$$p(x) = c(x+3)(x-2)^2$$

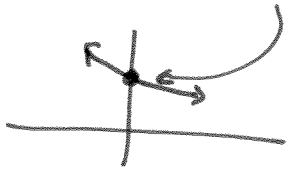
↑ you can use any negative number

3.2 & 3.3 Rational Functions

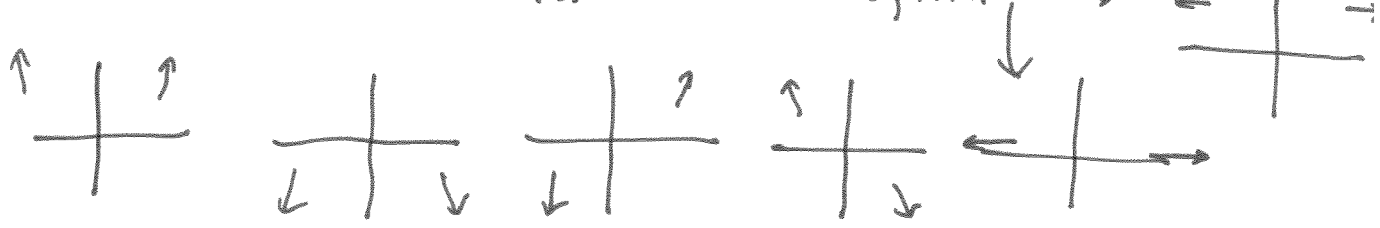
Ex $f(x) = \dfrac{2x+1}{x^2+x+10}$ $\quad g(x) = \dfrac{1}{x^2+3}$

$$\frac{\text{polynomial}}{\text{polynomial}}$$

- still have zeros → $f(x) = 0$ ← such x-values are zeros / roots
- still have y-intercepts → $f(0) = \#$



- still have LTB... two more options →



Ex f is a cubic polynomial with zeros at 3, 8, −2. Also $f(1) = 5$.

Find a formula for f.

$(x-3)$ is a factor

$(x-8)$ is a factor

$(x+2)$ is a factor

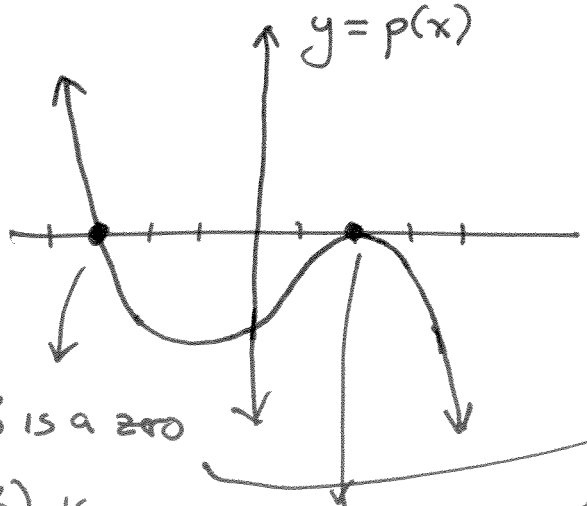
$$f(x) = c\cdot(x-3)\cdot(x-8)(x+2)$$ ← based on f being cubic and those 3 zeros...

Let $x=1$ ...

$$5 = c\,(1-3)(1-8)(1+2)$$
$$5 = c\,(-2)(-7)(3)$$
$$5 = c\,(42)$$

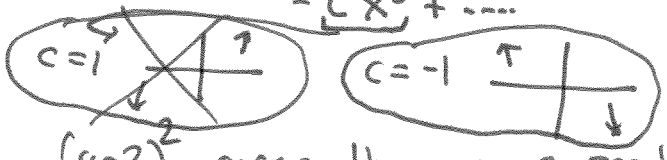
$$\Rightarrow c = \frac{5}{42} \Rightarrow f(x) = \frac{5}{42}(x-3)(x-8)(x+2)$$

Ex



what might be a formula for p?

$$p(x) = c\cdot(x+3)(x-2)^2$$
$$= c(x\ldots)(x\ldots)^2$$
$$= cx^3 + \ldots$$



since there is a parabola

— Now bad x-values are possible

ex: $f(x) = \dfrac{x^2+2x+3}{x-8}$

↑ 8 is not a valid input

<u>Ex</u> A factory spends \$10,000 on equipment to make DVD players. Each DVD player costs \$30 in parts, labor, electricity.

So $C(x) = 30x + 10000$

↑ how many DVD players they make

What is the <u>average cost</u> per DVD player after you've produced x...

$$A(x) = \frac{C(x)}{x} = \frac{30x+10000}{x}$$

a rational function.

