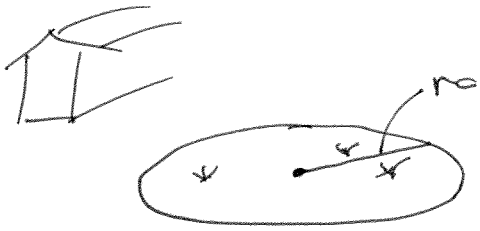
Section 4.1 Composition of Functions
(1.5 from open-source)

Ex A woman has a circular patch of lawn



A ← a function to compute area from radius

radius: r ⟼ area: A

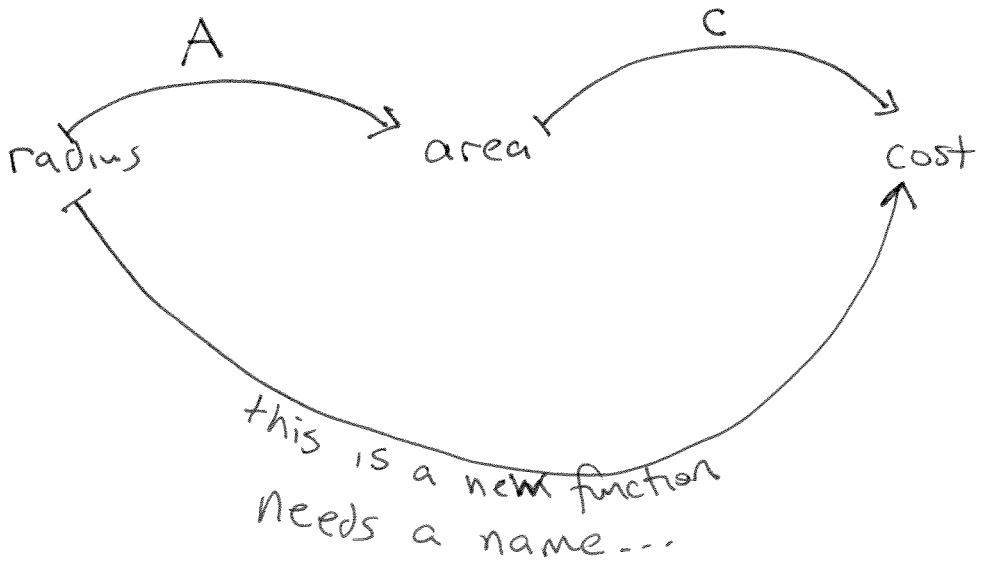
$A(r) = \pi \cdot r^2$

At the garden store, fertilizer costs $\$0.075$ / square meter

c ← a function to compute cost

area: A ⟼ cost to fertilize area...

$c(A) = A \cdot 0.075$



$c \circ A$ ⟶ $(c \circ A)(x) = (c \circ A)(r) = c(A(r))$

the composition of c and A

$(f \circ g)(x) = f(g(x))$

Ex Listening to crickets at night...
can tell you the temperature

if R is the chirp rate (one cricket chirping R times per minute)

then $T(R) = \frac{1}{4}R + 40$ (in °F)

$$T(16) = \frac{1}{4}(16) + 40$$
$$= 44$$

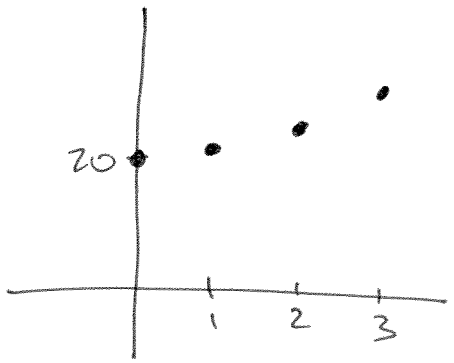
tells us its 44°F if we are hearing 16 chirps per minute.

