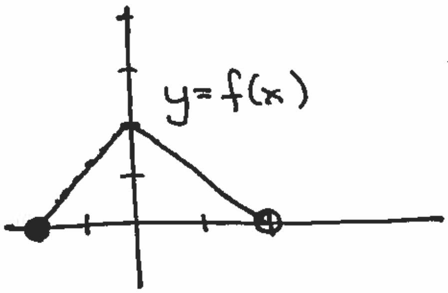
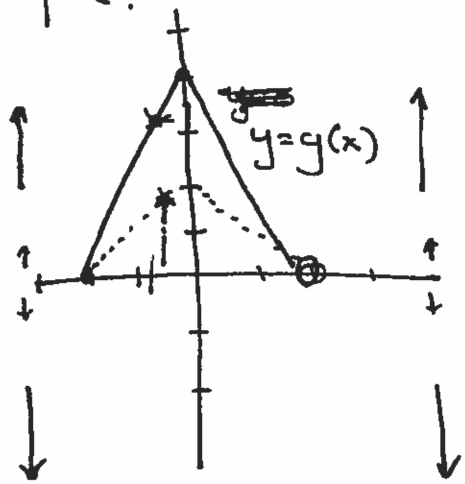
Scaling and Compressing

"Make bigger" — "Make smaller"

<u>Ex</u> Let f have this graph.



we decide to scale $\longrightarrow$ vertically by 2.



Task: Find an equation for the new graph.

- $\text{old}y = f(\text{old}x)$
- $\text{new}x = \text{old}x$
- $\text{new}y = 2\cdot\text{old}y$

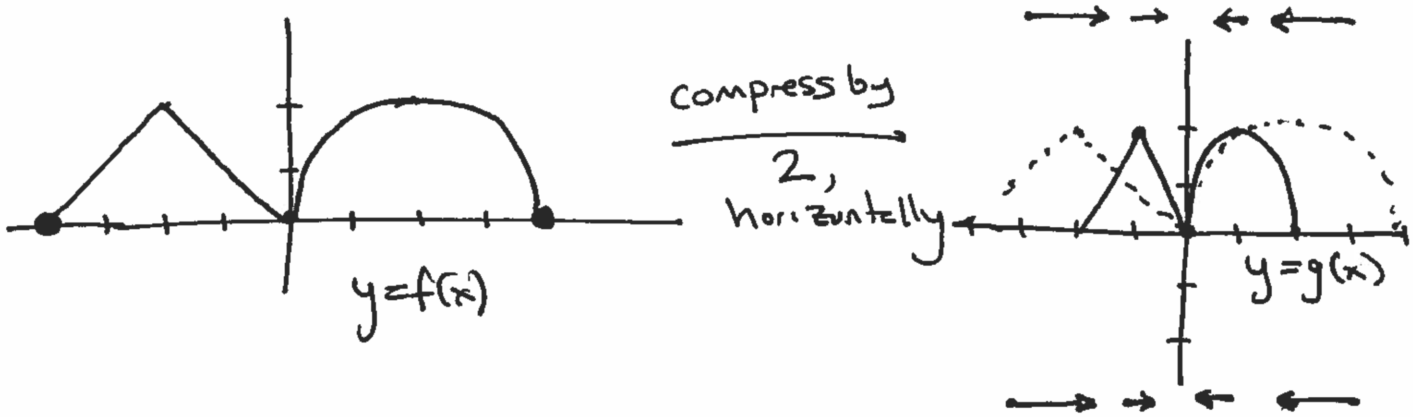
$\text{new}y = 2\cdot\text{old}y$
$\text{new}y = 2\cdot f(\text{old}x)$
$\text{new}y = 2\cdot f(\text{new}x)$

$\downarrow$ drop "new"

$y = 2f(x)$

So $g(x) = 2\cdot f(x)$

Ex Let f have graph



Task: Find a rule for g in terms of f

- $\text{old}\,y = f(\text{old}\,x)$
- $\text{new}\,x = \frac{1}{2}\text{old}\,x$
- $\text{new}\,y = \text{old}\,y$

$2\cdot\text{new}\,x = \text{old}\,x$

$\text{new}\,y = \text{old}\,y$

$\text{new}\,y = f(\text{old}\,x)$

$\text{new}\,y = f(2\cdot\text{new}\,x)$

↓ drop "new"

$y = f(2x)$

So $g(x) = f(2\cdot x)$

Function Transformations

(old x, old y)
(new x, new y)