- zeros of A $\longrightarrow$ $\frac{30x+10000}{x} = 0$

$30x + 10000 = 0$

(denom. is irrelevent for finding zeros)

$x = -10000/30 = -1000/3$

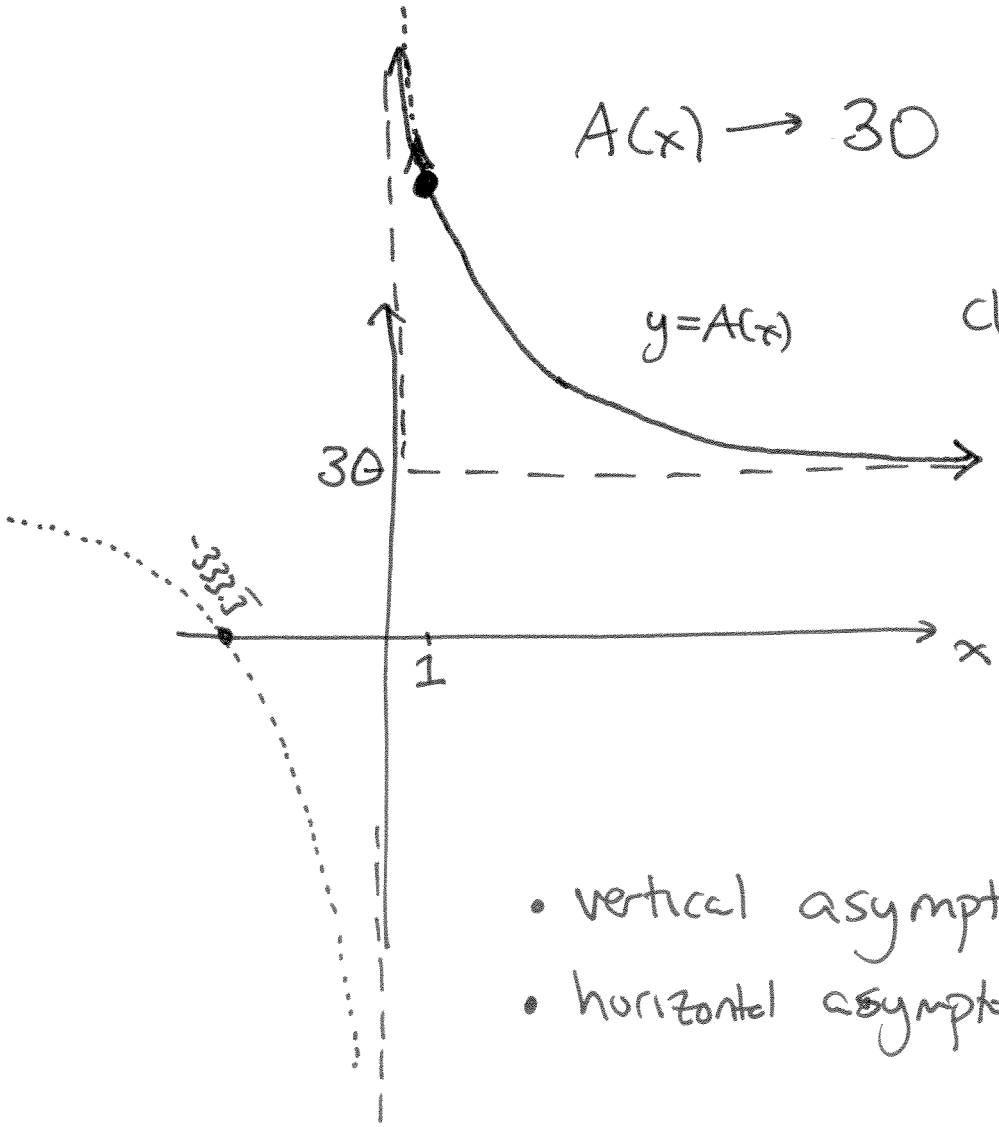
$= -333.\overline{3}$

- bad x-value(s) for A $\longrightarrow$ $x \neq 0$

- LTB $\longrightarrow$ When $x \to \infty$, what happens to $A(x)$?

