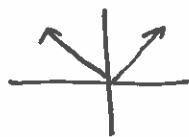


1.5 Function Transformations

Big Picture Ideas

Basic Function



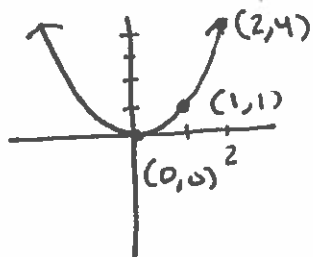
Transform



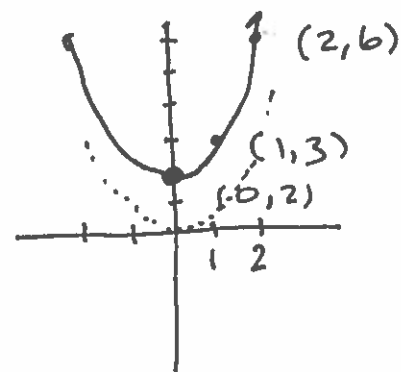
we hope to
give formula
for this...

- Take a complicated formula like
 $f(x) = 2\sqrt{\frac{x}{5}} - 3 + 7$
and be able to plot it "quickly".

Ex Take $f(x) = x^2$



shift up
2 units



this is $y = g(x)$

- (description of original curve)
 $y = x^2$
 $\text{old } y = (\text{old } x)^2$
- (how x-values changed)
 $\text{new } x = \text{old } x$
- (how y-values changed)
 $\text{new } y = \text{old } y + 2$

Write a formula for g ...

combine to get a "new" equation

$$\begin{aligned}\text{new } y &= \text{old } y + 2 \\ \text{new } y &= (\text{old } x)^2 + 2 \\ \text{new } y &= (\text{new } x)^2 + 2 \\ y &= x^2 + 2\end{aligned}$$

$$g(x) = x^2 + 2$$

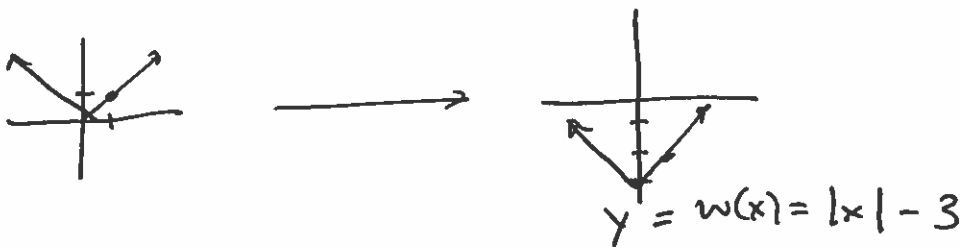
Shortcut... if you have f
and you want to shift up by k
to get a new function g ,

$$g(x) = f(x) + k$$

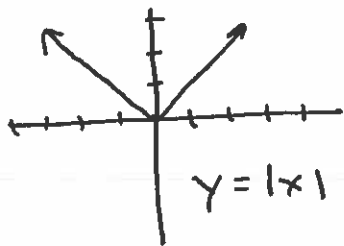
Ex Shift $y = |x|$ down 3 units...

Get a function w ... find its formula

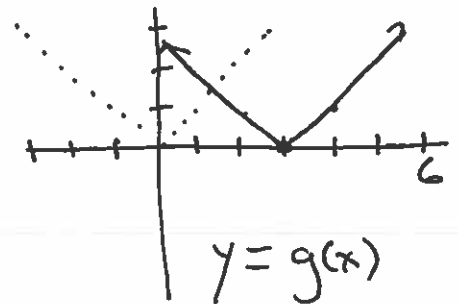
$$w(x) = |x| - 3$$



Ex Take $f(x) = |x|$



Shift right
by 3 units



Find a formula for g .