Vertical change? (intuitive): altering outside of f's parentheses

shift ~~right~~ up by k	scale by c
$f(x)+k$	$c\cdot f(x)$

or

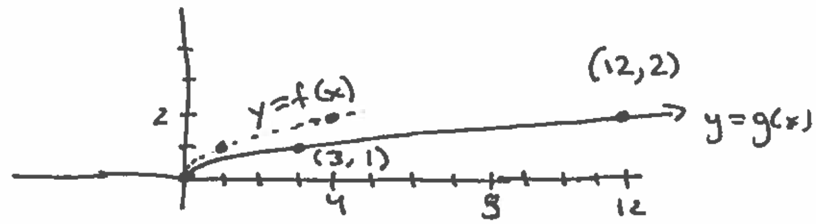
Horizontal change? (counter-intuitive): altering inside f's parentheses

shift by h	scale by c
$f(x-h)$	$f(x/c)$

Ex Take $f(x) = \sqrt{x}$ and stretch horizontally (away from y-axis) by a factor of 3 to get g.

Find a formula for g.

Visually:



- ~~$y = \sqrt{x}$~~

  $\text{old}y = \sqrt{\text{old}x}$

- $\text{new}x = 3 \cdot \text{old}x$

  $\frac{1}{3}\text{new}x = \text{old}x$

- $\text{new}y = \text{old}y$

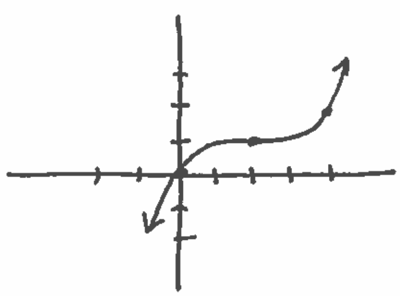
$\text{new}y = \text{old}y$
$\text{new}y = \sqrt{\text{old}x}$
$\text{new}y = \sqrt{\frac{1}{3}\text{new}x}$

$y = \sqrt{\frac{1}{3}x}$

$g(x) = \sqrt{\frac{1}{3}x}$

Note: 3 times longer horizontally, but $\frac{1}{3}$ shows up in the formula.

Ex Find a formula for $q(x)$:



Resembles $y = x^3$,
stretched away from y-axis by factor of 2,
shifted right 2,
shifted up 1

horizontal change affects input "x", and "counterintuitively"

again...

vertical change affects output, and "intuitively"

Using shortcuts

$y = x^3$
$y = \left(\frac{1}{2}x\right)^3$
$y = \left(\frac{1}{2}(x-2)\right)^3$
$y = \left(\frac{1}{2}(x-2)\right)^3 + 1$

So

$q(x) = \left(\frac{1}{2}(x-2)\right)^3 + 1$

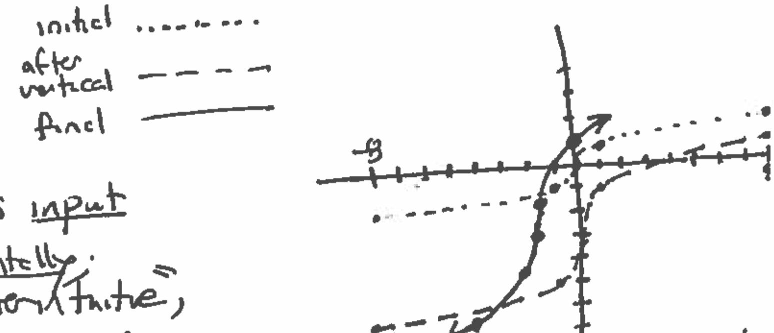
Ex Make a graph of f,

$f(x) = 2\sqrt[3]{4x+8} - 3$

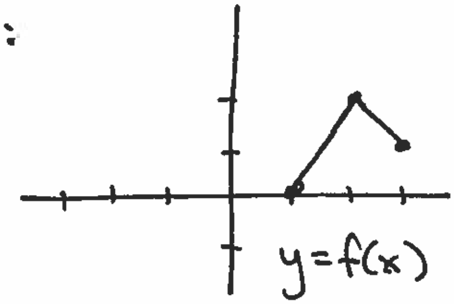
basically $\sqrt[3]{x}$

vertically stretch by 2, then shift down 3

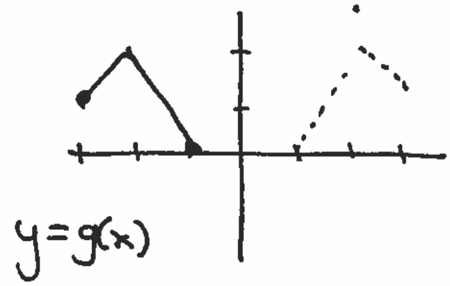
changes to $\sqrt[3]{\ }$'s input affect it horizontally: everything is "counterintuitive", including order of operations: first shift left 8, then compress horizont. by a factor of 4



Ex Take f:



Reflect over the y-axis:
(a horizontal change)



Find g's formula.