$\frac{10000 + 30x}{x}$

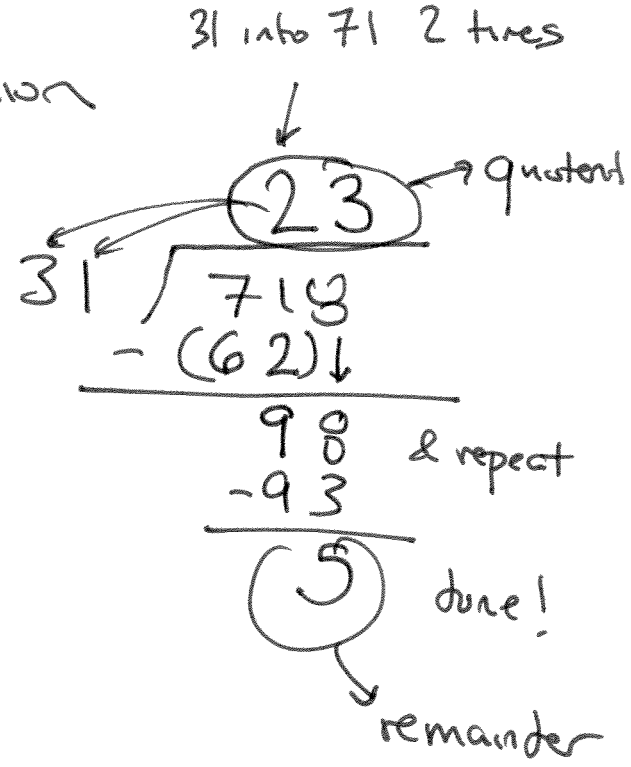
$A(x) \to 30$ (making more & more DVD players brings avg cost to closer to \$30/DVDplayer)



- vertical asymptote at $x = 0$
- horizontal asymptote at $y = 30$

Review of Long Division

Ex $\frac{718}{31}$ →



Ex $f(x) = \frac{x+1}{x-3}$

Task: plot this by hand.

- Zeros → (only numerator is relevant)

$$\frac{x+1}{x-3} = 0$$

$$x+1 = 0 \implies x = -1$$

- bad x-value → $x = 3$

- y-intercept → $f(0) = \frac{0+1}{0-3} = -\frac{1}{3}$

- LTB → to long division! (shortcuts later)

$\frac{x+1}{x-3}$ →

$$x-3 \overline{)\, x+1}$$

1 ← quotient

$x-3 \mid x+1$

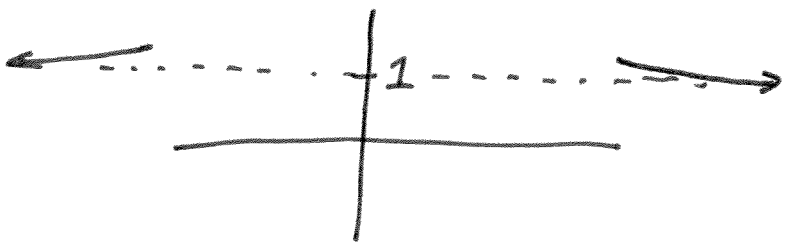
$-(1 \cdot x - 3)$

4 ← remainder can be ignored!

gives you LTB

what to mult. x by, to get x

So $y = 1$ ← quotient is the LTB

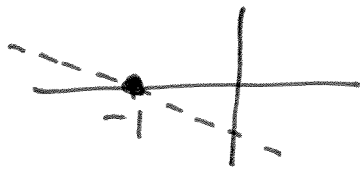


- At $x = -1$ ....

$$\frac{\sqrt{x+1}}{x-3} \approx \frac{x+1}{-1-3}$$

$$= \frac{x+1}{-4}$$

$$= -\frac{1}{4}(x+1)$$

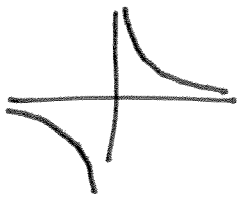


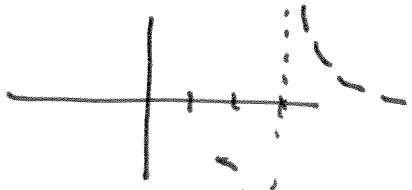
At $x = 3$ ....

