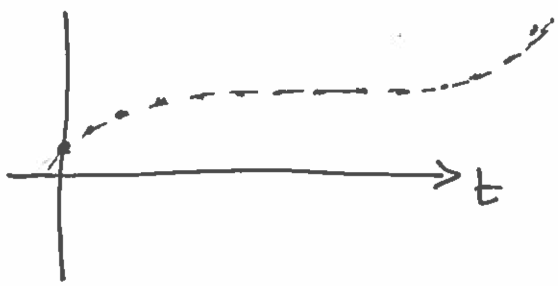
Section 3.1 Polynomial Functions

Motivational Example

Year	U.S. Natural Gas burned in a power plant (millions of ft^3)
1997	4,064,803
98	⋮
⋮	

graph



resembles transformed t^3 $\cancel{x^3}$ graph

$c\cdot(at+b)^3 + d$

multiply/expand this..

$$= \underbrace{\underbrace{A\cdot t^3} + \underbrace{Bt^2} + \underbrace{Ct} + \underbrace{D}}_{\text{a polynomial.}}$$

called **terms** (a piece added to other pieces)

terms have **coefficients** (the numerical part)

poly → multiple

nomial → "name" "noun" "thing"

Ex $2x + 5$

a **linear** polynomial

a **binomial**

degree 1 polynomial

leading term: $2x$

leading coefficient: 2

degree: the highest exponent on a variable

Ex $4x - 2x^2 + 1$

a quadratic polynomial

a trinomial

degree 2

leading term: $-2x^2$

leading coefficient -2

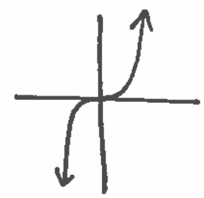
Not Polynomials		Are Polynomials
$\sqrt{x^2+2}$	(can't have variables inside radicals)	$x^2 + \sqrt{2}$
$\frac{1}{3x+8}$	(can't have variables in denominators)	$\frac{1}{3}x + \frac{1}{8}$
$5x^4 + 7x^{1.5}$	(~~can't~~ can only have 0, 1, 2, 3, 4, ... as exponents on variables)	$5x^4 + 7^{1.5}x$

Big Goal: Plot a polynomial by had.

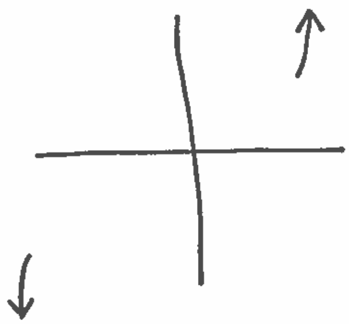
Ex Consider $f(x) = 2x^3 + 3x^2 - 4x + 7$

Obs: leading term $2x^3$

looks like



f will have same long term behavior as

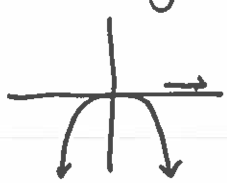


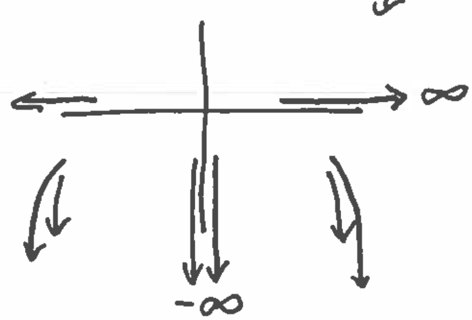
how the curve behaves on the far left/right ends...

Ex If $g(x) = 3x + 2x^3 - 8x^4$

what is g's Long Term Behavior.

