1.1 continued

Familiar with "2 + 5"

combine two numbers — result 7, another number

Now we look at "f + g".

Adding functions to get a new function.

New function's name: $f+g$

What $f+g$ does to numbers: $\underbrace{(f+g)}_{\text{adding functions}}(x) = \underbrace{f(x)}_{\text{number}} + \underbrace{g(x)}_{\text{number}}$ — add numbers

Also, we have $f-g$, fg, f/g.

For ex: $(fg)(x) = f(x)\cdot g(x)$

Ex Factory makes toy cars...

To make x cars... costs $L(x)$ (dollars) in labor.

Also, let $P(x)$ be the cost of materials for making x cars.

Look at $\underbrace{(L+P)}_{\text{the function that gives total cost}}(x) = L(x) + P(x)$

Ex At PCC, if x students plan to take MTH 111, let $n(x)$ be the number of sections offered

$$(n(582) = 20)$$

Also let $p(x)$ be the price of one new book in the bookstore. $(p(582) = 138)$

Library wants one copy per section. Not free. If x students take 111, what does this cost?

$$\underbrace{(np)}_{\text{total cost function...}}(x) = n(x)\cdot p(x)$$

$$(np)(582) = n(582)\cdot p(582)$$
$$= 20 \cdot 138$$
$$= 2760$$

Ex $f(x) = x+1$
$g(x) = x^2 + x + 5$

Find a formula for $f+g$.

$$(f+g)(x) = f(x) + g(x)$$
$$= x+1 + x^2+x+5$$
$$= x^2 + 2x + 6$$

Find a formula for fg.

$$(fg)(x) = f(x)\cdot g(x)$$
$$= (x+1)(x^2+x+5)$$
$$= x^3+x^2+5x+x^2+x+5$$
$$= x^3+2x^2+6x+5$$

Ex $f(x) = x + \frac{1}{x}$

$g(x) = x - \frac{1}{x}$

Find a formula for $f+g$:

$(f+g)(x) = f(x) + g(x)$

$= x + \frac{1}{x} + x - \frac{1}{x}$

$(f+g)(x) = 2x$

Find $f+g$'s domain.

It appears $f+g$'s domain is $\cancel{(-\infty, \infty)}$.

Actually,

$(f+g)(x) = f(x) + g(x)$

to be good... have to be good...

Here, 0 is bad for f (and g) so 0 can't be a good input for $f+g$.

So $f+g$'s domain: $(-\infty, 0) \cup (0, \infty)$

$\boxed{\text{domain of } f+g \text{ is: } (\text{dom of } f) \cap (\text{dom of } g)}$

intersection symbol

Ex $h(x) = \sqrt{x-2}$ and $k(x) = \frac{1}{x-4}$

Find the domain of hk.

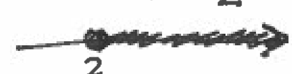
$(hk)(x) = h(x) \cdot k(x)$

looking for valid input...

got to have valid input for both

h's domain: need $x - 2 \geq 0$

$x \geq 2$



$[2, \infty)$

k's domain: need $x \neq 4$

$(-\infty, 4) \cup (4, \infty)$

hk's domain:

$[2, 4) \cup (4, \infty)$

Ex $\quad p(x) = \dfrac{1}{x+7} \qquad q(x) = (x+7)^2$

Find a formula for pq.

$$(pq)(x) = p(x)\cdot q(x)$$
$$= \frac{1}{x+7}(x+7)^2$$
$$= x+7$$

$\Downarrow$

So $(pq)(x) = x+7, \quad x \neq -7$

[$(pq)(-7)$ is undefined]

Find pq's domain.

appears to be ~~$(-\infty, \infty)$~~

p's dom:

q's dom:

pq's dom

$(-\infty,-7)\cup(-7,\infty)$

Ex $\quad f(x) = \dfrac{x-2}{x+1} \qquad g(x) = \dfrac{x-2}{x+2}$

Find a formula for f/g.

$$(f/g)(x) = \frac{f(x)}{g(x)}$$
$$= \frac{\frac{x-2}{x+1}}{\frac{x-2}{x+2}}$$
$$= \frac{x-2}{x+1}\cdot\frac{x+2}{x-2}$$
$$= \frac{x+2}{x+1}$$

[looks like f/g's domain is $x \neq -1$]

Find f/g's domain.

$$(f/g)(x) = \frac{f(x)}{g(x)}$$

- x must be valid for f.
- x must be valid for g.
- x cannot be something where $g(x) = 0$.

worried about
$$\frac{x-2}{x+2} = 0$$
$$\Rightarrow x-2 = 0$$
$$\Rightarrow x = 2$$

$x \neq -1$

$x \neq -2$

$x \neq 2$

$(-\infty,-2)\cup(-2,-1)\cup(-1,2)\cup(2,\infty)$

Domain of...