$$\frac{x+1}{\sqrt{x-3}} \approx \frac{3+1}{x-3}$$

$$= \frac{4}{x-3}$$

looks like $\frac{1}{x}$ ... right 3, vert stretched factor 4

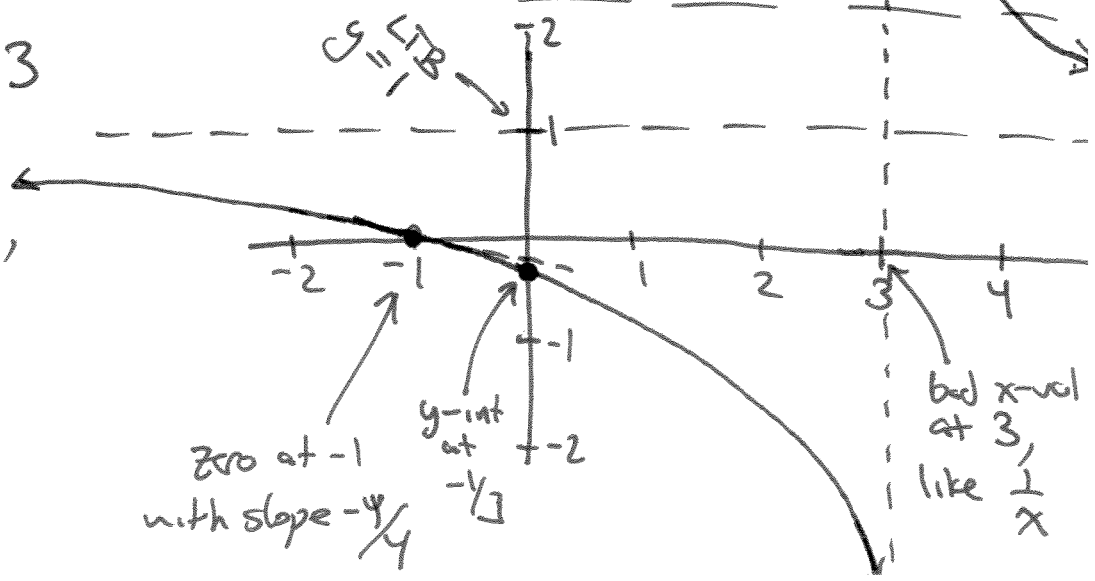


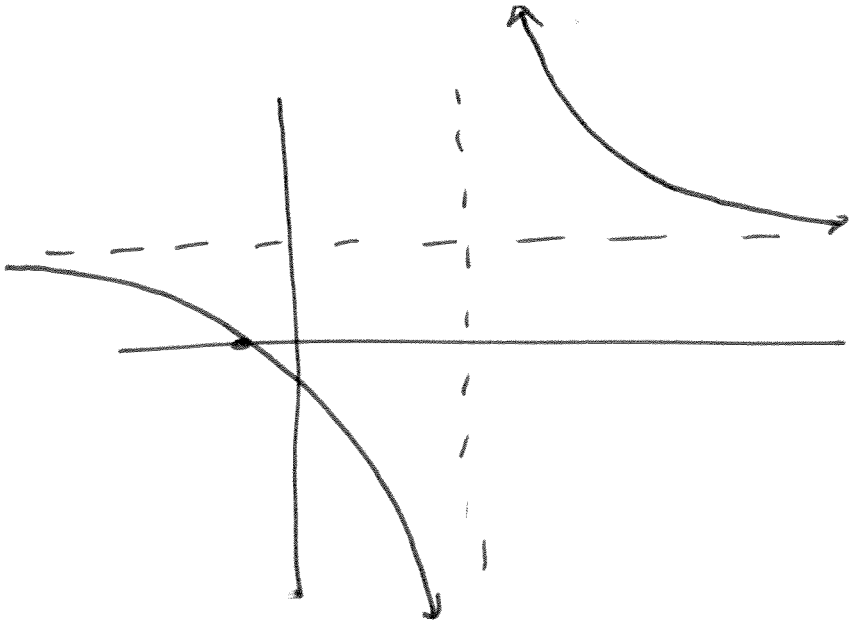


x-values: $-1, 3$

y-values: $-\frac{1}{3}, -\frac{1}{4}$,

y-int — slope





Ex $f(x) = \dfrac{(x+1)^2(x-2)}{(x-4)(x+3)^2}$ Sketch f's graph

- zeros $\longrightarrow$ $-1, 2$
- bad x $\longrightarrow$ $4, -3$
- y-intercept $\longrightarrow$ $f(0) = \dfrac{(0+1)^2(0-2)}{(0-4)(0+3)^2} = \dfrac{-2}{-4 \cdot 9} = \dfrac{1}{18}$
- LTB (sometimes there's a shortcut....

  if deg(top) = deg(bottom)

  $\Longrightarrow$ LTB: $y = \dfrac{\text{leading coef (top)}}{\text{leading coef (bottom)}} = \dfrac{1}{1}$

