# 4.2 Inverse Functions

We've seen that functions turn numbers into other numbers

**Ex** Grocery, buying apples

weight of a bag of apples in lb $\xrightarrow{f}$ cost of bag of apples

$$f(w) = 1.19 \cdot w$$

We can turn this around

weight $\xleftarrow{f^{-1}}$ cost

"f inverse" is a new function undoes what f did.

$$f^{-1}(c) = \frac{c}{1.19}$$

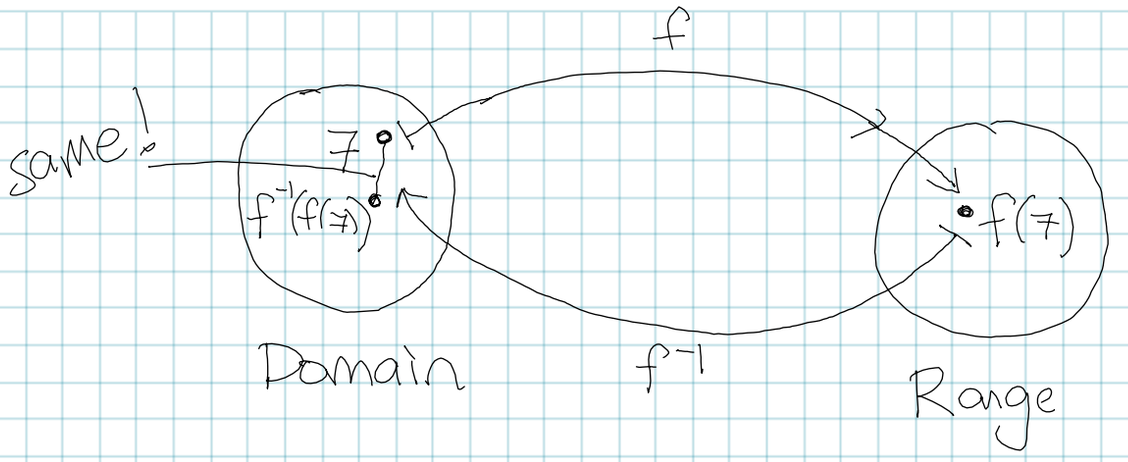
with numbers:

$$3^{-1} = \frac{1}{3}$$

$$x^{-1} = \frac{1}{x}$$

with a function

$$f^{-1} \neq \frac{1}{f}$$

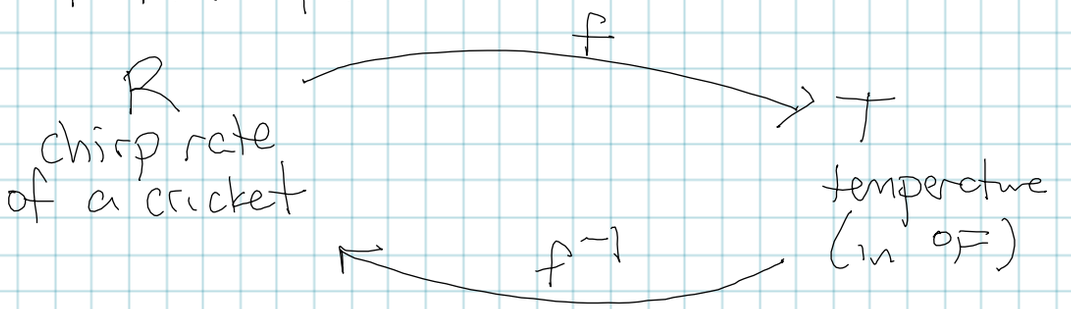



## Feature of $f^{-1}$

$$f^{-1}(f(x)) = x \qquad (f^{-1} \circ f)(x) = x$$
$$f(f^{-1}(x)) = x \qquad (f \circ f^{-1})(x) = x$$

$$f \circ f^{-1} = \text{identity}$$
$$f^{-1} \circ f = \text{identity}$$

**Ex** $f(R) = \frac{1}{4}R + 40$




A very common task is finding a formula for $f^{-1}$.

