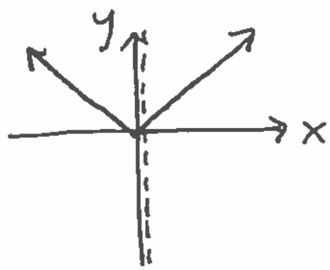
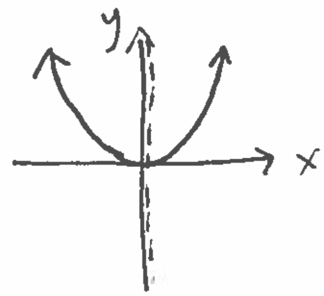
Section 1.3

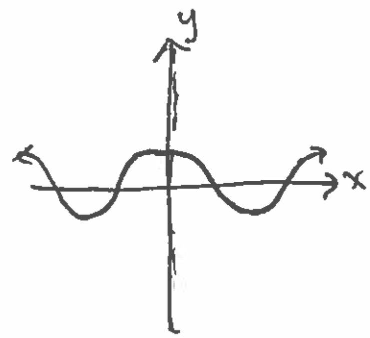
Symmetric Functions



$y = f(x) = |x|$



$y = g(x) = x^2$



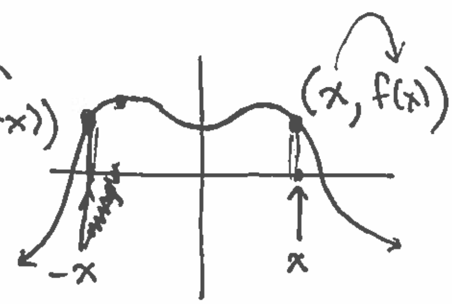
$y = h(x) = \cos(x)$

These have "y-axis symmetry"

We call these **even** functions...

Algebraically...

Generic Even function



$y = f(x)$

$$\boxed{f(-x) = f(x)}$$

Two y-values should be equal

definition of f being <u>even</u>.

<u>Ex</u> $g(x) = x^2$

Is g even?

well $g(-x) = (-x)^2$

$= x^2$

$g(-x) = g(x)$

So <u>g is even.</u>

Ex $k(x) = 12x^3 + 4x^2 - 18$

Is k even? (wondering whether $k(-x) \overset{?}{=} k(x)$)

Well, $k(-x) = 12(-x)^3 + 4(-x)^2 - 18$

$= -12 \cdot x^3 + 4 \cdot x^2 - 18$ same as $k(x)$?

It appears $k(-x) \neq k(x)$.

So k is not even.

Ex Is $f(x) = \dfrac{x^3 - 6x^5}{\sqrt[3]{x^3 + x^7}}$ even?

Well $f(-x) = \dfrac{(-x)^3 - 6(-x)^5}{\sqrt[3]{(-x)^3 + (-x)^7}}$

$= \dfrac{-x^3 + 6x^5}{\sqrt[3]{-x^3 - x^7}}$ doesn't look like original...

$= \dfrac{-x^3 + 6x^5}{\sqrt[3]{-(x^3 + x^7)}}$

$\sqrt[3]{8} = 2$

$\sqrt[3]{-8} = -2$

$\Downarrow$

$\sqrt[3]{-8} = -\sqrt[3]{8}$

$= \dfrac{-x^3 + 6x^5}{-\sqrt[3]{x^3 + x^7}}$

$= \dfrac{-(x^3 - 6x^5)}{-\sqrt[3]{x^3 + x^7}}$

$\dfrac{x^3 - 6x^5}{\sqrt[3]{x^3 + x^7}}$

$f(-x) = f(x)$

$\Downarrow$

f is even.

Ex Let $p(x) = \dfrac{x^3 - x^2}{x+1}$ Is p even?

Well, $p(-x) = \dfrac{(-x)^3 - (-x)^2}{-x + 1}$

$$= \frac{-x^3 - x^2}{-x+1}$$

are these equal?

Does $\dfrac{-x^3 - x^2}{-x+1} = \dfrac{x^3 - x^2}{x+1}$?

