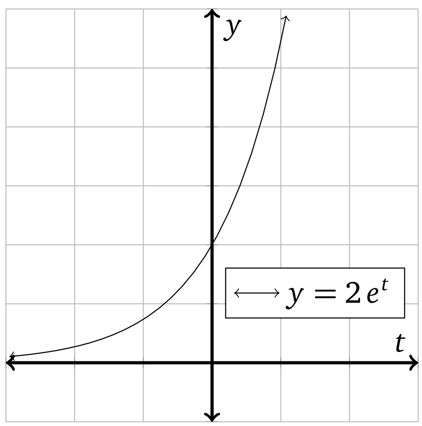
Introduction to the number e and the natural logarithm ln

In these exercises, we will familiarize ourselves more with e and what it means to have e as the growth factor in an exponential function. Also we will use the logarithm with base e, which is usually denoted ln.

1. Here is the graph of f, where $f(t) = 2\,e^t$.



   (a) What is the initial value of this function?

   (b) Since the function has its growth factor $b = e$, what is the initial slope?

   (c) Sketch a straight line passing through the graph's y-intercept that has the function's initial slope. Make sure that your sketch agrees with your answers to 1a and 1b.

2. How many digits of e have you memorized? Spend a minute studying them. You can see them if you use your calculator to show you e (you may need to enter `e^1`). Later in this worksheet you will be asked to see how many you have memorized.

3. The heat energy in a chemical reaction is growing exponentially: $E(t) = a\,b^t$. Initially, there is 2300 J of heat energy. Also, for the first second of the reaction, the heat energy is growing by 2300 $\frac{\text{J}}{\text{h}}$.

   (a) Give a formula for $E(t)$, using the observation that the initial energy *amount* is equal to the initial *rate of change* (per hour).

   (b) Use your answer from 3a to give the growth factor per unit time (as an exact value) and the relative growth rate per unit time (as a rounded percentage).

   (c) How long will it take until there is 5000 J of heat energy?

4. Simplify/expand each of the following expressions, using the rules of logarithms and the rules of exponents. Do not use a calculator.

   (a) $\ln\left(e^5\right)$
   (b) $e^{\frac{1}{2}\ln 25}$
   (c) $\ln\left(\frac{10}{e}\right)+\ln\left(\frac{e^2}{10}\right)$

5. Solve the equations using ln or e somehow. Find *exact* answers without the help of a calculator. Then *also* use your calculator to find decimal approximations. Get practice using your calculator's `ln` command rather than any solving or graphing tool. Always check your solutions, since sometimes these equations can lead to extraneous solutions.

   (a) $12^t = 40$
   (c) $\ln(x-3) = \frac{1}{3}$
   (b) $5(3)^{2x-7} = 12\left(\frac{1}{2}\right)^{x+2}$
   (d) $\ln(x+3)+\ln(x-3) = 2\ln(3x-11)$

6. Without using any reference, how many digits of e do you remember?

7. Pick a large value for n—like one million or more. Have your calculator compute $\left(1+\frac{1}{n}\right)^n$. What is the result?

8. Remember what $n!$ means? As an example, $5! = 5\cdot 4\cdot 3\cdot 2\cdot 1 = 120$. Have your calculator compute $\frac{1}{0!}+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\cdots+\frac{1}{9!}$. (You can find a ! button on your calculator in the Math menu if it is a TI calculator.) What is the result?