Formula for $f$

$f(R) = \frac{1}{4}R + 40$

drop function notation

$T = \frac{1}{4}R + 40$

isolate this

$T - 40 = \frac{1}{4}R$

$4(T-40) = R$

a formula for $f^{-1}$

$f^{-1}(T) = 4(T-40)$

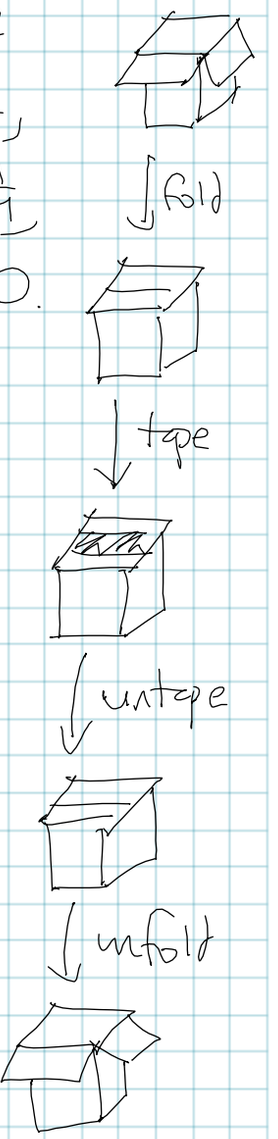
Same!

Verbally, $f$ takes input, multiplies by $\frac{1}{4}$, then adds 40.

$f^{-1}$ does opposite things in opposite order

- subtract 40
- multiply by 4

$f^{-1}(x) = 4(x-40)$




Ex $f(x) = 4x^3 + 7$ Find formula for $f^{-1}$.