- $\text{old } y = |\text{old } x|$

- $\text{new } x = \text{old } x + 3$

- $\text{new } y = \text{old } y$

$\text{new } x - 3 = \text{old } x$

$\text{new } y = \text{old } y$

$\text{new } y = |\text{old } x|$

$\text{new } y = |\text{new } x - 3|$

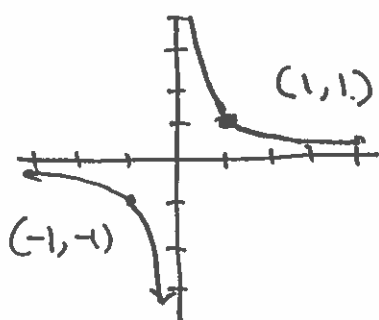
$y = |x - 3|$

$g(x) = |x - 3|$

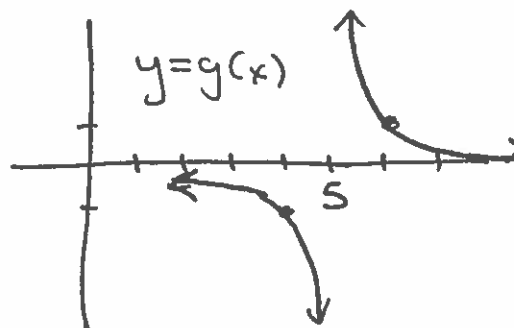
Shortcut: if you have f ...
and you want to shift right by h
to get a new function g ...

$$g(x) = f(x-h)$$

Ex $y = f(x) = \frac{1}{x}$

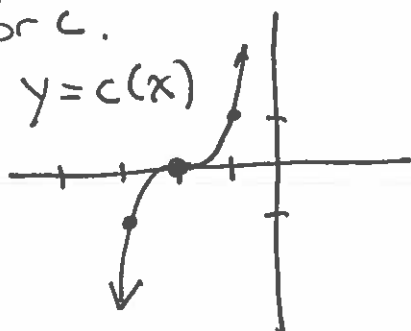


Define g by shifting
 f to the right by 5 units..



Find a formula for g .

Ex Find a formula
for c .



$$g(x) = f(x-5)$$

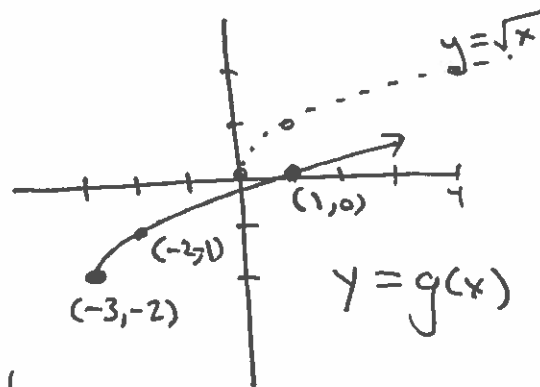
$$g(x) = \frac{1}{x-5}$$

Resembles $y = x^3$, shifted left 2
old function $f(x) = x^3$...

$$c(x) = f(x+2)$$

$$c(x) = (x+2)^3$$

Ex Take $f(x) = \sqrt{x}$ and make



Give a formula
for g ...

- $\text{old } y = \sqrt{\text{old } x}$

- $\text{new } x = \text{old } x - 3$

- $\text{new } y = \text{old } y - 2$

$\text{new } x + 3 = \text{old } x$

$\text{new } y = \text{old } y - 2$

$\text{new } y = \sqrt{\text{old } x} - 2$

$\text{new } y = \sqrt{\text{new } x + 3} - 2$

$y = \sqrt{x + 3} - 2$

$\Rightarrow g(x) = \sqrt{x + 3} - 2$

Check $g(-3) = \sqrt{-3 + 3} - 2 \checkmark$

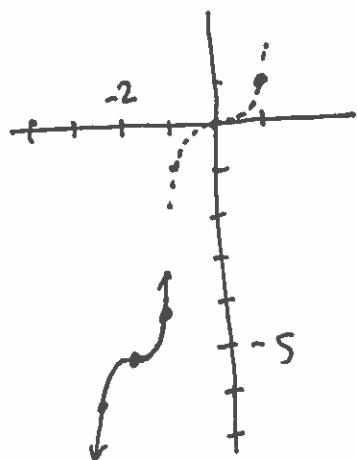
$g(-2) = \sqrt{-2 + 3} - 2$
 $= -1 \checkmark$