Unsure... plug in x!

$x = 2 \Longrightarrow$

$$\frac{-8-4}{-2+1} \overset{?}{=} \frac{8-4}{2+1}$$

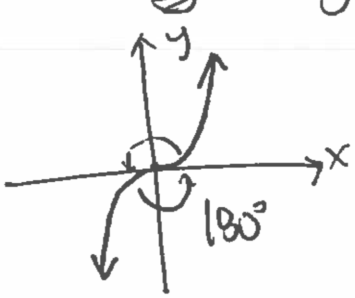
$$12 \overset{?}{=} 4/3 \text{ No!}$$

$p(-x) \neq p(x)$

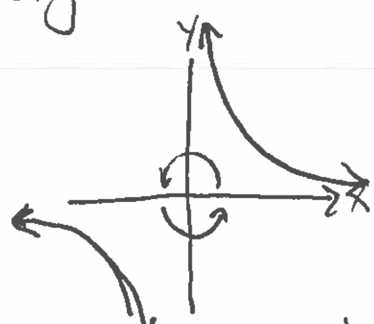
$\Downarrow$

p is not even.

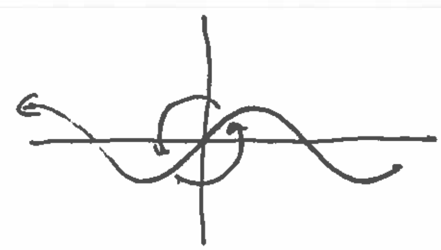
Another kind of symmetry:



$y = f(x) = x^3$



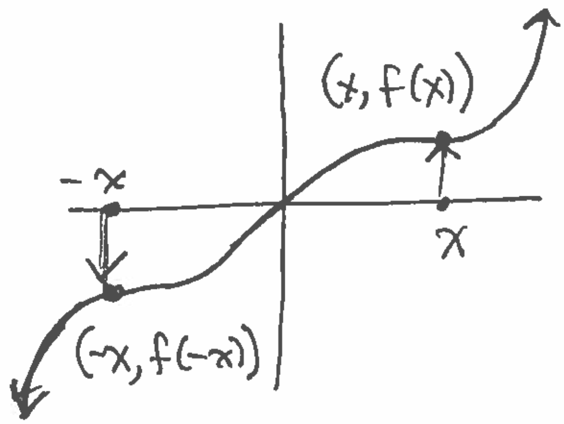
$y = g(x) = \dfrac{1}{x}$



$y = h(x) = \sin(x)$

These are odd functions.

Generic odd function:



$y = f(x)$

$$\boxed{f(-x) = -f(x)}$$

definition of <u>f being odd</u>.

two y-values are negatives!

<u>Ex</u> $g(x) = \frac{1}{x}$. Is g odd?

Well, $g(-x) = \frac{1}{-x}$

$= -\frac{1}{x}$

$g(-x) = -g(x)$ ✓ $\Longrightarrow$ g <u>is odd</u>.

<u>Ex</u> $q(x) = 5x^3 - \frac{1}{x} + \sqrt[3]{2x}$

Is q even, odd, or neither?

Well $q(-x) = 5(-x)^3 - \frac{1}{-x} + \sqrt[3]{2(-x)}$

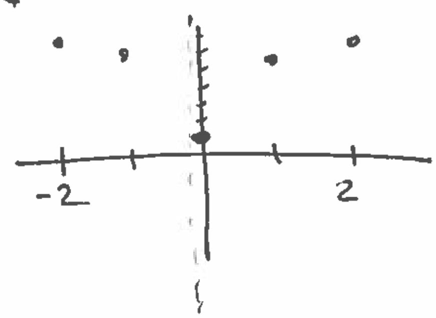
$= -5x^3 + \frac{1}{x} - \sqrt[3]{2x}$

$= -\left(5x^3 - \frac{1}{x} + \sqrt[3]{2x}\right)$

$q(-x) = -q(x)$ $\Longrightarrow$ q <u>is odd</u>.