- $\text{old}y = f(\text{old}x)$
- $\text{new}x = -\text{old}x$
- $\text{new}y = \text{old}y$

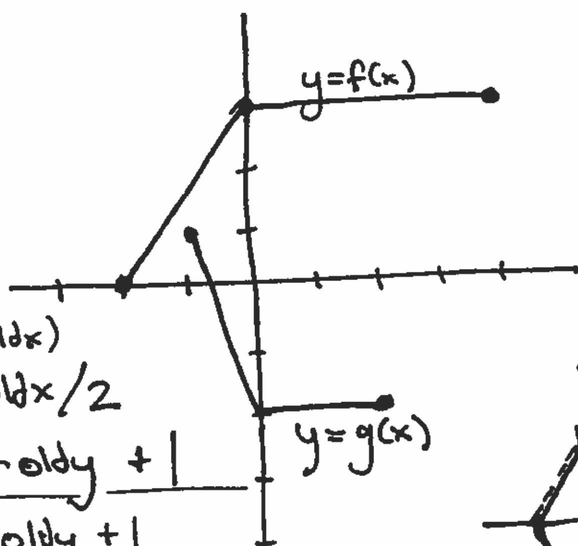
$-\text{new}x = \text{old}x$

$\text{new}y = \text{old}y$
$\text{new}y = f(\text{old}x)$
$\text{new}y = f(-\text{new}x)$

$\downarrow$

$y = f(-x)$

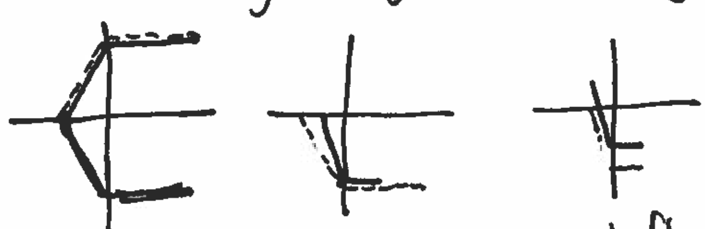
Ex



It appears f can transform into g--

Task: Find a rule for g in terms of f.

1) Catalog changes as $f \to g$.



flip over x-axis (vert.)

compressed horiz. by 2

shift up 1.

- $\text{old}y = f(\text{old}x)$
- $\text{new}x = \text{old}x/2$
- $\text{new}y = -\text{old}y + 1$

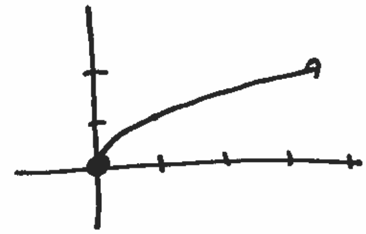
$$\begin{aligned}\text{new}y &= -\text{old}y + 1\\ &= -f(\text{old}x) + 1\\ &= -f(2\cdot\text{new}x) + 1\end{aligned}$$

$g(x) = -f(2x) + 1$

__Ex__ Plot

$$y = -\sqrt{\frac{x}{3} + 2}$$

$y = \sqrt{x}$



Let our old function be sqrt(x).

Let our new function be f(x).

$$\text{So...} \quad \overbrace{f(x)}^{\text{new } y} = -\underbrace{\text{sqrt}\Big(\underbrace{\tfrac{x}{3} + 2}_{}\Big)}_{}$$

new x

__Have to__ recognize which transformations...

old output
old y

gets fed to sqrt... our old function

$$\text{old}x = \frac{\text{new}x}{3} + 2$$

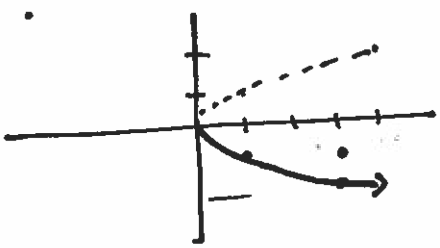
Not telling a story until you solve for new x