$y = 4x^3 + 7$

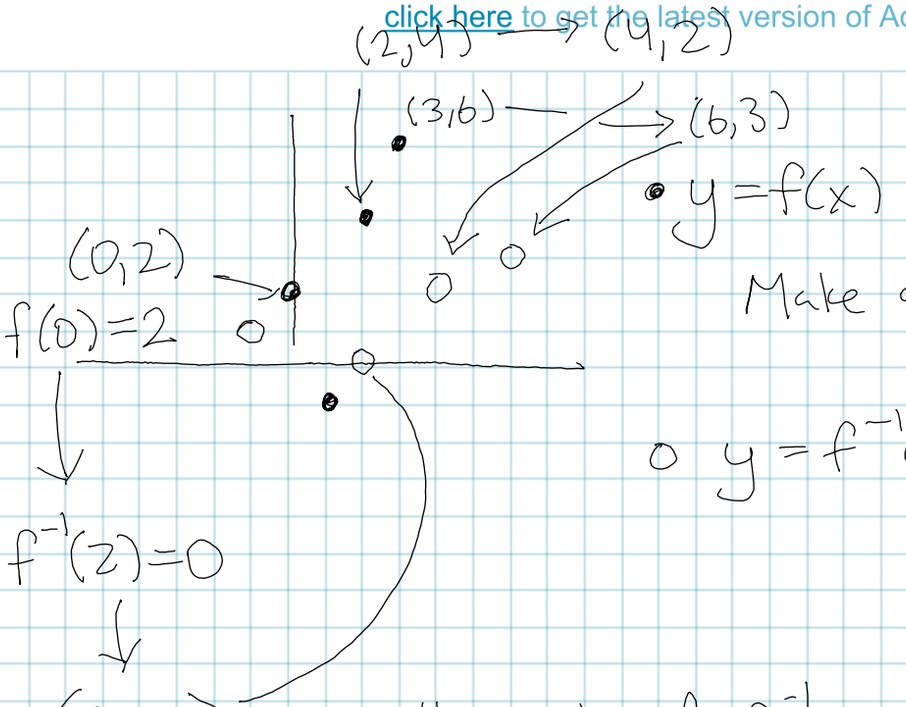
$y - 7 = 4x^3$

$\frac{y-7}{4} = x^3$

$\sqrt[3]{\frac{y-7}{4}} = x$

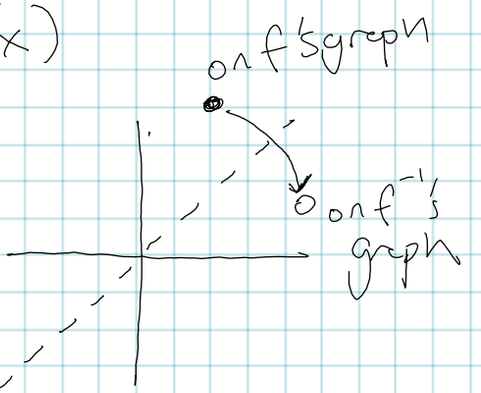
$f^{-1}(y) = \sqrt[3]{\frac{y-7}{4}}$

or $f^{-1}(x) = \sqrt[3]{\frac{x-7}{4}}$

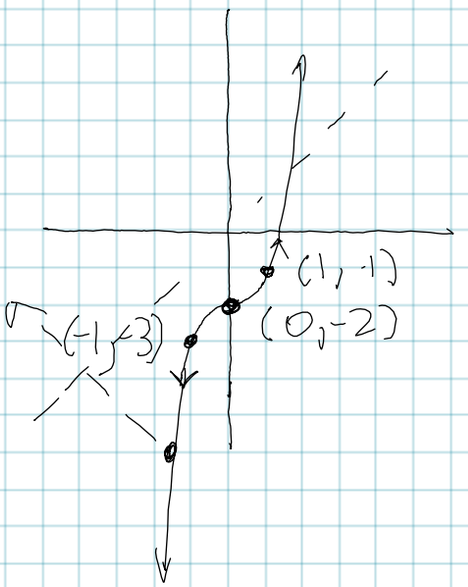


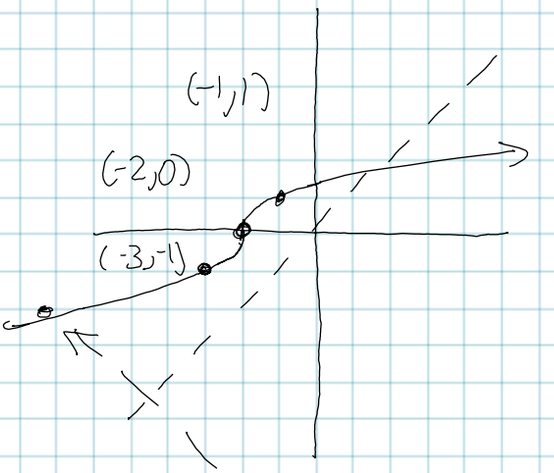

Make a graph for $f^{-1}$.

$y = f^{-1}(x)$




Ex $h(x) = x^3 - 2$ Find graph for $h^{-1}$



Ex

| $x$ | $f(x)$ |
|---|---|
| 0 | 2 |
| 1 | −1 |
| 2 | 4 |
| 3 | 6 |

Make a table for $f^{-1}$...

| $x$ | $f^{-1}(x)$ |
|---|---|
| 2 | 0 |
| −1 | 1 |
| 4 | 2 |
| 6 | 3 |

Ex Let $f(x) = \dfrac{3x-4}{x-8}$

Find an formula for $f^{-1}$.

drop function notation: $y = \dfrac{3x-4}{x-8}$ ← clear denom.

$y(x-8) = 3x-4$

$x \cdot y - 8y = 3x - 4$

$xy - 3x = 8y - 4$

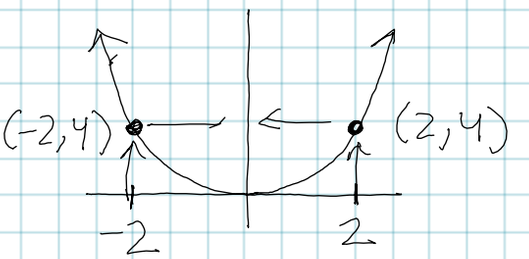
$x(y-3) = 8y-4$

$x = \dfrac{8y-4}{y-3}$

$f^{-1}(y) = \dfrac{8y-4}{y-3}$

Not every function has an inverse

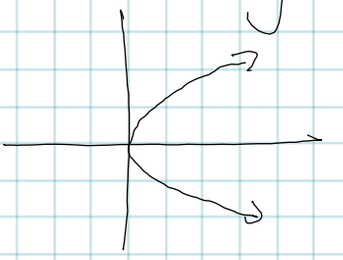
Ex $f(x) = x^2$




$f(2) = 4$
$f(-2) = 4$

What is $f^{-1}(4)$?