$-8x^4$



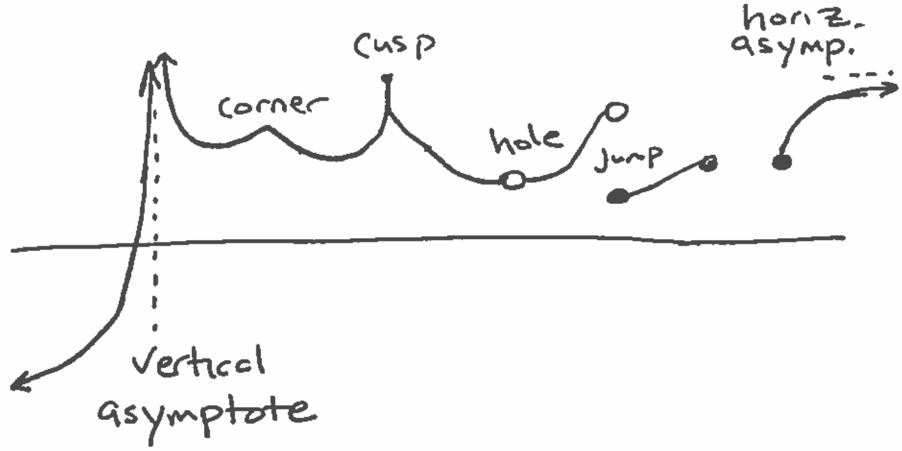


We write: $\lim\limits_{x \to \infty} g(x) = -\infty$

"The limit as x goes to ∞ of g(x) is $-\infty$."

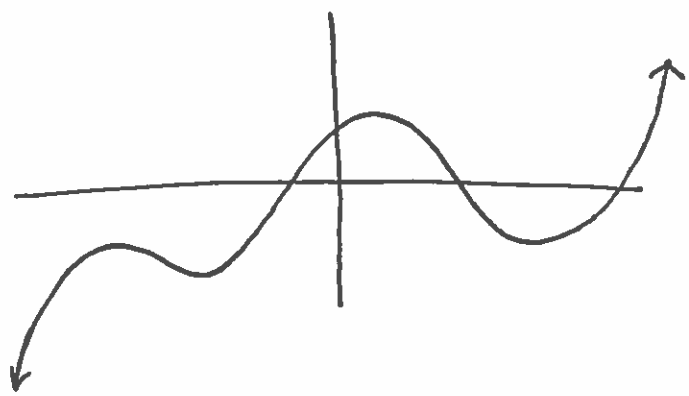
$\lim\limits_{x \to -\infty} g(x) = -\infty$

Things <u>not</u> to put in a polynomial graph



polynomial graph arms are predominantly pointing up/down

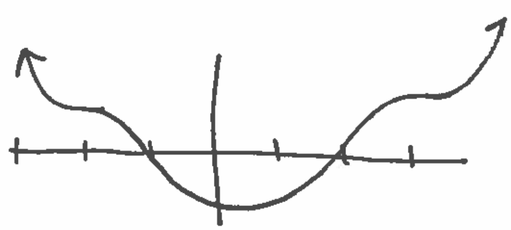
Good, typical polynomial



Next Feature : <u>Zeros</u>

a zero is an x-value that makes $f(x) = 0$

a.k.a. <u>roots</u>



has zeros at -1 and 2.

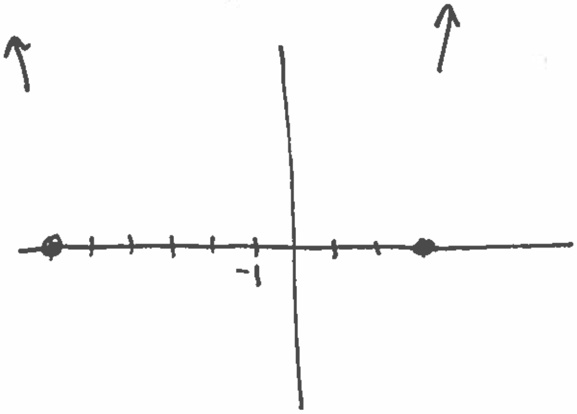
has roots at -1 and 2

has x-intercepts at $(-1,0)$ and $(2,0)$

LTB:

Ex $f(x) = x^2 + 3x - 18$

Find f's zeros.