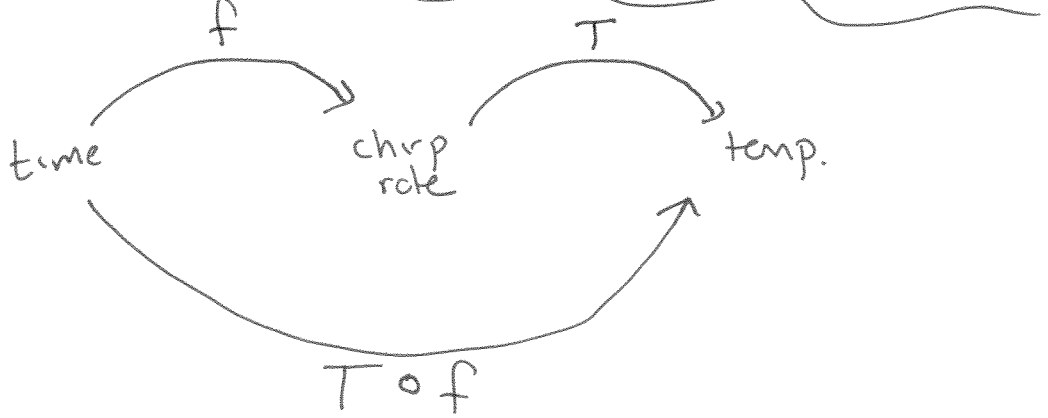
We gather data over time recording chirp rate

t	$f(t)$
0	20
1	21
2	24
3	29

every hour

chirps per minute





What is $(T \circ f)(1)$?

$= T(f(1)) = T(21)$

$= \frac{1}{4}(21) + 40$

$= 5.25 + 40$

$= 45.25$

$T(R) = \frac{1}{4}R + 40$

After 1 hr, it was 45.25°F

Note $f \circ T$ .... $(f \circ T)(x) = f(T(x))$

chirp rates go into T...

temps come out....

but f takes time as input

Ex Let $f(x) = \frac{1}{x}$ and $g(x) = x^2 + 2$.

Find a formula for $f \circ g$.

$(f \circ g)(x) = f(g(x))$

$= f(x^2 + 2)$

$= \frac{1}{x^2 + 2}$

Domain: $(-\infty, \infty)$

Find a formula for $g \circ f$...

$(g \circ f)(x) = g(f(x))$

$= g\left(\frac{1}{x}\right)$

$= \left(\frac{1}{x}\right)^2 + 2$

$= \frac{1}{x^2} + 2$

Domain: $(-\infty, 0) \cup (0, \infty)$

Ex $f(x) = \frac{x+1}{2x-3}$ (domain: $x \neq 3/2$)

$g(x) = \frac{1}{x-8}$ (domain: $x \neq 8$)

Find formula for $f \circ g$...

$(f \circ g)(x) = f(g(x))$

$= f\left(\frac{1}{x-8}\right) = \frac{(\ ) + 1}{2(\ ) - 3}$

$= \frac{\frac{1}{x-8} + 1}{2\left(\frac{1}{x-8}\right) - 3} \cdot \frac{(x-8)}{(x-8)}$

$= \frac{1 + x - 8}{2 - 3(x-8)}$

$= \frac{x - 7}{-3x + 26}$

What is $f \circ g$'s domain?

- exclude any x that is bad for g...
- also worried about $\frac{1}{x-8}$ working out to be bad for f

$\frac{1}{x-8} = \frac{3}{2}$

$2 = 3x - 24$

$26 = 3x$

$\frac{26}{3} = x$

So $f \circ g$ excludes 8, and $\frac{26}{3}$.

$(-\infty, 8) \cup (8, \frac{26}{3}) \cup (\frac{26}{3}, \infty)$

$f \circ g$'s domain

- exclude bad x-values for g
- exclude x-values that make $g(x)$ into a bad input for f.

$f+g$
$f-g$
$f \cdot g$ $\longrightarrow$ domain excludes bad input for f, also bad input for g.
Intersection of domain of f with domain of g

f/g $\longrightarrow$ same as above, but also ~~not~~ exclude any x-values that cause $g(x)=0$

$f \circ g$ $\longrightarrow$ exclude bad inputs for g, and any x-values that make $g(x)$ bad for f.

$(f \circ g)(x) = f(g(x))$

is x bad?