Ex Given

$f(x) = (x + 2)^3 - 5$

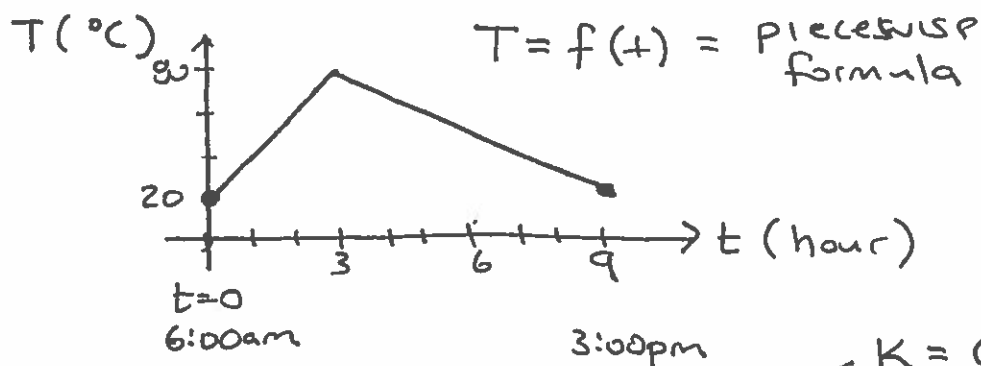
Can we plot this?

Looks like x^3 , shifted left 2,
down 5



we can graph "quickly"

Ex Work in a lab... record temperatures over time.



Boss says:

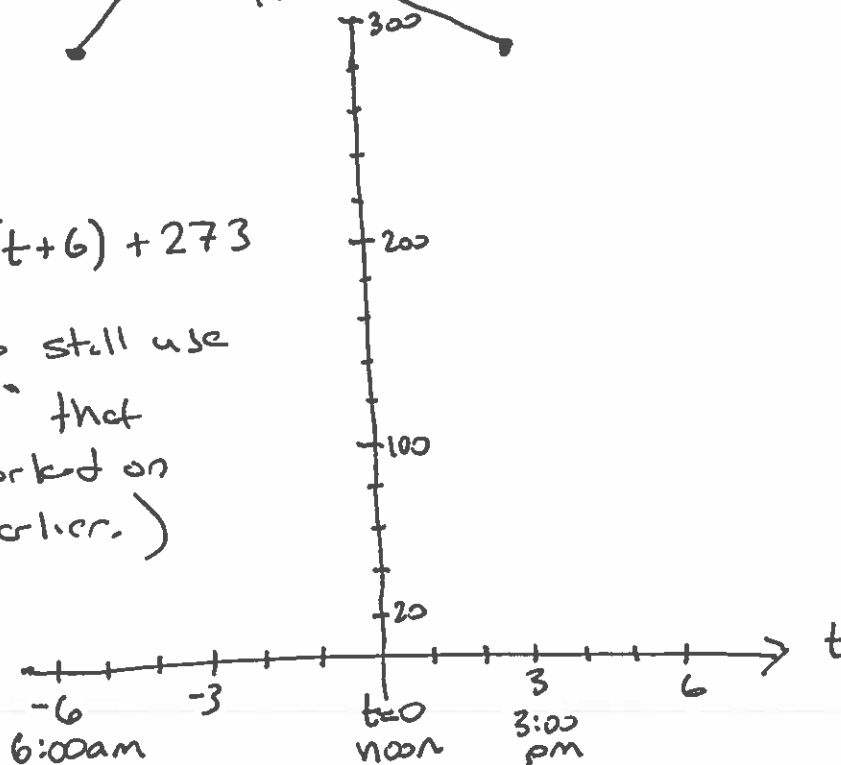
• we use Kelvin (not Celsius)

• the standard is $t=0 \leftrightarrow 12:00\text{pm}$ (noon)

$20^{\circ}\text{C} = 293\text{K}$

$T = f(t+6) + 273$

(Get to still use the "f" that we worked on earlier.)