Set $f(x) = 0$

$$x^2 + 3x - 18 = 0$$

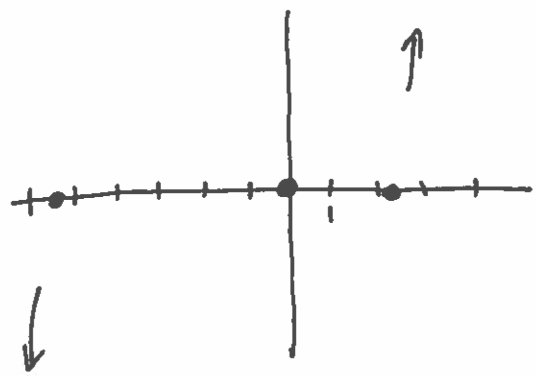
$$(x+6)(x-3) = 0$$

-6 $\quad$ 3

LTB:

Ex $g(x) = x^3 + 3x^2 - 12x$

Find g's zeros.



$$g(x) = 0$$

$$x^3 + 3x^2 - 12x = 0$$

$$x \cdot (x^2 + 3x - 12) = 0$$

$\Downarrow$

$x = 0$ or $x^2 + 3x - 12 = 0$

Quadratic Formula $\quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-3 \pm \sqrt{9 - 4(1)(-12)}}{2}$$

$x \approx 2.275\ldots$ or $x \approx {}^-5.275\ldots$

Ex $h(x) = x^4 - 3x^3 + x^2 - 1$

Find h's zeros...

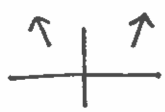
set $h(x) = 0$

$x^4 - 3x^3 + x^2 - 1 = 0$

Calculator!

Same place where MAX, MIN, has ZERO.

Ex Plot $y = f(x) = x^2 + 3x - 18$

- LTB: x^2  
- y-intercept: $f(0) = -18 \longrightarrow (0,-18)$
- x-intercepts: $x^2 + 3x - 18 = 0$

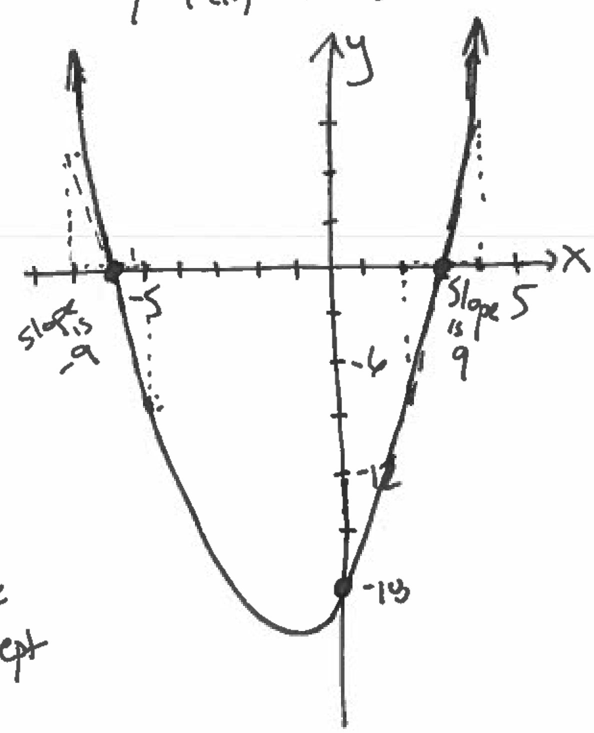
$$(x+6)(x-3) = 0$$

$$\downarrow \qquad \downarrow$$

$$-6 \qquad 3$$

- local behavior at each zero.

$y = f(x) = x^2 + 3x - 18$



At -6....

Factored: $\overbrace{(x+6)}(x-3)$

$(x+6)(-6-3)$

$-9(x+6)$

a linear function with slope -9 and x-intercept at -6.

At 3

$(x+6)(x-3)$

$(3+6)(x-3)$

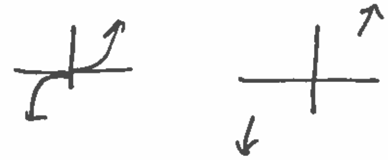
$9(x-3)$

line with slope 9 and x-intercept at 3

Ex $g(x) = 3(2x+1)(x-4)(x+2) = 6x^3 + \cdots$

Sketch g.