$y = 1$

- Local behavior → At $x = -1$

$$\frac{(x+1)^2(x-2)}{(x-4)(x+3)^2} \approx \frac{(x+1)^2(-1-2)}{(-1-4)(-1+3)^2}$$

$$= \frac{-3}{-5 \cdot 4}(x+1)^2$$

$$= \frac{3}{20}(x+1)^2$$

parabola, upward opening

$x = 2$

$$\frac{(x+1)^2(x-2)}{(x-4)(x+3)^2}$$

$$\approx \frac{(2+1)^2(x-2)}{(2-4)(2+3)^2}$$

$$\approx \frac{9}{(-2)(25)}(x-2)$$

$$= -\frac{9}{50}(x-2)$$

line with slope $\frac{-9}{50}$

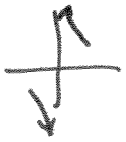
$= -0.18$

At $x = 4$

$$\frac{(x+1)^2(x-2)}{(x-4)(x+3)^2} \approx \frac{(4+1)^2(4-2)}{(x-4)(4+3)^2}$$

$$= \frac{25 \cdot 2}{49 \cdot (x-4)}$$

$$= \frac{50}{49} \cdot \frac{1}{(x-4)}$$

looks like $\frac{1}{x}$ 

At $x = -3$

$$\frac{(-3+1)^2(-3-2)}{(-3-4)(x+3)^2}$$

$$= \frac{4 \cdot (-5)}{-7(x+3)^2} = \frac{20}{7} \cdot \frac{1}{(x+3)^2}$$

looks like $\frac{1}{x^2}$ ... Atari logo 

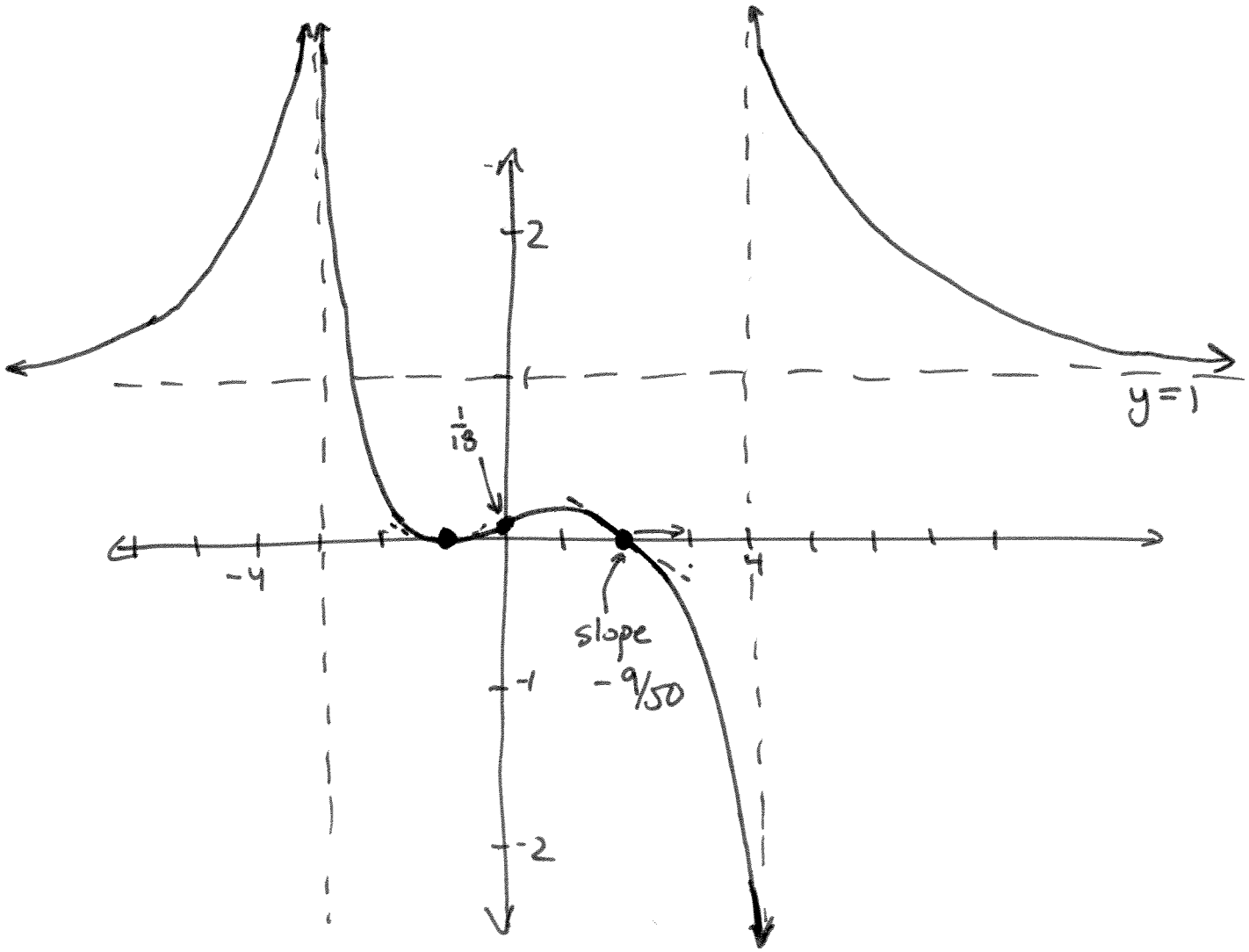
x-values: $-1, 2, -3, 4$

y-values: $\frac{1}{18}$, 1, $\frac{-9}{50}$

y-int

LTB

slopes

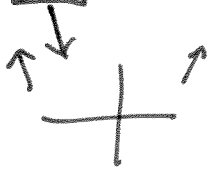


Ex $f(x) = \dfrac{(x^2-1)(x+2)}{x+1}$ Sketch a graph

- zeros → $-2, 1, \cancel{-1}$ (numerator matters)
- bad x-values → -1
- y-intercept → $f(0) = \dfrac{(0^2-1)(0+2)}{0+1} = -2$
- LTB → .... Actually, we're not fully factored yet $\dfrac{(x-1)\cancel{(x+1)}(x+2)}{\cancel{(x+1)}}$

$\approx (x-1)(x+2)$ ← a polynomial! (quadratic)