Ex

x	$f(x)$
-2	7
-1	6
0	1
1	6
2	7



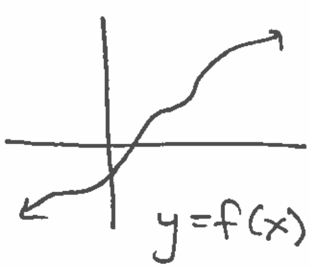
Is f even, odd, or neither?

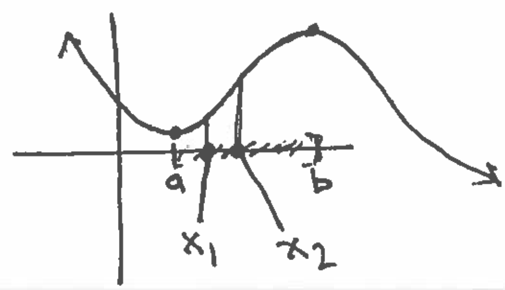
$f(-2) = f(2)$ ✓

$f(-1) = f(1)$ ✓

$\Longrightarrow f$ even

Increasing / Decreasing

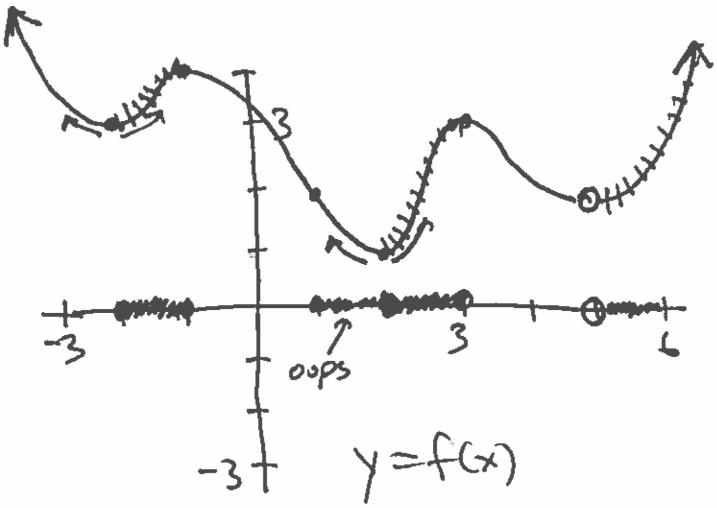




Intuitively, "f is increasing"

To say "f is increasing on $[a,b]$" means whenever $x_1 < x_2$ within $[a,b]$, then $f(x_1) < f(x_2)$

Ex



Where is f increasing?

$[-2,-1] \cup [2,3] \cup (5,\infty)$

Where is f decreasing?

$(-\infty,-2] \cup [-1,2] \cup [3,5)$

There are local minima at -2 and at 2.
(with value 3) (with value 1)

There are local maxima at -1 and 3
(with value 4) (with value 3)

There are absolute minima at 2 (with value 1)

There are no absolute maxima.

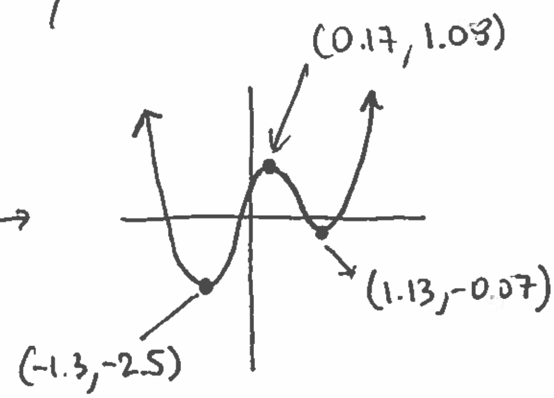
Ex $f(x) = x^4 - 3x^2 + x + 1$

Find where f is increasing / decreasing.

Find f's local / absolute minima / maxima.

aka relative — aka global

Use graphing calculator →



Inc on $[-1.3, 0.17] \cup [1.13, \infty)$

Dec on $(-\infty, -1.3] \cup [0.17, 1.13]$