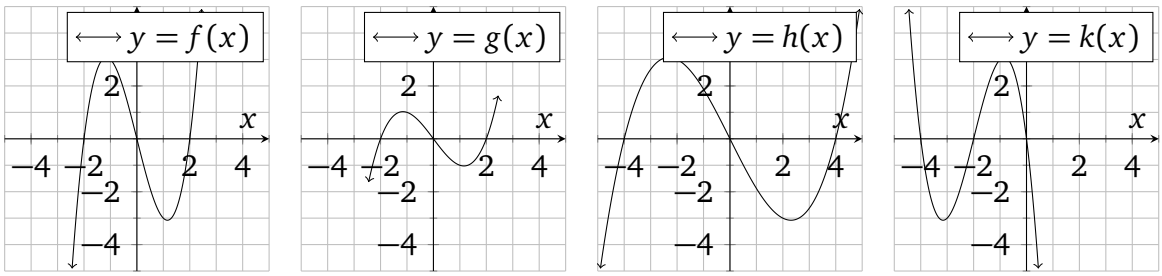
Transforming Functions and Graphs

In these exercises, we will practice using the rules for transforming a function in all of the various ways that is done. We will use a verbal description of a transformation of a given function to create a representation of a new function in various ways. We will also compare two functions where one has been transformed from the other, and identify how it has been transformed.

1. First on the left we have the graph of f, where $f(x) = x^3 - 4x$. Then there are three transformations of f.



For *each* of the functions g, h, and k, do the following:

   (a) Verbally, how has the graph been changed from f's graph?

   (b) Write a formula for the function.

2. For *each* of the functions below, make a graph of this function on separate paper (maybe graph paper if you have some), using your understanding of the basic function that each function resembles and the transformations that have been made.

   a) $p(x) = -3\,|x-4|$ b) $q(x) = \left(\frac{1}{2}x + 5\right)^2 + 1$ c) $r(x) = 2\sqrt{-2x} - 3$

3. The function f defined by $f(x) = 2x^5 + x^2$ had its graph shifted left 3 units and scaled vertically by a factor of 7. Write a formula for the resulting function.

4. The function g defined by $g(x) = \frac{2^x}{x}$ had its graph compressed vertically by a factor of $\frac{1}{3}$, compressed horizontally by a factor of 7, and lastly shifted up by 10 units. Write a formula for the resulting function.

5. The curve $y = k(x)$ was reflected horizontally, shifted right 5 units, shifted up 3 units, compressed vertically by a factor of 2, and lastly scaled horizontally by a factor of 3. Write an equation for the new curve. (The equation will still involve the function "k".)

6. A function f has domain $(-\infty, 3) \cup (3, \infty)$ and range $[4, \infty)$. If g is defined by $g(x) = -3f(2x) + 4$, what is the domain and range of g?

7. The function f has the given table. If the function g is defined by $g(x) = \frac{1}{2}f(-2x) + 1$, make a table for g. Hint: what kinds of transformations have been made to f's graph to get g's graph? Convert the table into something graphical and then use the visual transformations to help.

x	$f(x)$
4	2
5	2
9	6
13	12
14	16