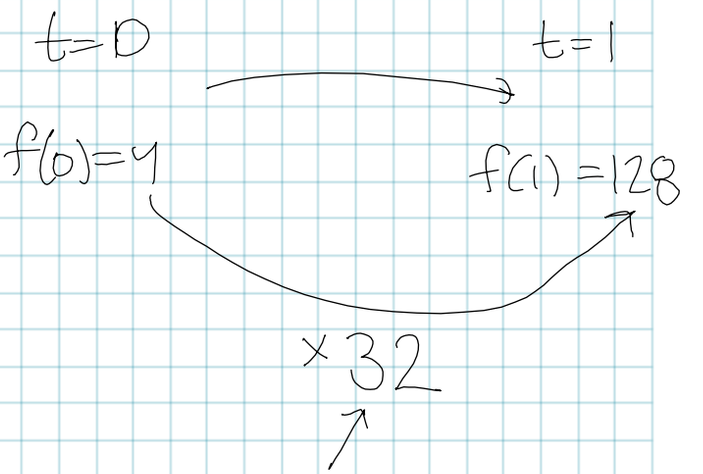
$f(0) = 4$ $\xrightarrow{\times b}$ $f(1) =$

$f(1) = 4^{1+1} \cdot 2^{3 \cdot 1}$

$= 4^2 \cdot 2^3$

$= 16 \cdot 8$

$= 128$



So $a = 4$

$b = 32$

b is what we multiply by each unit of time

And $f(t) = 4 \cdot 32^t$

Ex $f(t) = 4^{\frac{t+1}{2}} \cdot 3^{2t}$ (this is an exponential function)

Rewrite in $a \cdot b^t$ form

Find a: $f(0) = 4^{\frac{0+1}{2}} \cdot 3^{2 \cdot 0}$

$= 4^{1/2} \cdot 3^0 = 2$

Find b: $f(1) = 4^{\frac{1+1}{2}} \cdot 3^{2 \cdot 1}$

$= 4^1 \cdot 3^2$

$= 4 \cdot 9 = 36$

$\times b = \times 18$

$b = 18$

So $f(t) = 2 \cdot 18^t$

Solve exponential equations, not using logarithms

Ex Solve $3^t = 81$

$\Rightarrow t = 4$

(solve using mult. tables)

$3^1 = 3$
$3^2 = 9$
$3^3 = 27$
$3^4 = 81$

Ex Solve $2^{(x^2)} = 16$.

$\Rightarrow 2^{(x^2)} = 2^4$

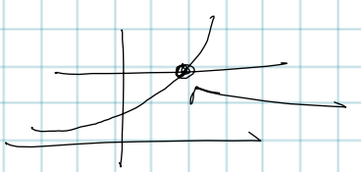
$\Rightarrow x^2 = 4 \rightarrow x = 2$ or -2

Ex Solve $2^t = 13$

↖ 13 is not a clean power of 2.

For this, use technology.

$Y1 = 2^x$
$Y2 = 13$

→  intersection tool

$xc = 3.7004...$

$t \approx 3.7004...$

Ex Solve $16^x - 3 \cdot 4^x - 4 = 0$

3 terms

2 have x in exponent....

It would be nice if we used same base.

$$(4^2)^x - 3(4^x) - 4 = 0$$

same base!

$$4^{2x} - 3 \cdot 4^x - 4 = 0$$

$$(4^x)^2 - 3(4^x) - 4 = 0$$

$$(Y)^2 - 3(Y) - 4 = 0$$

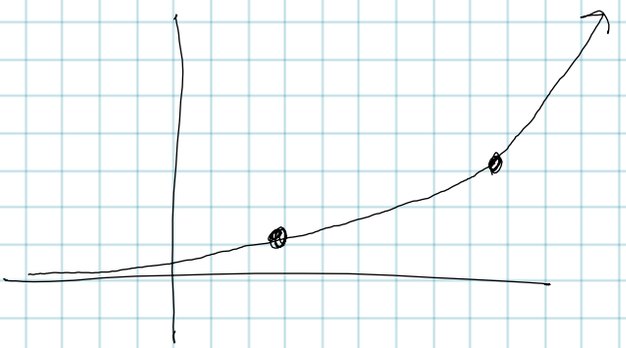
$$(Y - 4)(Y + 1) = 0$$

$Y = 4$ $\qquad$ $Y = -1$

$4^x = 4$ $\qquad$ $4^x = -1$

$\Rightarrow x = 1$ $\qquad$ not possible

Task: Given two points, find the exponential curve that connects them



Find a and b in $f(t) = a \cdot b^t$.

Ex $(-2, 3/4)$ and $(2, 12)$.

Find the exp. curve that fits these.

Algebraic way

$f(t) = a \cdot b^t$

use one point → $\frac{3}{4} = a \cdot b^{-2}$

$12 = a \cdot b^2$

Divide left by left, right by right

$\frac{3/4}{12} = \frac{a \cdot b^{-2}}{a \cdot b^2}$

$\frac{3}{48} = \frac{b^{-2}}{b^2} = b^{-4}$

$\frac{1}{16} = b^{-4} = \frac{1}{b^4}$

$b^4 = 16$

$b^4 = 2^4$

$\Rightarrow b = 2$

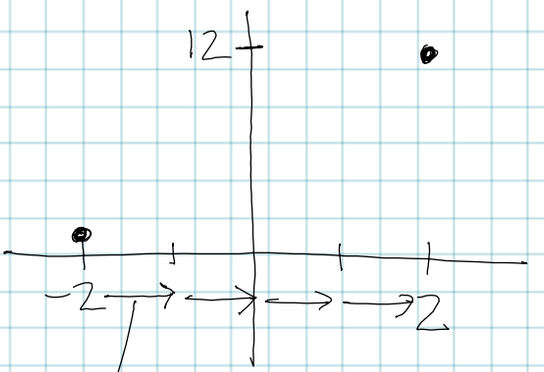
And a...