↘ reflect over $y=x$ diagonal

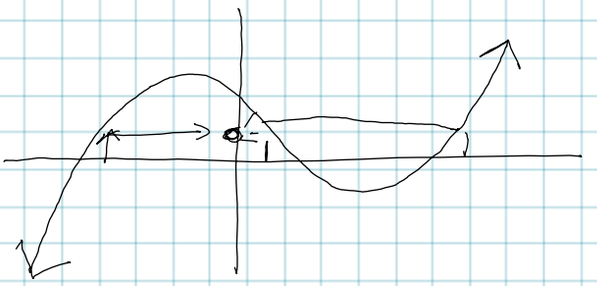


not a function!

A function has to be one-to-one for the inverse function to exist

$f(x) = x^2$ is <u>two</u>-to-one.

Ex



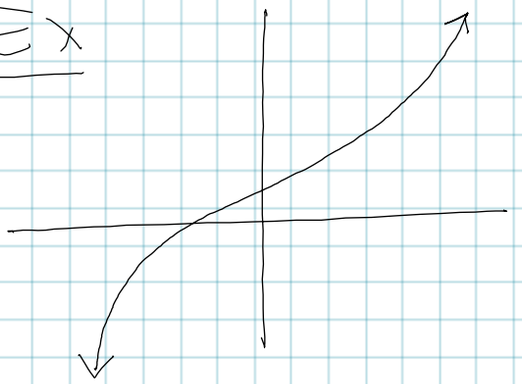
$y = g(x)$

Does g have an inverse?

3 different x-values lead to the same y-value

three-to-one ⇒ not invertible.

Ex

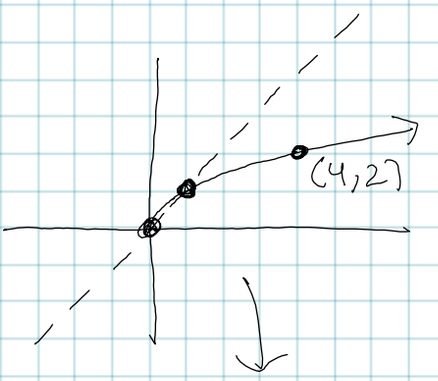


$y = h(x)$

h is one-to-one.