$$\text{old}x - 2 = \frac{\text{new}x}{3}$$

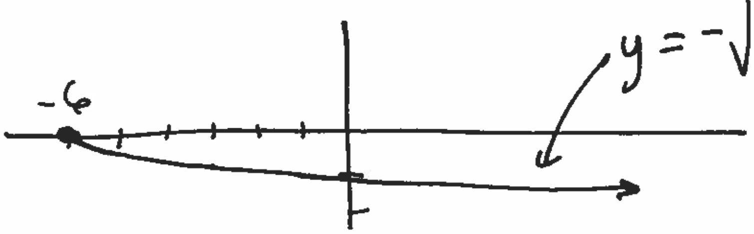
$$\text{new}x = 3 \cdot \text{old}x - 6$$

$$\text{new}y = -\,\text{old}y$$

reflected over the x-axis



x-values get tripled, then shifted left by 6.



$y = -\sqrt{\frac{x}{3} + 2}$

Ex $f(x) = \sqrt{x}$, shifted right 3, flipped over x-axis, scaled hor by 12.

Find the rule for this new function, g.

- $\text{old}y = \sqrt{\text{old}x}$
- $\text{new}x = 12 \cdot (\text{old}x + 3)$ ← tracking the actual changes as we went old function to new.
- $\text{new}y = -\text{old}y$

$$\frac{\text{new}x}{12} = \text{old}x + 3$$

$$\left(\frac{\text{new}x}{12} - 3\right) = \text{old}x$$

$\text{new}y = -\text{old}y$

$\text{new}y = -\sqrt{\text{old}x}$

$\text{new}y = -\sqrt{\frac{\text{new}x}{12} - 3}$ ← An "new"

$y = -\sqrt{x/12 - 3}$

$g(x) = -\sqrt{x/12 - 3}$

- shift right 3 $\quad f(x-3)$
- flip over x-axis $\quad -f(x)$
- scale hor. by 12 $\quad f(x/12)$

Apply to $\sqrt{x}$

$\sqrt{x}$

$\sqrt{x-3}$

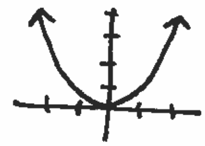
$-\sqrt{x-3}$

$-\sqrt{x/12-3}$

Ex Plot $y = -4(x-1)^2 + 5$

using graph transformations.

Recognize which basic function this comes from. $(\ )^2$



$f(x) = -4\boxed{(x-1)^2} + 5$

an input for f, the new function "x" = new x

old x = x − 1

old x = new x − 1

Solve for new x . . .

$\boxed{\text{new } x = \text{old } x + 1}$

Horizontal Story: Shifted 1 right.