$f+g$, $f-g$, fg $\Big\}\longrightarrow$ exclude bad input for f
exclude bad input for g
((domain of f) $\cap$ (domain of g))

f/g $\Big\}\longrightarrow$ same as above, but exclude any x that make $g(x)=0$.
((domain of f) $\cap$ (domain of g) $\cap$ $\{x \mid g(x)\neq 0\}$)

<u>Ex</u>

$f(x) = x^2 - 3x$
$f(\ ) = (\ )^2 - 3(\ )$
$f(z) = z^2 - 3z$
$f(\Delta) = \Delta^2 - 3\Delta$

"x" is not important

all say the same thing.

So... simplify $f(-x)$....

$f(\ ) = (\ )^2 - 3(\ )$
$f(-x) = (-x)^2 - 3(-x)$
$f(-x) = x^2 + 3x$

And... $f(2x)$

$f(\ ) = (\ )^2 - 3(\ )$
$f(2x) = (2x)^2 - 3(2x)$
$f(2x) = 4x^2 - 6x$

And still using $f(x) = x^2 - 3x \ldots$

Simplify $f(\underbrace{x+2}_{\text{input}}) \ldots$

$$f(\ ) = (\ )^2 - 3(\ )$$
$$f(x+2) = (x+2)^2 - 3(x+2)$$
$$f(x+2) = x^2 + 4x + 4 - 3x - 6$$
$$f(x+2) = x^2 + x - 2$$

The Difference Quotient

Cumbersome algebra expression; important in calculus

Here, we get a bit more comfortable...

If ~~f~~ f is a function, f's difference quotient is

$$\frac{f(x+h) - f(x)}{h}$$

Ex ~~$f(x) = x^2 - 3$~~ $g(x) = 2x+5$

Find, simplify g's difference quotient.

$$\frac{g(x+h) - g(x)}{h} = \frac{2x + 2h + 5 - (2x+5)}{h}$$

$$= \frac{\cancel{2x} + 2h + \cancel{5} - \cancel{2x} - \cancel{5}}{h} = \frac{2h}{h}$$

$$= 2$$

$g(x+h) = ?$

$g(\ ) = 2(\ ) + 5$

$g(x+h) = 2(x+h) + 5$

$= 2x + 2h + 5$

Ex $f(x) = x^2 - 3x$ Find, simplify its diff. quot.

$$\frac{\overbrace{f(x+h)} - f(x)}{h} = \frac{\overbrace{\cancel{x^2} + 2xh + h^2 - \cancel{3x} - 3h} - (\cancel{x^2} - \cancel{3x})}{h}$$

$$= \frac{2xh + h^2 - 3h}{h}$$

$$= \frac{\cancel{h}(2x + h - 3)}{\cancel{h}}$$

$$= 2x + h - 3$$

$f(x+h)$

$f(\ \ ) = (\ \ )^2 - 3(\ \ )$

$f(x+h) = (x+h)^2 - 3(x+h)$

$= x^2 + 2xh + h^2 - 3x - 3h$

Ex $k(x) = \frac{1}{x+3}$ Find, simplify k's diff quot.

$$\frac{\overbrace{k(x+h)} - k(x)}{h} = \frac{\overbrace{\frac{1}{x+h+3}} - \frac{1}{x+3}}{h}$$

$k(\ ) = \frac{1}{(\ )+3}$

$k(x+h) = \frac{1}{x+h+3}$

$$= \frac{\frac{1 \cdot (x+3)}{(x+h+3)(x+3)} - \frac{(x+h+3)}{(x+h+3)(x+3)}}{h}$$

$$= \frac{\frac{-h}{(x+h+3)(x+3)}}{h}$$

$$= \frac{-h}{(x+h+3)(x+3)} \cdot \frac{1}{h} = \frac{-1}{(x+h+3)(x+3)}$$