$12 = a \cdot 2^2$

$3 = a$

So $f(t) = 3 \cdot 2^t$

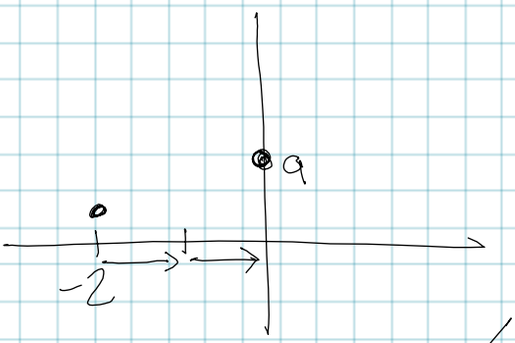
Same ex: $(-2, 3/4)$ and $(2, 12)$



mult by b $\Longrightarrow$ $\frac{3}{4} \cdot b^4 = 12$

$b^4 = 16$

$b = 2$

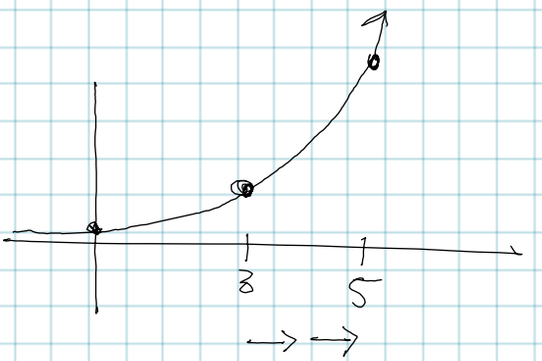


a

$f(t) = 3 \cdot 2^t$

$\frac{3}{4} \cdot 2^2 = 3$

Ex An exp. curve passes through $(3,8)$ and $(5,128)$. Find the formula.



algebra

$f(t) = a \cdot b^t$

$8 = a \cdot b^3$

$128 = a \cdot b^5$

$\frac{8}{128} = \frac{a \cdot b^3}{a \cdot b^5}$

$\frac{1}{16} = b^{-2} = \frac{1}{b^2}$

$b^2 = 16$

$\Rightarrow b = 4$

And $8 = a \cdot 4^3$

$8 = a \cdot 64$

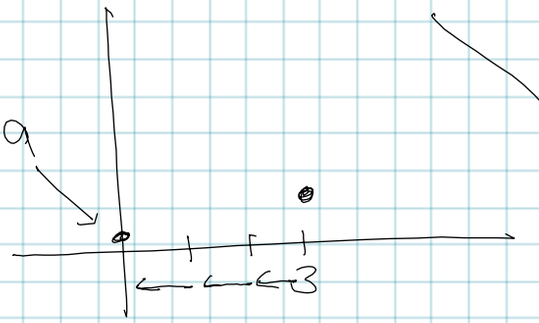
$\frac{1}{8} = a$

So $f(t) = \frac{1}{8} \cdot 4^t$

$8 \cdot b^2 = 128$

$b^2 = 16$

$b = 4$



$8 \div (4)^3 = a$

$\frac{8}{64} = a$

$\frac{1}{8} = a$

$f(t) = \frac{1}{8} \cdot 4^t$

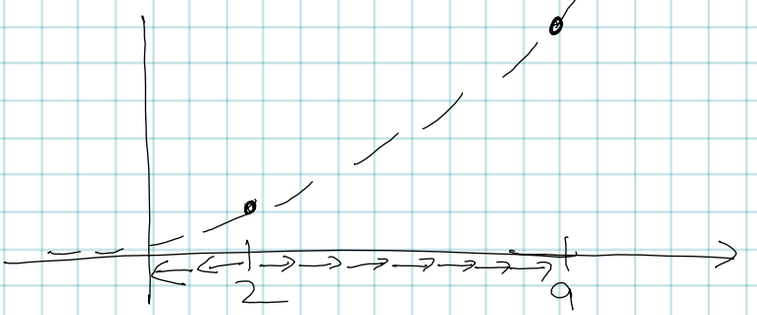
Ex An outbreak of a disease

On day ~~0~~ 2, doctors identify 25 patients

One week later 2391 people are infected.

- Find an exponential model for how many sick people
- How many were sick on day 5?
- How long until 1 million sick?

Points: $(2, 25)$ and $(9, 2391)$.



growing by 91.8% per day

Find b: $25 \cdot b^7 = 2391$

$b^7 = \frac{2391}{25}$

$b = \sqrt[7]{\frac{2391}{25}} \approx 1.918441\ldots$

Find a: $25 \div b^2$

$25 \div (1.918441\ldots)^2$

$= 6.79271\ldots$

$P(t) \approx 6.79\,(1.918441)^t$

- $P(5) = 6.79(1.918441)^5$

  $\approx 177 \Longrightarrow$ on day 5

  177 sick people.

- $P(t) = 1000000$

  $6.79(1.918441)^t = 1000000$

  Y1 — solve for t using calculator — Y2

  $t \approx 18.26\ldots$

  So on day 18~19 we have 100000 sick people.