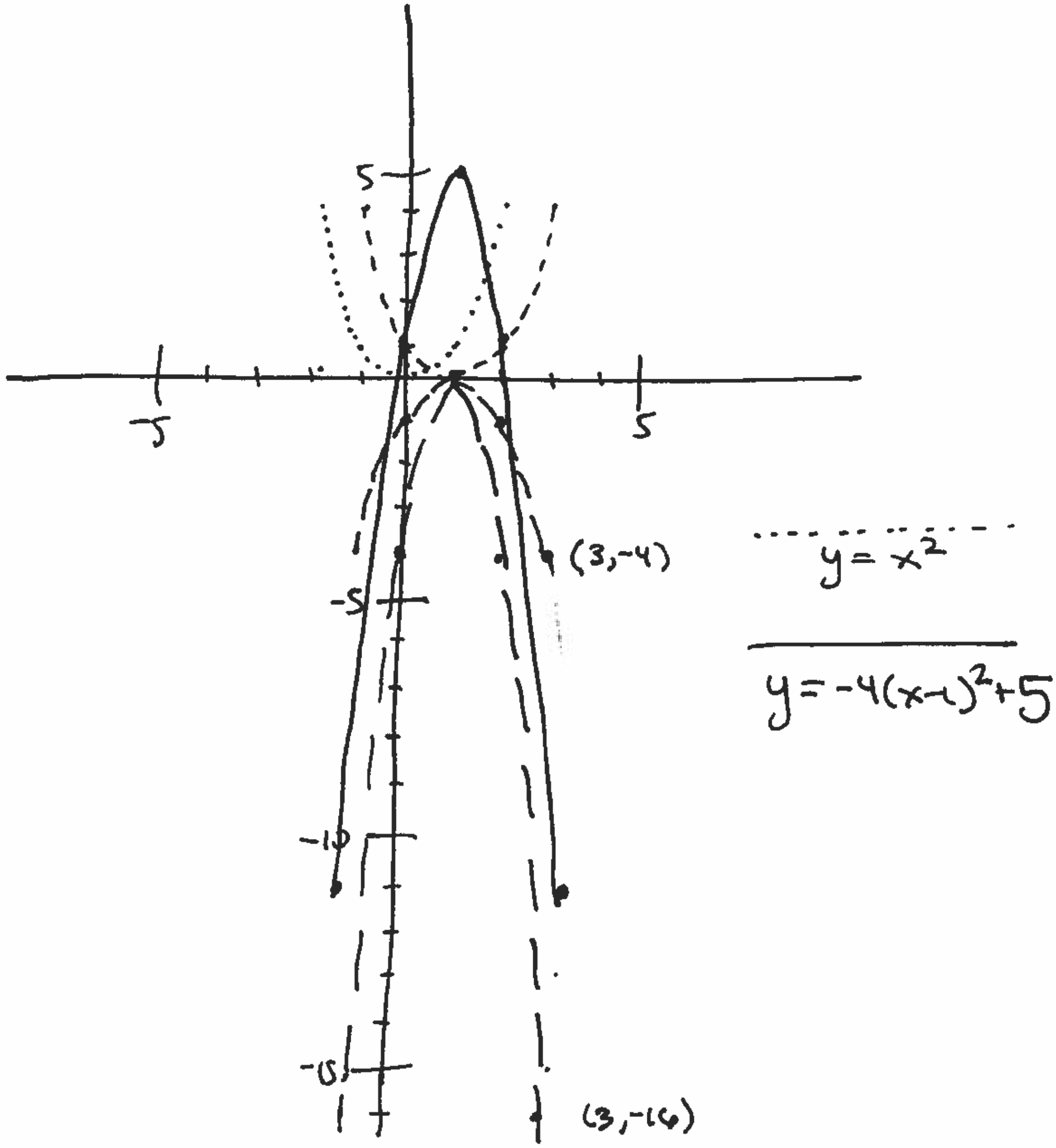
$(\ )^2$: old

$\boxed{f : \text{new}}$

$\boxed{\text{new } y = -4 \cdot \text{old } y + 5}$

Vertical story: Scaled by 4, flip over x-axis, shifted up 5.

away from x-axis.


5
-5
-10
-15
5
(3,-4)
(3,-16)
y = x^2
y = -4(x-1)^2+5

Ex Given:

x	f(x)
3	6
8	6
10	4
12	2

Let $g(x) = 2 \cdot f(-x)$

Find a table for g.

$$\underbrace{g(x)} = 2 \cdot \underbrace{f(\underbrace{-x})}$$

f: old

g: new

$\boxed{\text{new}y = 2 \cdot \text{old}y}$

vert stretch by 2

$\text{old}x = -\text{new}x$

$\boxed{\text{new}x = -\text{old}x}$

Hor flip

x	g(x)
-3	12
-8	12
-10	8
-12	4

Given a table for a function f...

x	f(x)
3	12
8	8
6	5
4	5

We are given ~~Define~~ g to be a new function

$$g(x) = \frac{1}{3} f(-2x+1) - 2$$

Task: Make a table for g.

f: old function
g: new function

$$\underbrace{g(x)}_{\text{new } y} = \underbrace{\frac{1}{3} \cdot f(\underbrace{-2x+1}_{\text{new } x}) - 2}$$

$$\text{new } y = \frac{1}{3} \cdot \text{old } y - 2$$

$$\text{old } x = -2 \cdot \text{new } x + 1$$

$$-1 \qquad -1$$

$$\frac{\text{old } x - 1}{-2} = \frac{-2 \text{ new } x}{-2}$$

$$-\frac{1}{2} \text{old } x + \frac{1}{2} = \text{new } x$$

$$\text{new } x = -\frac{1}{2} \text{old } x + \frac{1}{2}$$

	x	g(x)	
$-\frac{1}{2}(3)+\frac{1}{2}$	-1	2	$\frac{1}{3}(12)-2$
	-3.5	$2/3$	
	-2.5	$-1/3$	
$-\frac{1}{2}(4)+\frac{1}{2} =$	-1.5	$-1/3$	$\frac{1}{3}(5)-2$