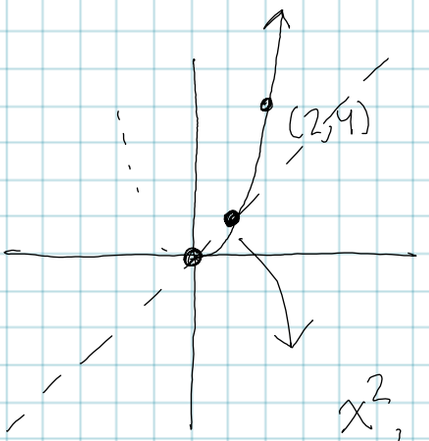
Ex $y = \sqrt{x}$

$f(x) = \sqrt{x}$




is one-to-one

So it has an inverse function

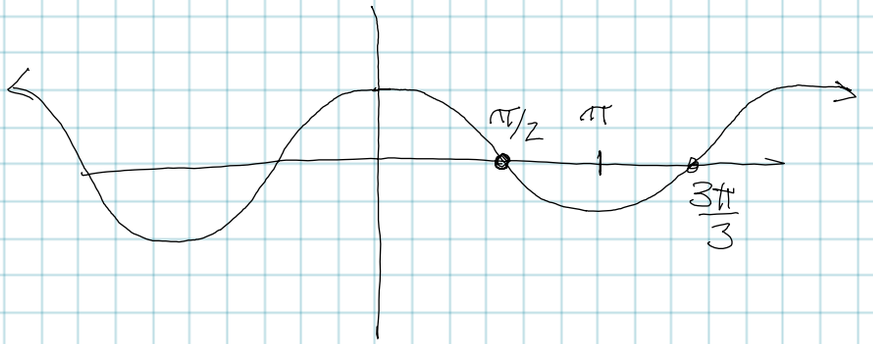



$x^2, \; x \geq 0$

$\sqrt{x}$ and $x^2, \; x \geq 0$

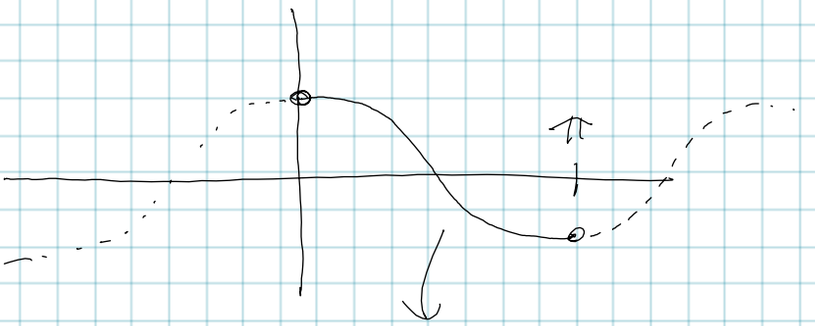
are inverse functions of each other

Ex $g(x) = \cos(x)$




Is g one-to-one? No!

We can restrict the domain!

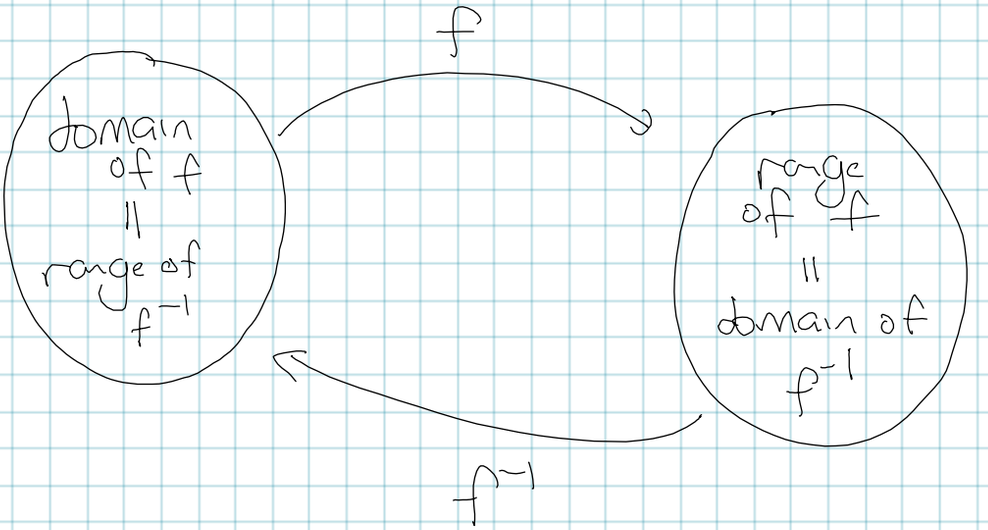



this is one-to-one

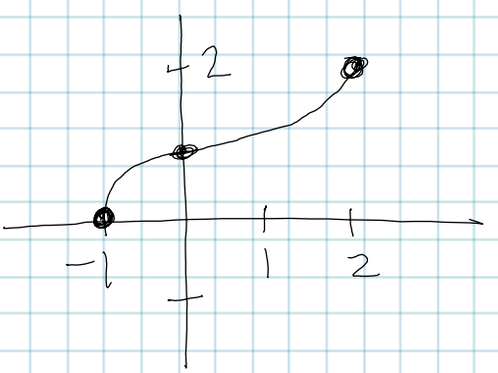
this does have an inverse function

$\cos(x)$, $0 \le x \le \pi$

does an inverse!




**Ex**




$y = f(x)$

What is $f^{-1}$'s domain and range?

domain of $f^{-1}$?
$=$ range of $f$
$= [0, 2]$

range of $f^{-1}$
$=$ domain of $f$
$= [-1, 2]$

**Ex** Suppose $f(x) = \frac{3x+1}{x-8}$

Find range of $f^{-1}$. ✓

$=$ domain of $f$

$= (-\infty, 8) \cup (8, \infty)$