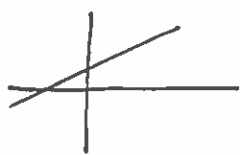


Concavity



~~flat~~
perfectly
straight

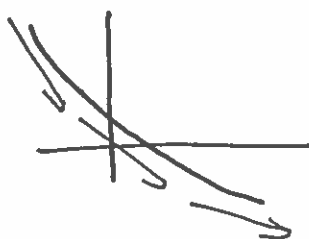
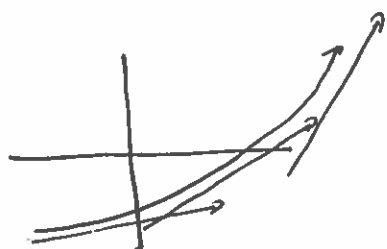


Concave up



Concave down

Formal Definition: f is concave up, if f is increasing at an increasing rate or if f is decreasing at a decreasing rate.



Ex

	x	$f(x)$
+1	1	5
+1	2	8
+1	3	11
+1	4	14

Arrows indicate the change in $f(x)$ for each unit increase in x : $5 \rightarrow 8 (+3)$, $8 \rightarrow 11 (+3)$, $11 \rightarrow 14 (+3)$.

Figure out f 's concavity
(up, down, neither?)

This is a line with slope $\frac{3}{1}$

No concavity!

Ex

	x	$f(x)$
+1	1	1
+1	2	4
+1	3	9
+1	4	16

Arrows indicate the change in $f(x)$ for each unit increase in x : $1 \rightarrow 4 (+3)$, $4 \rightarrow 9 (+5)$, $9 \rightarrow 16 (+7)$.

increasing at an increasing rate

Same task

Concave up!

3.4, 3.5 Rational Functions (3.2, 3.3 in 2nd edition)

Thinking through
making a calculation
Do a division

Everyday

"Makes sense"

"Reasonable"

"Well thought out"

Math

Comes from a ratio

Dividing two things

Rational Functions:

ratio of two polynomials

Ex $\frac{2x+1}{x^2+x+10}$, $\frac{1}{x^2+1}$, $\frac{4x-8}{1}$

Ultimate Goal: Be able to graph....

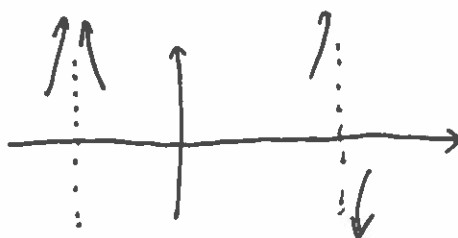
- Long Term Behavior , etc.
- y-int: $f(0)$ 

- x-int 

- Local behavior

- Vertical Asymptotes

- Local behavior



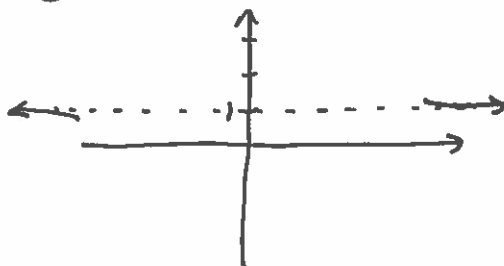
Ex $f(x) = \frac{x+1}{x-3}$ Plot this.

- Long Term Behavior $\rightarrow$ all about when x is large, how do outputs behave...

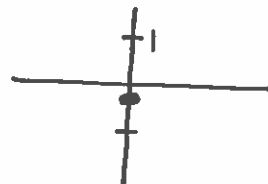
For huge x , $\frac{x+1}{x-3} \approx 1$

$\begin{matrix} \text{deg } 1 \\ \text{deg } 1 \end{matrix}$
 $\begin{matrix} \text{Leading coef} \\ \text{Leading coef} \end{matrix}$
 $\frac{x+1}{x-3} \Rightarrow \frac{1}{1} = 1$

So $y=1$ is the Long Term Behavior



- y-int: $f(0) = \frac{0+1}{0-3} = -\frac{1}{3}$



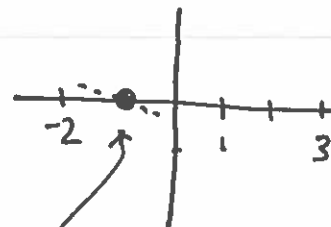
- x-int: $f(x) = 0$

$$\frac{x+1}{x-3} = 0$$

$\cdot (x-3)$

$$x+1 = 0$$

$$x = -1$$



Just use numerator

line with slope $-\frac{1}{4}$

- Local Behavior: $\frac{x+1}{x-3} \xrightarrow{\text{At } x=-1} \frac{x+1}{-1-3} = \frac{x+1}{-4} = -\frac{1}{4}(x+1)$