$\approx x^2 + \cdots$



- Near $x = -2$

$\dfrac{(x-1)(x+1)\overbrace{(x+2)}}{(x+1)}$

$\approx \dfrac{(-2-1)(-2+1)(x+2)}{(-2+1)}$

$\approx -3(x+2)$

a line with slope -2

Near $x = 1$

$\dfrac{\overbrace{(x-1)}(x+1)(x+2)}{(x+1)}$

$\approx \dfrac{(x-1)(1+1)(1+2)}{(1+1)}$

$= 3(x-1)$

a line with slope 3

Near $x = -1$

$\dfrac{(x-1)\overbrace{(x+1)}(x+2)}{\underbrace{(x+1)}}$

$\approx \dfrac{(-1-1)(x+1)(-1+2)}{(x+1)}$

$= -2 \cdot \dfrac{(x+1)}{(x+1)}$

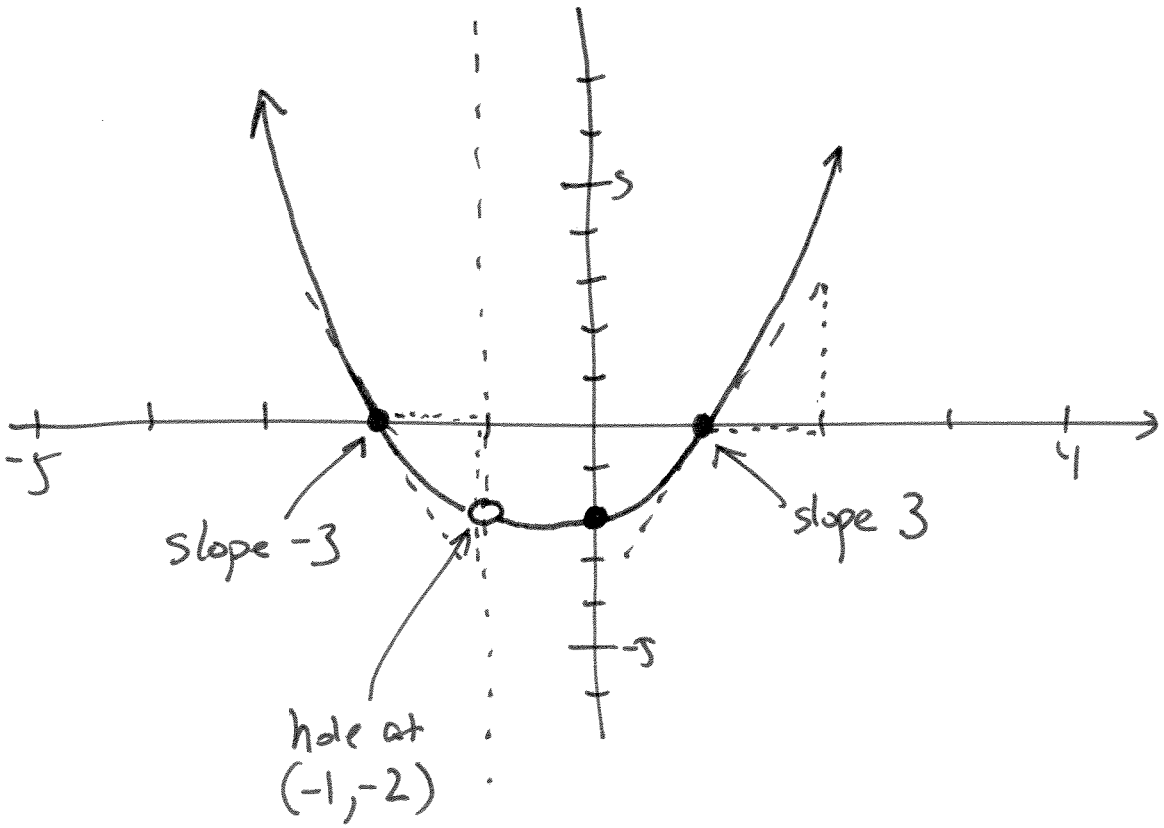
$= -2$

a value

x-values: $-2, 1, -1$

y-values: $-2, 3, -3, -2$

- -2: y-intercept
- 3, -3: slopes
- -2: value near $x = -1$



Ex $f(x) = \dfrac{(x+2)(x-2)}{x+3}$ Sketch graph

- zeros → $-2, 2$
- bad x → -3
- y-intercept → $f(0) = \dfrac{2(-2)}{3} = -\dfrac{4}{3}$
- LTB Need expand to do long division $\dfrac{x^2-4}{x+3}$ →

$$x+3 \;\overline{)\; x^2 + 0x - 4}$$

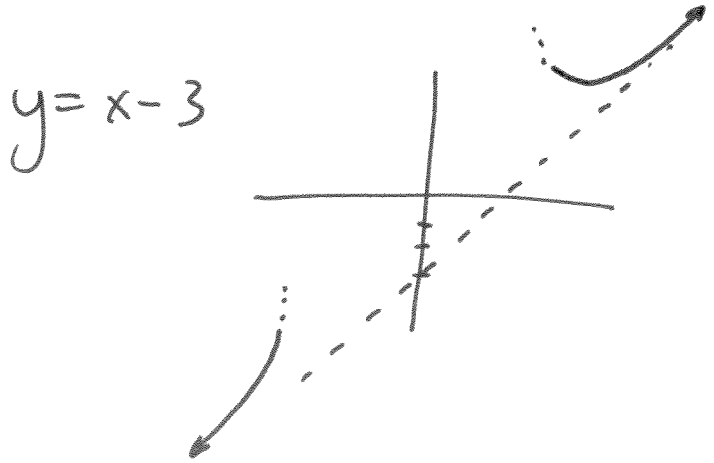
$x - 3$ → LTB! $y = x - 3$

$-(x^2 + 3x)$ ← any necessary place holders...!