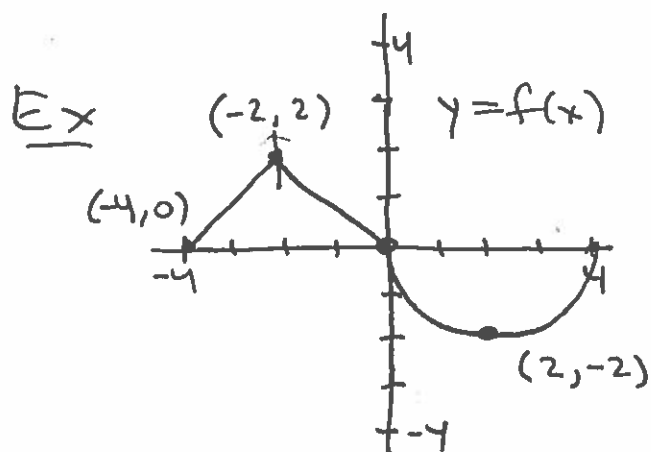
Scaling, stretching, compressing

↓ ↓ ↓

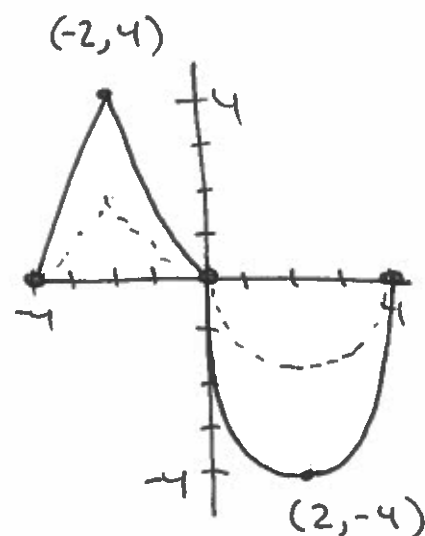
generic make make

word bigger smaller

for either →



Stretch
away from
x-axis
by a
factor of 2



Call this g .

Find a formula for g .
(in terms of f).

- $\text{old } y = f(\text{old } x)$

- $\text{new } x = \text{old } x$

- $\text{new } y = 2 \cdot \text{old } y$

$$\text{new } y = 2 \cdot \text{old } y$$

$$\text{new } y = 2 \cdot f(\text{old } x)$$

$$\text{new } y = 2f(\text{new } x)$$

$$y = 2 \cdot f(x)$$

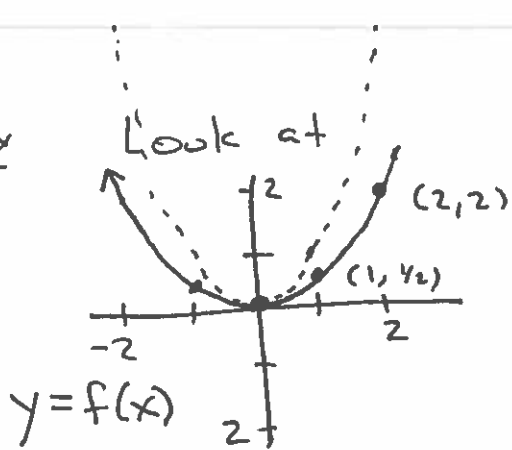
$$g(x) = 2 \cdot f(x)$$

Shortcut: take f and

stretch away from x-axis (a vertical change) by $c \dots$
to get $g \dots$

$$g(x) = c \cdot f(x)$$

Ex



Find a formula for f

Came from $y = x^2$,
compressed toward x-axis
by a factor of 2
(or $\frac{1}{2}$?)