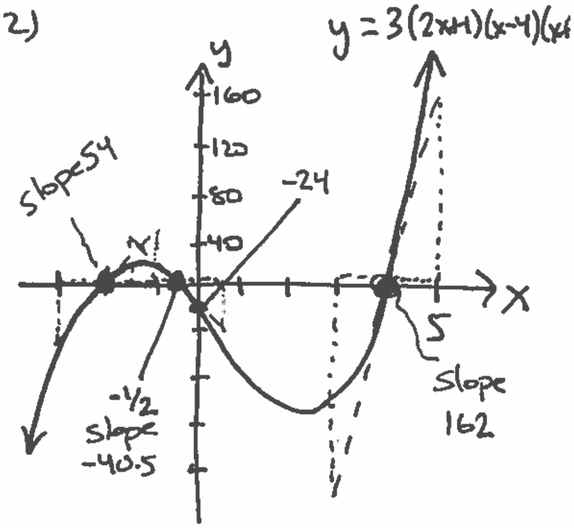
- Long Term Behavior: $6x^3$



- y-intercept: $g(0) = 3(2\cdot 0+1)(0-4)(0+2)$
$= 3(1)(-4)(2)$
$= -24$

- x-intercepts: $4, \ -2, \ -\frac{1}{2}$

$2x+1=0$
$2x=-1$
$x=-\frac{1}{2}$



- Local behavior at x-intercepts

At 4...

$3(2x+1)\overline{(x-4)}(x+2)$

$3(\ 9\ )(x-4)(\ 6\ )$

$162(x-4)$

a line with slope 162

At -2...

$3(2x+1)(x-4)\overline{(x+2)}$

$3(-3)(-6)(x+2)$

$54(x+2)$

a line with slope 54

At $x=-\frac{1}{2}$

$3\overline{(2x+1)}(x-4)(x+2)$

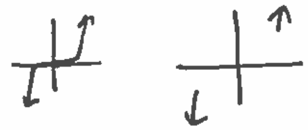
$3(2x+1)(-\frac{1}{2}-4)(-\frac{1}{2}+2)$

$\frac{-81}{4}\cdot(2x+1)$

a line with slope $\frac{-81}{4}\cdot 2 = -40.5$

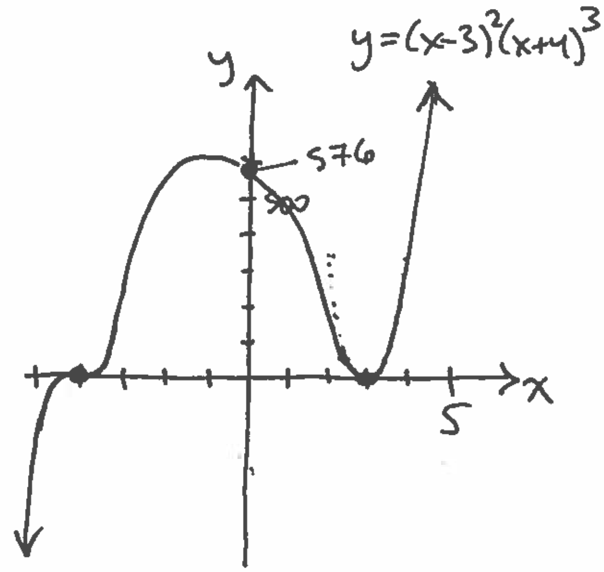
Ex $\ell(x) = (x-3)^2(x+4)^3 = x^5 + \cdots$

- L.T.B:



- y-int: $\ell(0) = (0-3)^2(0+4)^3$

$$= 9 \cdot 64$$
$$= 576$$



- x-int: $3, \; -4$

- local behavior....

At $x = 3$

$$\overbrace{(x-3)^2}(x+4)^3$$
$$(x-3)^2(3+4)^3$$
$$343 \cdot (x-3)^2$$

a parabola shape upward opening

At $x = -4$

$$(x-3)^2\overbrace{(x+4)^3}$$
$$(-4-3)^2(x+4)^3$$
$$49(x+4)^3$$

a serpentine shape