$-3x - 4$

$-(-3x - 9)$

5 → remainder irrelevant



- local behavior

at -2 $\quad \dfrac{(x+2)(-2-2)}{-2+3} = -4(x+2)$

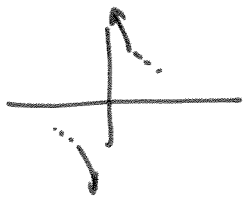
line slope -4

at 2 $\quad \dfrac{(2+2)(x-2)}{2+3} = \dfrac{4}{5}(x-2)$

line slope $\frac{4}{5}$

at -3 $\quad \dfrac{(-3+2)(-3-2)}{(x+3)} = 5 \cdot \dfrac{1}{x+3}$

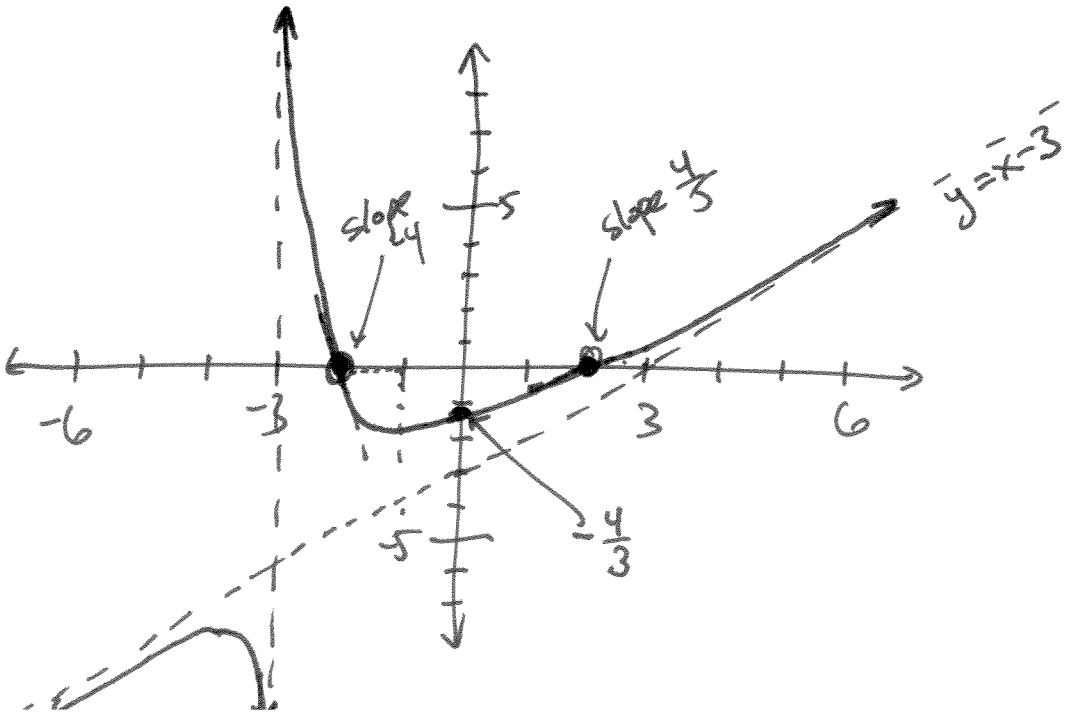
like $\frac{1}{x}$



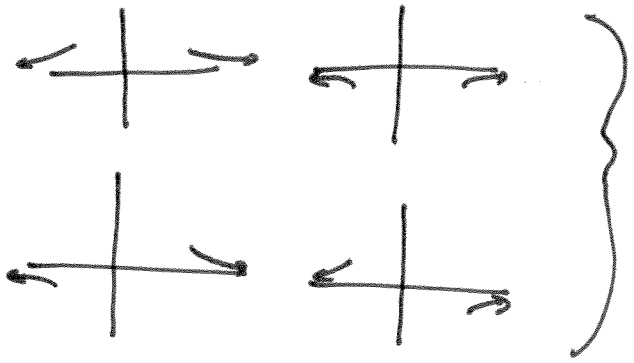
x-vals: $-2, 2, -3$

y-vals: $-\frac{4}{3}, -3, -4, \frac{4}{5}$