$$f(x) = \frac{1}{2} \cdot x^2$$

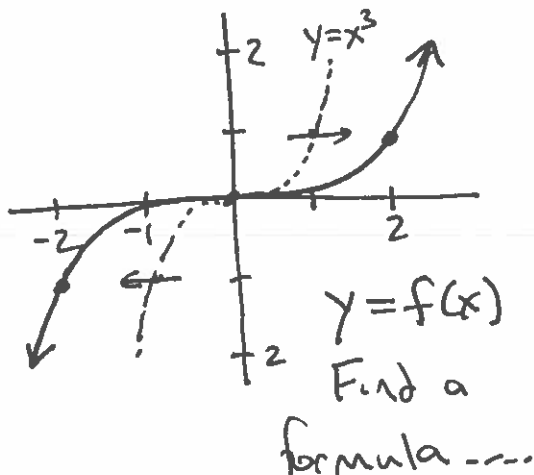
Shortcut: Given f , to compress toward x-axis
by a factor of c :

$$g(x) = \frac{1}{c} \cdot f(x)$$

some number < 1 .

$$\frac{\text{new } x}{2} = \text{old } x$$

Ex



$$y = \left(\frac{x}{2}\right)^3$$
$$f(x) = \left(\frac{x}{2}\right)^3$$

- old $y = (\text{old } x)^3$
- $\text{new } x = 2 \cdot \text{old } x$
- $\text{new } y = \text{old } y$

$$\text{new } y = \text{old } y$$
$$\text{new } y = (\text{old } x)^3$$
$$\text{new } y = \left(\frac{\text{new } x}{2}\right)^3$$

Shortcut: Start with f , stretch horizontally by $c \dots$ to get $g \dots$

$$g(x) = f\left(\frac{x}{c}\right)$$

Start with f , compress horizontally by c to get $g \dots$

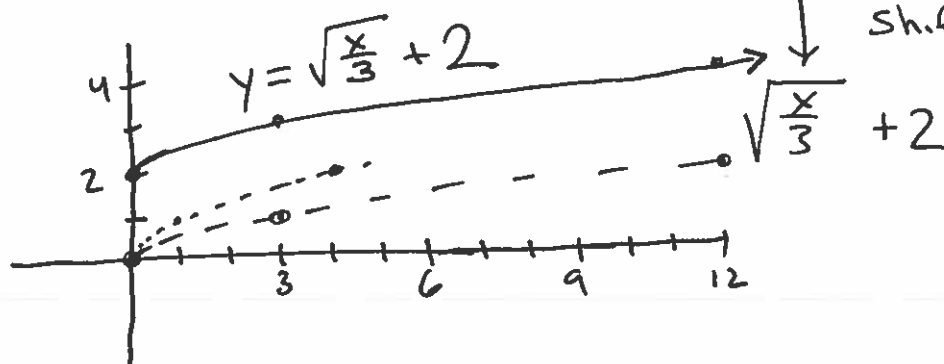
$$g(x) = f(c \cdot x)$$

some number > 1 .

Ex Suppose you have

$$f(x) = \sqrt{\frac{x}{3}} + 2$$

plot this...



$$\sqrt{x}$$



stretch hor by 3

$$\sqrt{\frac{x}{3}}$$



shift -p by 2

$$\sqrt{\frac{x}{3}} + 2$$

Ex $f(x) = \frac{1}{(2x-3)^2} + 2$

$$\frac{1}{x^2}$$

↓ shift right 3

$$\frac{1}{(x-3)^2}$$

↓ comp. toward y-axis
by a factor of 2

$$\frac{1}{(2x-3)^2}$$

↓ shift up 2

$$\frac{1}{(2x-3)^2} + 2$$