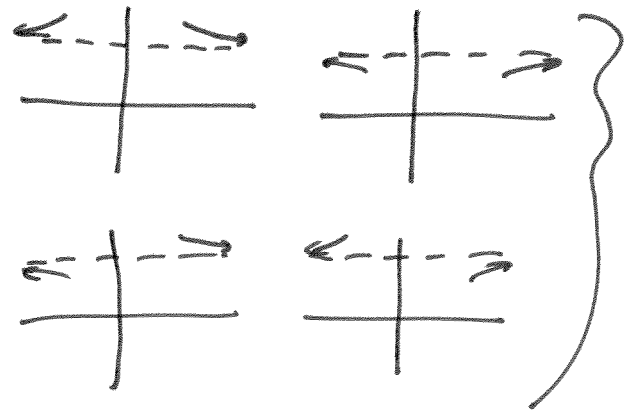
↑ y int ↑ LTB ↑ ↑ slopes



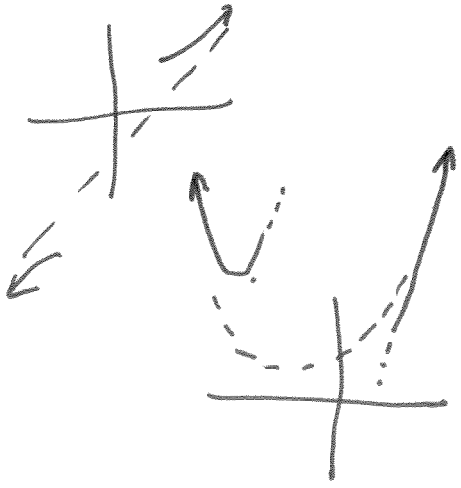
Notes on LTB for Rational Functions

Visually	equation for LTB	when?
	$y = 0$	deg (below) is greater than deg (above)

Ex $\dfrac{x^2-3x+2}{2x^3+8x-7}$

	$y =$ some # ↑ leading coef (top) / leading coef (bottom)	deg (below) equals deg (above)

Ex $\dfrac{2x^3+3x-7}{5x^2-8}$

	$y =$ ~~remainder~~ quotient from long division ↑	deg (below) is smaller than deg (above)

No shortcut!